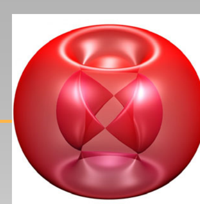
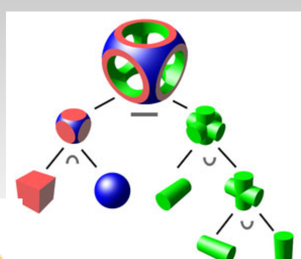
CPSC 424
Implicits

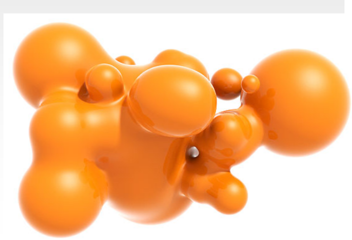
(deck derived from CMU 15-462/662/Keenan Crane)

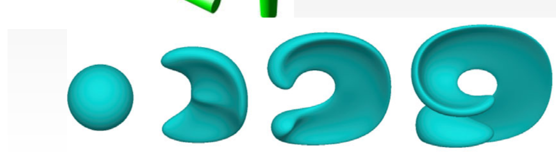
Implicit Representations



- algebraic surfaces
- constructive solid geometry
- level set methods
- blobby surfaces
- fractals
- ...



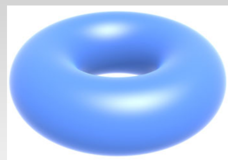




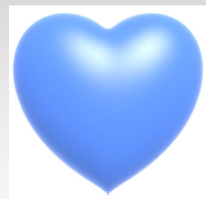
Algebraic Surfaces (Implicit)

- Surface is zero set of a polynomial in x, y, z
- Examples:

$$x^2+y^2+z^2=1$$



$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$



$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

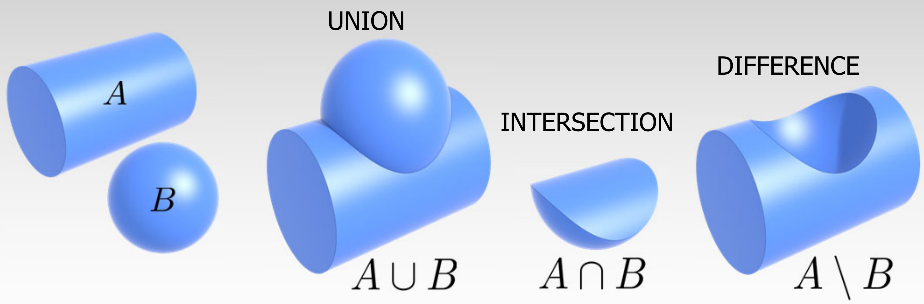
Algebraic Surfaces (Implicit)

- What about more complicated shapes?

- Very hard to come up with polynomials!

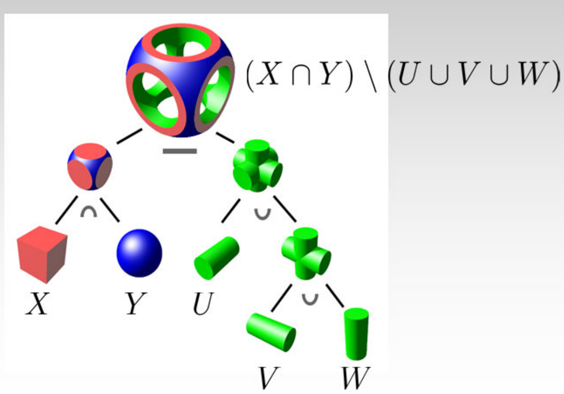
Constructive Solid Geometry (Implicit)

- Build more complicated shapes via Boolean operations
- Basic operations:



Constructive Solid Geometry (Implicit)

