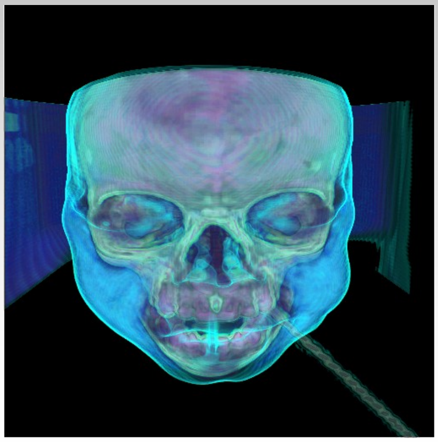
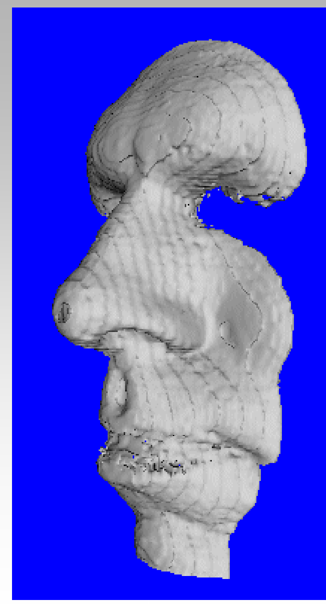
Grids & Marching Cubes





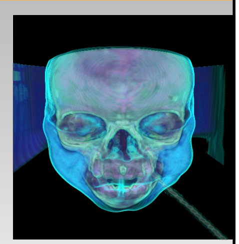
Grid Implicits

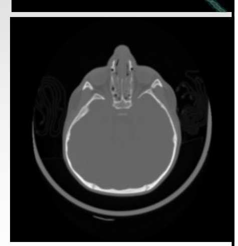
Voxel grid with values at vertices

- Defines implicit function in 3D
  - *interpolate grid values*

Shape defined by isosurface

- isosurface = set of points with constant isovalue α
- separates values above α from values below





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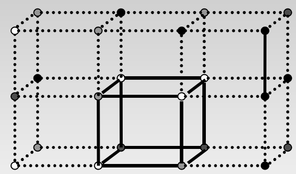
Voxels

Voxel – cube with values at eight corners

- Each value is above or below isovalue α

Each voxel is either:

- Entirely inside isosurface
- Entirely outside isosurface
- Intersected by isosurface



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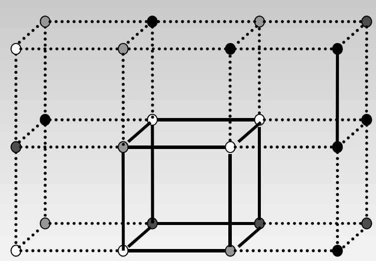
Reconstruction

Extract triangulation approximating isosurface

MC main observation: Can extract triangulation independently per voxel



2^8=256 possible configurations (per voxel)

- reduced to 15 (symmetry and rotations)

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Basic MC Algorithm

For each voxel produce set of triangles

- Based on above/below corner configuration
- Empty for non-intersecting voxels
- Approximate surface inside voxel

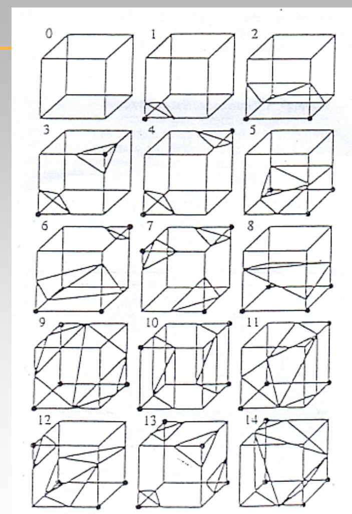

Figure 2. Configurations.

