1

Notes

- Assignment 0 marks should be ready by tonight (hand back in class on Monday)

2

Review

- **Motion Curve**: A curve describing how an animation parameter changes in time (or abstract parameter u)
- **Retiming Curve**: An extra curve which indicates what u values to evaluate at for each frame time. Use to change how fast a motion takes place without changing the motion trajectory.
- **Arclength Parameterization**: For more intuitive retiming of an object position, user's timing curve specifies arclength s, not abstract parameter u.

3

Calculating Arc Length

- Recall definition of arc length:

$$s(u) = \int_0^u \left| \frac{d\vec{x}}{du} \right| du$$

  - Where x(u) is the 3D position of the curve at parameter value u
    - Really three curves: X(u), Y(u), Z(u)
- Approximate by splitting curve up into linear segments (sample u finely), add up lengths of those segments
- Build a table of approximate u,s(u) pairs

4

Computing Inverse Map

- From these pairs of u,s(u) values, how to calculate u(s)?
- Treat the s values as knots, the corresponding u values as control points
- Pass a Catmull-Rom spline through it
- Evaluate spline for any value of s you want (from the retiming curve)

5

Kinematics

6

Kinematics

- The study of how things move
- Usually boils down to describing the motion of articulated rigid figures
  - Things made up of rigid "links" attached to each other at movable joints (articulation)
- There are many mathematical approaches to this description
- Text: sections 2.2 and 4.2
  (and more advanced: 6.1 and 6.2)

7

Links

- The actual geometry of each link is irrelevant for kinematics
  - But typically corresponds to a bone, or a solid piece of metal, or …
- All that matters is that they have an attached coordinate system
  - That is an "origin" -- the base point for doing measurements relative to the link -- and a set of basis vectors -- three orthogonal directions relative to the link

8

Joints

- A joint connects two links
  - Attachment point has to be specified in both coordinate systems
  - Often simplified by making sure attachment is the origin of one of the links
  - Also specify how basis vectors of the two are related (rotation)
- A joint can have up to six degrees of freedom (DOF) - three translations, three rotations
- For animation, usually just concerned with rotations (for robotics, maybe not): revolute
  - That is, how are the basis vectors of the second link rotated from the basis vectors of the first?

9

Reality Check

- This boils down to how we model real joints in the body
- E.g. from a distance:
  - Elbow has 1 DOF: the angle the forearm makes with the upper arm (rotation in plane)
  - Wrist has 3 DOF
  - ...
- Up close, life can be much more complex: no simple attachment

10

Joint Decomposition

- Often write multiple-DOF joints as a sequence of 1-DOF joints connecting imaginary links between them
- D-H notation
  - For each joint-link sequence, specify displacement and angles in standard format

11

Hierarchical Skeletons

- Usually a character (or a robot) has no loops --- link/joint graph is a tree
- Pick an arbitrary link to be the root
- Settle on its coordinate system with respect to the world
  - 6 DOF: position + orientation (more on this later)
- For attached links, get translation and rotation from root's coordinate system to neighbours' coordinate systems
- Go through the tree to the tips ("end-effectors") concatenating transformations
- This is "Forward Kinematics" (FK)