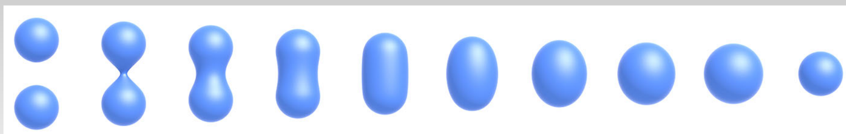
- Then chain together expressions:



https://iquilezles.org/articles/distfunctions/

Blobby Surfaces (Implicit)

- Instead of Booleans, gradually blend surfaces together:

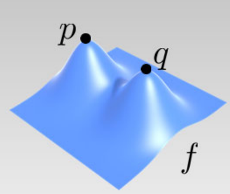


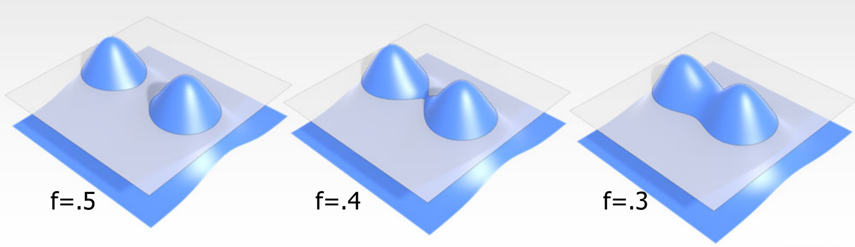
$\phi_p(x) := e^{-|x-p|^2}$ (Gaussian centered at p)

$f := \phi_p + \phi_q$ (Sum of Gaussians centered at different points)

Blobby Surfaces (Implicit)

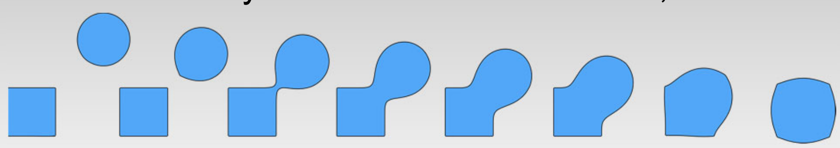
- Easier to understand in 2D:





Blending Distance Functions (Implicit)

- A distance function gives distance to closest point on object
- Can blend any two distance functions d1, d2:



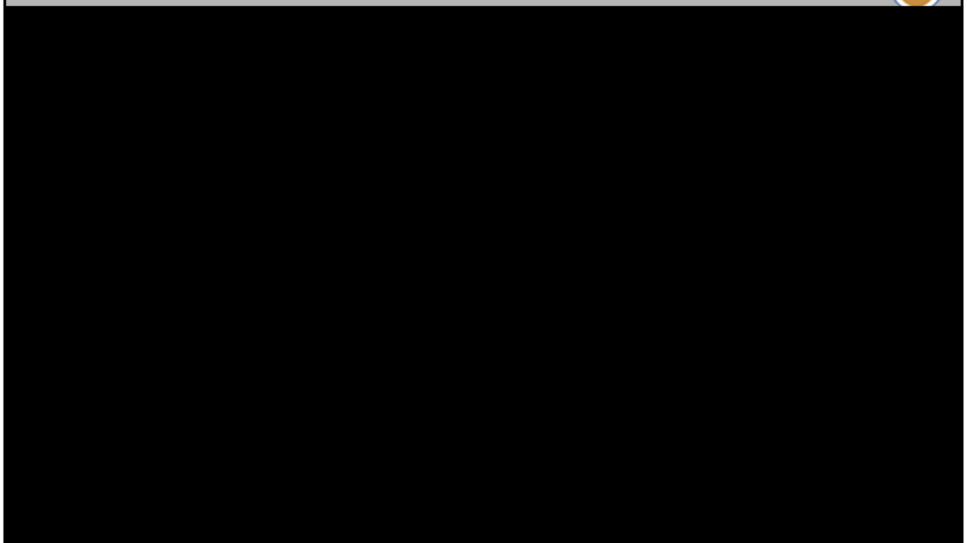
- Similar strategy to points, though many possibilities. E.g.,

$$f(x) := e^{-d_1(x)^2} + e^{-d_2(x)^2} - \frac{1}{2}$$

Blending Distance Functions (Implicit)

