Bezier curve is defined by the following equation:

$$F(t) = \sum_{i=0}^{m} B_i^m(t) \cdot \mathbf{b_i} \tag{1}$$

We want to find its derivative:

$$F'(t) = \sum_{i=0}^{m} B_i'^m(t) \cdot \mathbf{b_i} \tag{2}$$

Let's start first by finding the derivative of a Bernstein polynomial (I omit (t) for clarity).

$$B_i^m = \binom{m}{i} t^i (1-t)^{m-i} \tag{3}$$

$$B_i'^m = \binom{m}{i} [it^{i-1}(1-t)^{m-i} - (m-i)t^i(1-t)^{m-i-1}] = \tag{4}$$

$$= \frac{m!}{i!(m-i)!} \cdot it^{i-1}(1-t)^{m-i} - \frac{m!}{i!(m-i)!} \cdot (m-i)t^i(1-t)^{m-i-1} = \tag{5}$$

$$= m\frac{(m-1)!}{(i-1)!(m-i)!}t^{i-1}(1-t)^{m-i} - m\frac{(m-1)!}{i!(m-1-i)!}t^i(1-t)^{m-i-1} = \tag{6}$$

$$mB_{i-1}^{m-1} - mB_i^{m-1}. \tag{7}$$

This is true for $i \geq 1$, and derivative of B_0^m is just $-mB_0^{m-1}$. Therefore, we have

$$B_i'^m = m(B_{i-1}^{m-1} - B_i^{m-1}), i \geq 1. \tag{8}$$

Then our initial sum (2) can be rewritten in the following way:

$$F' = -mB_0^{m-1}\mathbf{b_0} + m\sum_{i=1}^{m}(B_{i-1}^{m-1} - B_i^{m-1}) \cdot \mathbf{b_i} = m[B_0^{m-1}b_1 - B_1^{m-1}b_1 + B_1^{m-1}b_2 - B_2^{m-1}b_2 + \ldots] = \tag{9}$$

$$= m \cdot \sum_{i=0}^{m-1} B_i^{m-1}[\mathbf{b_{i+1}} - \mathbf{b_i}]. \tag{10}$$

This concludes the proof.