CPSC 424
Simplification

Syllabus

Curves in 2D and 3D

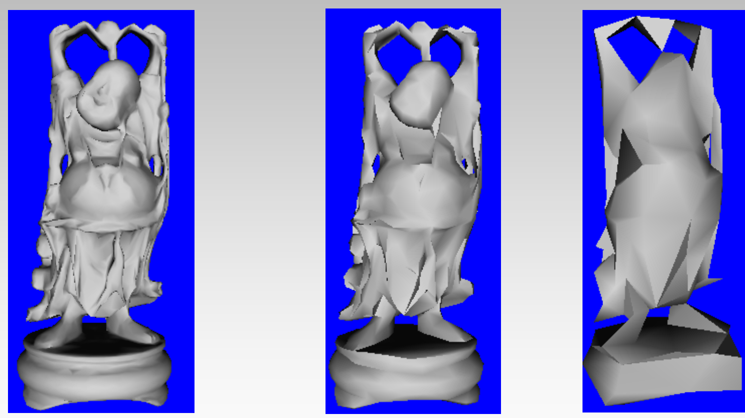
Properties of Curves and Surfaces

Surfaces

Polygonal meshes

- Definitions & Data Structures
- Subdivision (Loop, Butterfly, …)
- **Simplification**
- Acquisition
- Smoothing

Mesh Simplification



12,000 2,000 300

Simplifi
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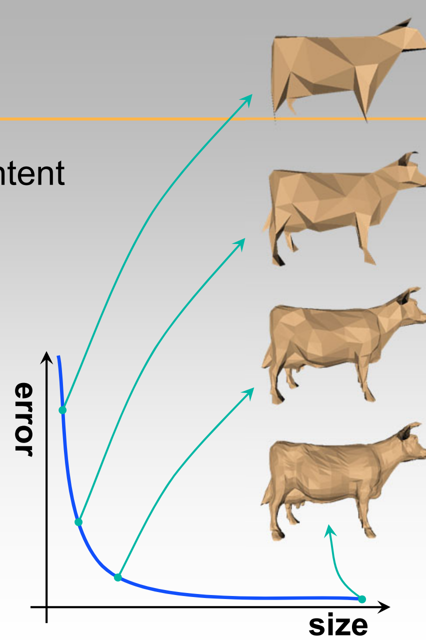
Motivation

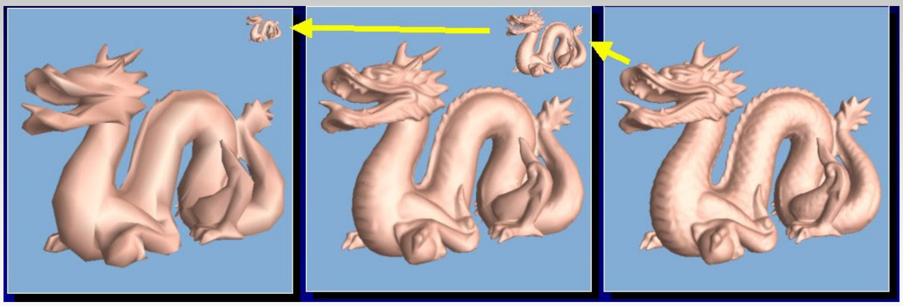
- Reduce information content
- Accelerate rendering
- Multi-resolution models



Level of Detail (LOD)

Refined mesh for close objects

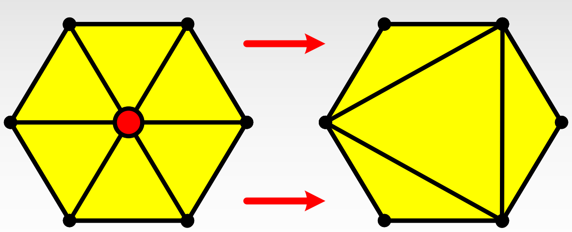
Simplified mesh for far



Methodology

Sequence of local operations

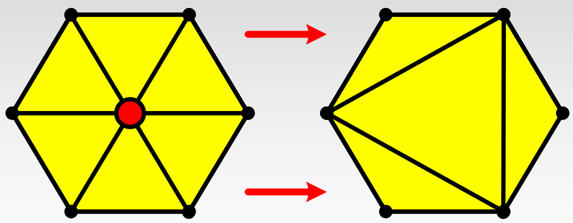
- Involve near neighbors - only small *patch* affected in each operation
- Each operation introduces error
- Find and apply operation which introduces the least error