- Apearance depends on how we combine functions
- Q: How do we implement a Boolean union of $d_1(x)$, $d_2(x)$?
- A: Just take the minimum: $f(x) = min(d_1(x), d_2(x))$

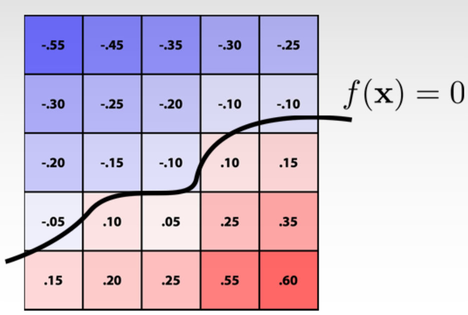
Scene of pure distance functions



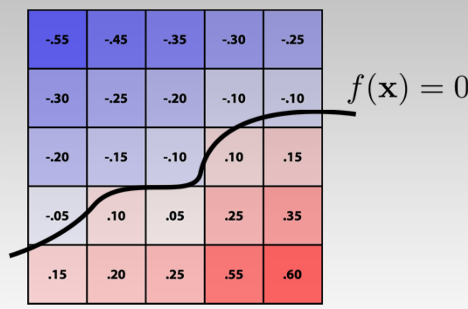
see
http://iquilezles.org/

Level Set Methods (Implicit)

- Implicit surfaces have some nice features (e.g., merging/splitting)
- But, hard to describe complex shapes in closed form
- Alternative: store a grid of values approximating function



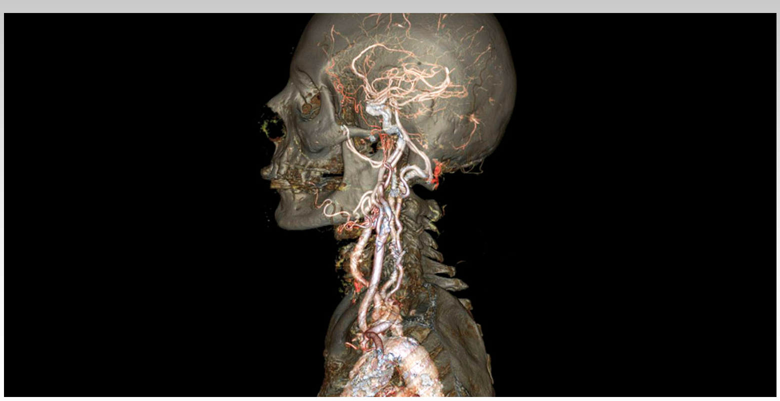
Level Set Methods (Implicit)



- Surface is found where interpolated values equal zero
- Provides much more explicit control over shape (like a texture)
- Unlike closed-form expressions, run into problems of aliasing!

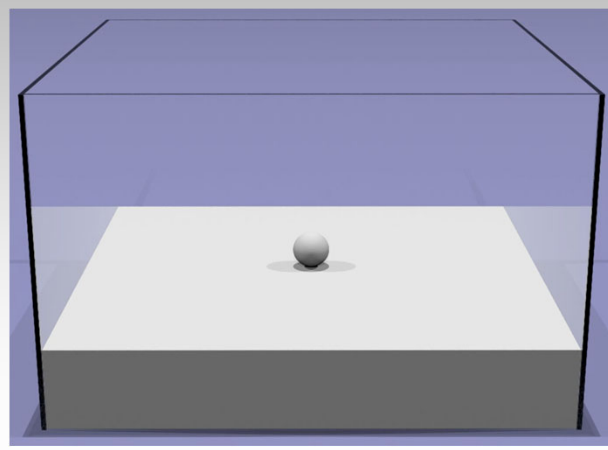
Level Sets from Medical Data (CT, MRI, etc.)

- Level sets encode, e.g., constant tissue density



Level Sets in Physical Simulation

Level set encodes distance to air-liquid boundary:



see http://physbam.stanford.edu