

The initial control polygon is  $b_0, \dots, b_m$ , we want to find a new set of points  $b'_0, \dots, b'_{m+1}$  so that they describe the same Bezier curve.

To do that, let's express  $B_i^m(t)$  via  $B_i^{m+1}$ :

$$B_i^m(t) = (1-t)B_i^m(t) + tB_i^m(t) = \frac{m+1-i}{m+1}B_i^{m+1}(t) + \frac{i+1}{m+1}B_{i+1}^{m+1}(t). \quad (1)$$

Then we can rewrite the equation of our curve using the Bernstein polynomials of higher order:

$$F(t) = \sum_{i=0}^m B_i^m(t) \cdot \mathbf{b}_i = \sum_{i=0}^m \frac{m+1-i}{m+1} B_i^{m+1}(t) \cdot \mathbf{b}_i + \sum_{i=0}^m \frac{i+1}{m+1} B_{i+1}^{m+1}(t) \cdot \mathbf{b}_i = \quad (2)$$

$$= \sum_{i=0}^{m+1} \left( \frac{i}{m+1} \mathbf{b}_{i-1} + \left(1 - \frac{i}{m+1}\right) \mathbf{b}_i \right) B_i^{m+1}(t) = \sum_{i=0}^{m+1} \mathbf{b}'_i B_i^{m+1}(t) \quad (3)$$

Thus we have found an expression of the same curve via  $m+1$  control points, so we have elevated the degree.