CPSC 424
Meshes

Syllabus

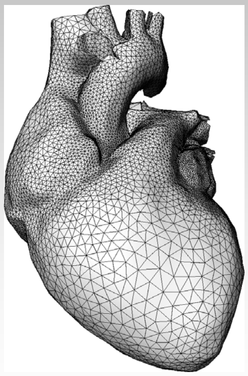
Curves in 2D and 3D

Properties of Curves and Surfaces

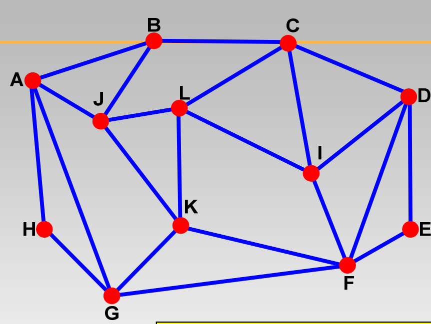
Surfaces

- Tensor-product surfaces
- Bézier triangles
- Differential Geometry
- **Polygonal meshes**
- Subdivision Surfaces

Meshes



Standard Graph Definitions



G = <**V**,**E**>
V = vertices =
{A,B,C,D,E,F,G,H,I,J,K,L}
E = edges =
{(A,B),(B,C),(C,D),(D,E),(E,F),(F,G),(G,H),(H,A),(A,J),(A,G),(B,J),(K,F),(C,L),(C,I),(D,I),(D,F),(F,I),(G,K),(J,L),(J,K),(K,L),(L,I)}

Vertex degree (valence) = number of edges incident on vertex
deg(J) = 4, deg(H) = 2

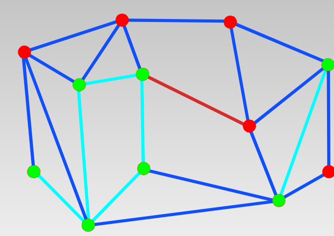
Face: cycle of vertices/edges which cannot be shortened
F = faces =
{(A,H,G),(A,J,K,G),(B,A,J),(B,C,L,J),(C,I,L),(C,D,I),(D,E,F),(D,I,F),(L,I,F,K),(L,J,K),(K,F,G)}

Connectivity

Graph is ***connected*** if there is a path of edges connecting every two vertices

Graph **G'**=<**V'**,**E'**> is a ***subgraph*** of graph **G**=<**V**,**E**> if **V'** is a subset of **V** and **E'** is the subset of **E** incident on **V'**

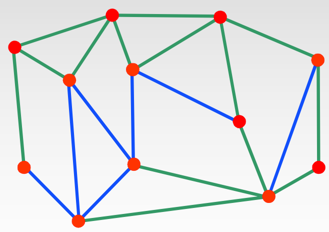
Connected component of a graph: maximal connected subgraph



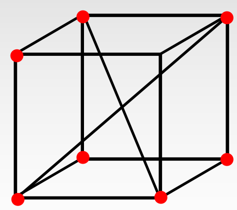
How do we find if a graph is connected?

Graph Embedding

Graph is ***embedded*** in R^d if each vertex is assigned a position in R^d and the edges are represented by straight lines

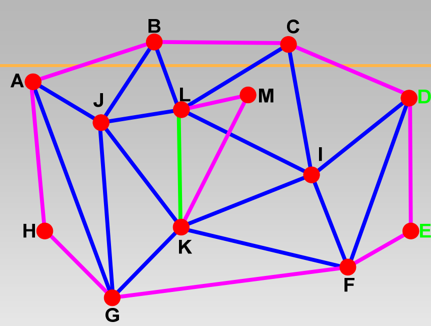


Embedding in R^2



Embedding in R^3

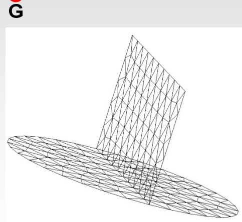
Meshes



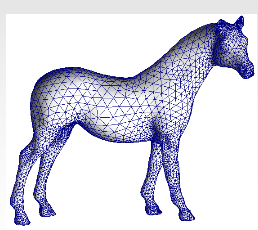
Mesh: graph embedded in R^3

Boundary edge: adjacent to exactly *one* face
Regular edge: adjacent to exactly *two* faces
Singular edge: adjacent to more than two faces