Simplification Operations (1)

Decimation

- Vertex removal:
  - $v \leftarrow v-1$
  - $f \leftarrow f-2$



Remaining vertices - subset of original vertex set

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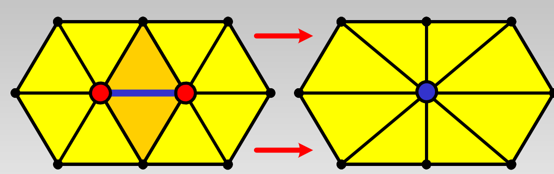
Simplification Operations (2)

Decimation

- Edge collapse
  - $v \leftarrow v-1$
  - $f \leftarrow f-2$



Vertices may move

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Error Control

Local error: Compare new patch with previous iteration

- Fast
- Accumulates error
- Memory-less

Global error: Compare new patch with original mesh

- Slow
- Better quality control
- Can be used as termination condition
- Must remember the original mesh throughout the algorithm

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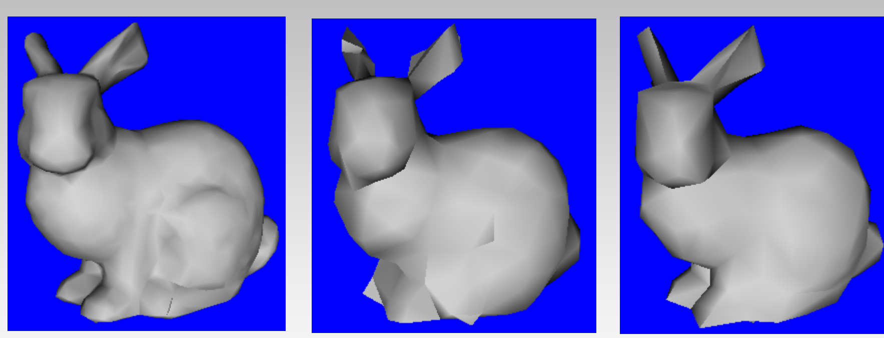
Local vs. Global Error



2000 faces **488 faces** **488 faces**

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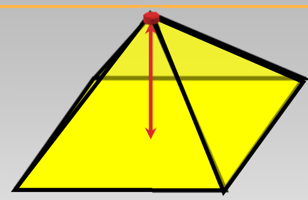
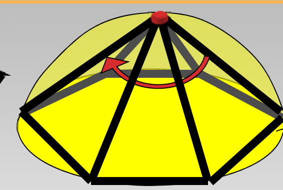
Simplification Error Metrics



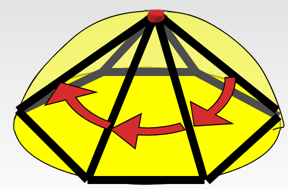


Measures

- Distance to plane
- Curvature

Usually approximated

- Average plane
- Discrete curvature



$\Sigma\alpha / 2\pi$

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The Basic Algorithm

Repeat

- Select the element with minimal error
- Perform simplification operation (remove/contract)
- Update error (local/global)

Until mesh size / quality is achieved

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Triangulating the Hole

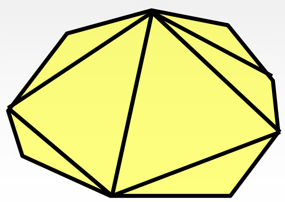
Vertex removal produces non-planar loop

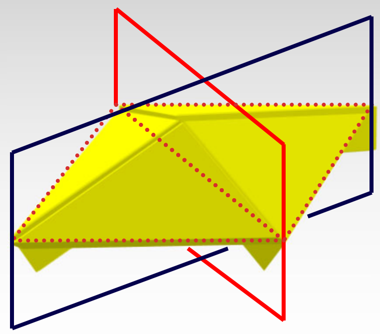
- Split loop recursively
- Split plane orthogonal to the average plane