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Configurations

For each configuration add 1-4 triangles to isosurface

Isosurface vertices computed by:

- Interpolation along edges (according to grid values)
  - *better shading, smoother surfaces*
- Default – mid-edges

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Example

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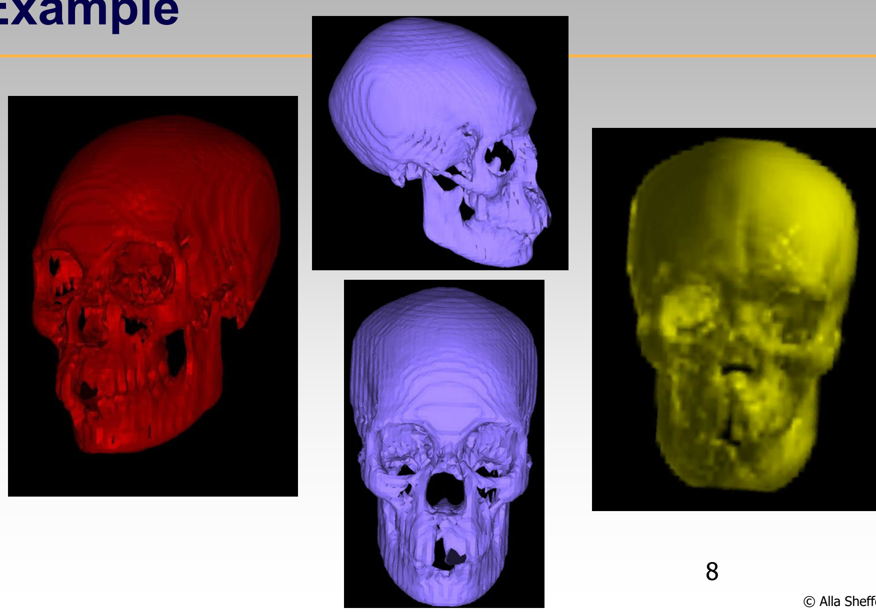
Example



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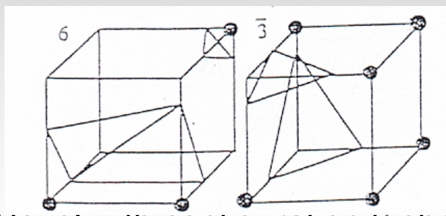
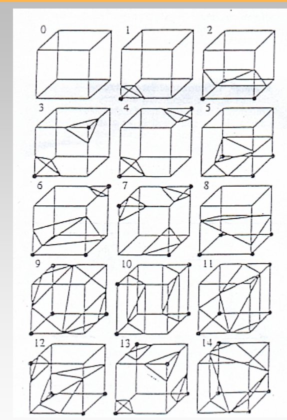
Consistency Problem

Can produce non-manifold results

- Isovalue surfaces with "holes"

Example:





- Voxel with configuration 6 sharing face with complement of configuration 3

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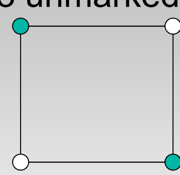
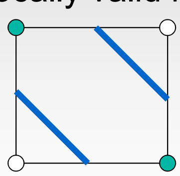
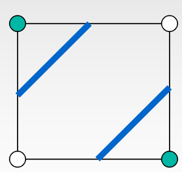
Ambiguous Faces

- Face containing two diagonally opposite marked grid points and two unmarked ones



- Two locally valid interpretations

- Source of MC consistency problem

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Consistency

Problem:

- Connection of isosurface points on shared face done one way on one face & another way on the other

Need consistency → use different triangulations

If choices are consistent get topologically correct surface

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Solution

For each problematic configuration have more than one triangulation

Distinguish different cases by choosing pairwise connections of four vertices on common face

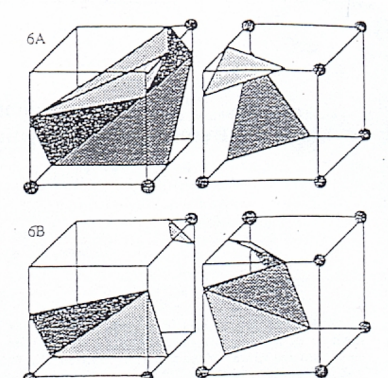

Figure 4. Two possible triangulations which yield a topologically correct isovalue surface.