Control aspect ratio

Triangulation may fail

- Vertex is not removed





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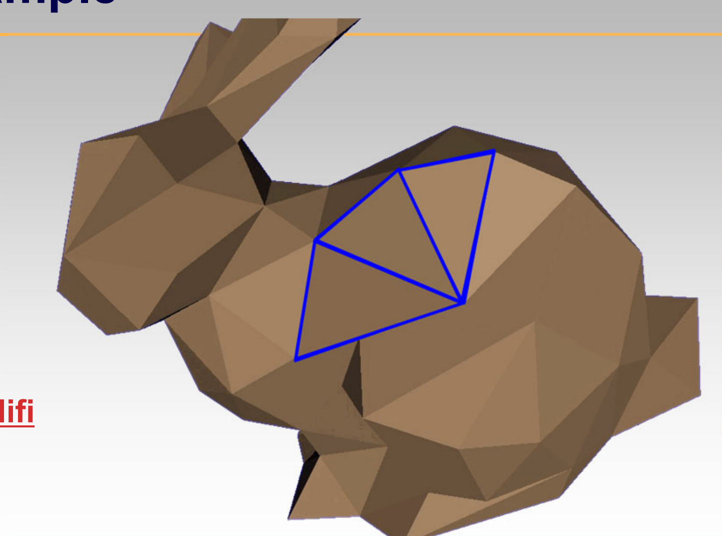
Example



Simplifier

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Pros and Cons

Pros:

- Efficient
- Simple to implement and use
  - *Few input parameters to control quality*
- Reasonable approximation
- Works on very large meshes
- Preserves topology
- Vertices are a subset of the original mesh

Cons:

- Error is not bounded
  - *Local error evaluation causes error to accumulate*

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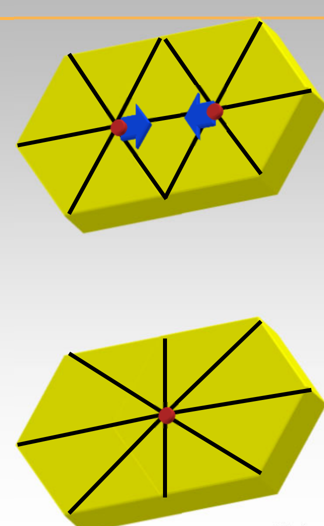
Edge Collapse Algorithm

Simplification operation:
Edge collapse (pair contraction)

Error metric:
distance, pseudo-global

Also simplifies topology



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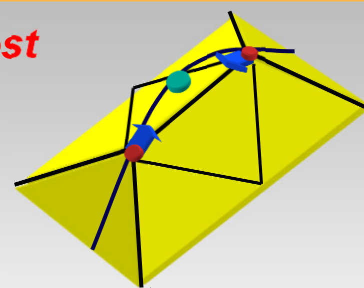
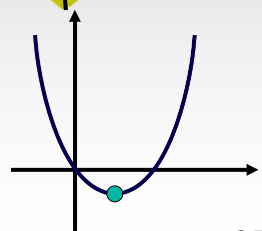
Distance Metric: Quadrics

Choose point closest to set of planes (triangles)



Sum of squared distances to set of planes is quadratic ⇒ has a minimum



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Quadrics

Plane

- $Ax + By + Cz + D = 0$, where $A^2 + B^2 + C^2 = 1$
- $p = [A, B, C, D]$, $v = [x, y, z, 1]$, $v\,p^T = 0$

Quadratic distance between v and p:

$$\Delta_p(v) = (v\,p^T)^2$$

$$= (v\,p^T)\,(p\,v^T) = v\,(p^T p)\,v^T$$

$$= v\,K_P\,v^T$$

$$K_P = \begin{bmatrix} A^2 & AB & AC & AD \\ AB & B^2 & BC & BD \\ AC & BC & C^2 & CD \\ AD & BD & CD & D^2 \end{bmatrix}$$

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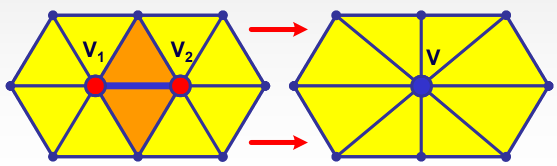
Distance to Set of Planes

$$\Delta(v) = \sum_{p \in planes(v)} \Delta_p(v) = \sum_{p \in planes(v)} (v K_p v^T) = v \left(\sum_{p \in planes(v)} K_p \right) v^T = v Q_v v^T$$

After v_1, v_2 are contracted to v,
$Q_v \leftarrow Q_{v1} + Q_{v2}$

Pseudo-global

All original planes persist during the entire simplification process



Contracting Two Vertices

- Goal: Given edge e = (v_1,v_2), find contracted v = (x,y,z,1) that minimizes $\Delta(v)$:
- $\partial\Delta/\partial x = \partial\Delta/\partial y = \partial\Delta/\partial z = 0$
- Solve system of linear normal equations:

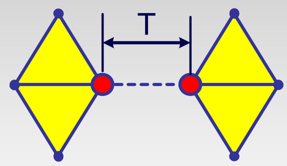
$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

- If no solution - select the edge midpoint

Selecting Valid Pairs for Contraction

Edges:
$\{(v_1,v_2) : (v_1 v_2)$ is in the mesh$\}$

Close vertices:
$\{(v_1,v_2) : ||v_1 - v_2|| < T\}$



- Threshold T is input parameter

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Algorithm

- Compute Q_V for all the mesh vertices
- Identify all valid pairs
- Compute for each valid pair (v_1,v_2) the contracted vertex v and its error $\Delta(v)$
- Store all valid pairs in a priority queue (according to $\Delta(v)$)
- While reduction goal not met
  - Contract edge (v_1,v_2) with the smallest error to *v*
  - Update the priority queue with new valid pairs

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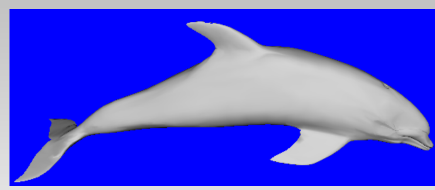
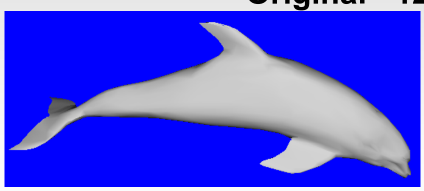
Examples

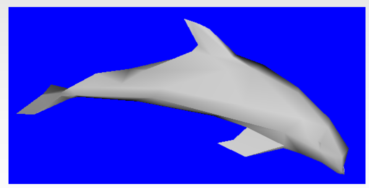
Dolphin (Flipper)



Original - 12,337 faces

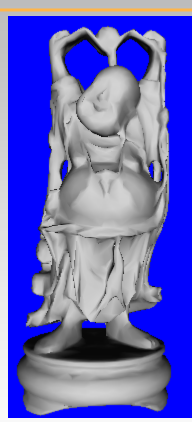


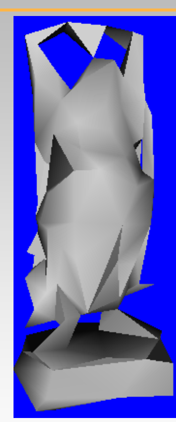
2,000 faces



300 faces (142 vertices)

Examples





Original - 12,000

2,000 faces

298 faces (140 vertices)

Simplifier

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Pros and Cons

Pros

- Error is bounded
- Allows topology simplification
- High quality result
- Quite efficient

Cons

- Difficulties along boundaries
- Difficulties with coplanar planes
- Introduces new vertices not present in the original mesh

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