2.0 Asymptotic Decider

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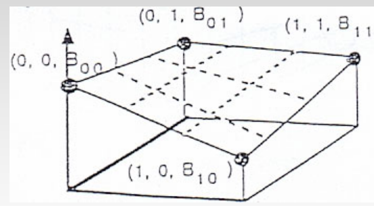
Asymptotic Decider

Select connectivity that better fits implicit function

Use bilinear interpolation to approximate function

- 2D extension of interpolation



$$B(s,t) = \begin{pmatrix} 1-s & s \end{pmatrix} \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \begin{pmatrix} 1-t \\ t \end{pmatrix}$$

$$\{(s,t) : 0 \le s \le 1, \quad 0 \le t \le 1\}$$

- B_{ij} - isovalues at face corners

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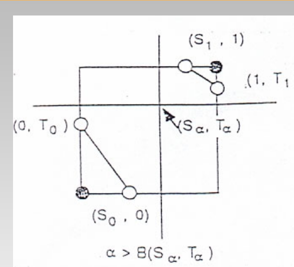
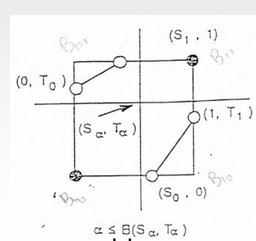
Asymptotic Decider

E.g. B_{00} & B_{11} above α

Test value at face "center" (S_α, T_α)

- If $\alpha > B(S_\alpha, T_\alpha)$
  - connect $(S_1,1)$-$(1,T_1)$ & $(S_0,0)$-$(0,T_0)$
- else
  - connect $(S_1,1)$-$(0,T_0)$ & $(S_0,0)$-$(1,T_1)$





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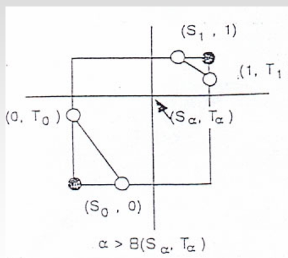
Asymptotic Decider

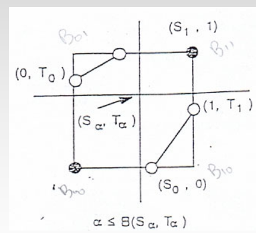
Choice of "center":

$$S_\alpha = \frac{B_{00} - B_{01}}{B_{00} + B_{11} - B_{01} - B_{10}}$$

$$B(S_\alpha, T_\alpha) = \frac{B_{00} B_{11} + B_{10} B_{01}}{B_{00} + B_{11} - B_{01} - B_{10}}.$$

$$T_\alpha = \frac{B_{00} - B_{10}}{B_{00} + B_{11} - B_{01} - B_{10}}$$





- Related to contour curves asymptotic behaviour

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Various Cases

Some configurations have no ambiguous faces → no modifications

Other configurations need modifications according to number of ambiguous faces

- Apply decoder to each face to decide on triangulation template

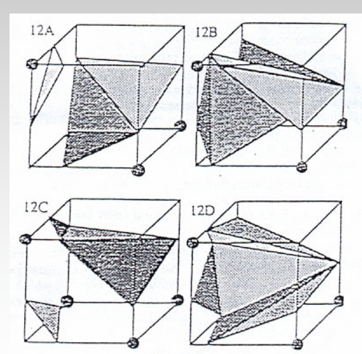

Figure 11.

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Remarks

Add considerable complexity to MC

No significant impact on running time or total number of triangles produced

New configurations occur in real data sets

- But not very often

Config.	Example 1	Example 2	Example 3
0	263,519	285,074	110,993
1	7,705	1,912	1,673
2	8,710	2,065	2,421
3A	60	0	6
3B	46	0	6
4	28	0	0
5	5,611	1,228	1,143
6A	20	0	0
6B	47	0	0
7A	3	0	0
7B,D	3	0	0
7C	3	0	0
8	4,637	906	1,146
9	1,003	304	261
10A,C	13	0	0
10B,D	1	0	0
11	36	0	0
12A,C	7	0	0
12B,D	4	0	0
13	0	0	0
14	69	0	0

Table 1. Frequency of configurations