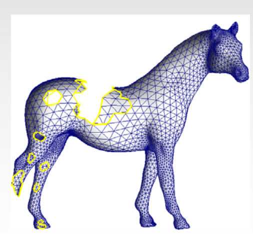
Closed mesh: mesh with no boundary edges
Manifold mesh: mesh with no singular edges



Non-Manifold



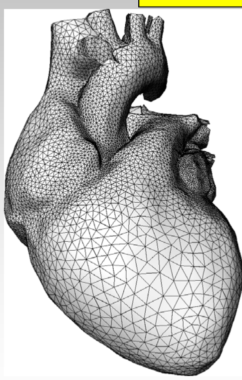
Closed Manifold



Open Manifold

Meshes

Triangular mesh – mesh all of whose faces have three vertices

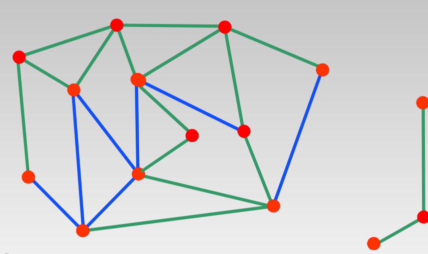


We will discuss connected (3-connected) manifold triangular meshes (closed or open)

Clicker question

Is this graph connected? Is it manifold?

A. Yes & Yes
B. Yes & No
C. No & Yes
D. No & No
E. Not enough information

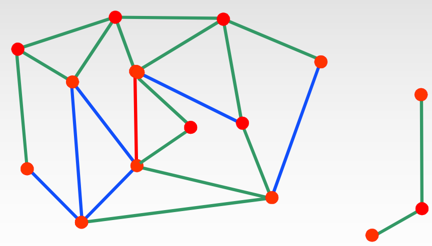


Clicker question

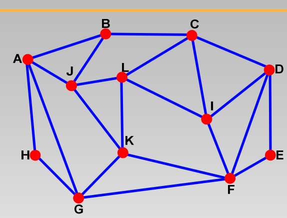
What if I tell you that the 'red' edge is part of 3 faces?

Is this graph connected? Is it manifold?

A. Yes & Yes
B. Yes & No
C. No & Yes
D. No & No
E. Not enough information



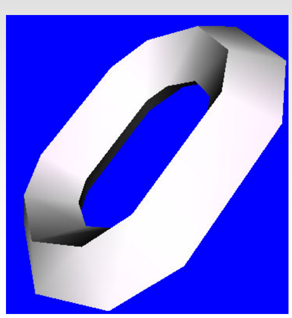
Topology

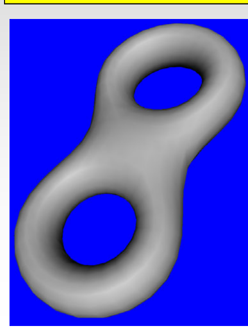


v =12
f = 14
e = 25
c = 1
g = 0
b = 1

Genus of graph: *half* of *maximal* number of closed paths that do *not* disconnect the graph (number of "holes")

Genus(sphere)= 0
Genus(torus) = 1

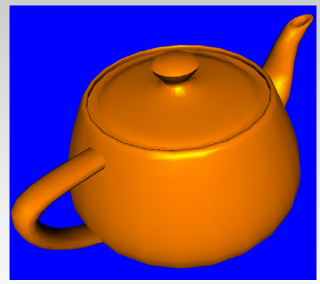




Topology Quiz

What is the genus of a teapot?

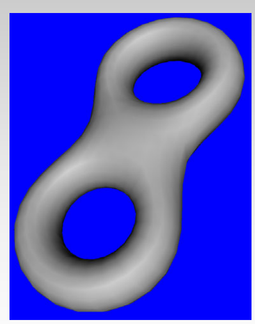
A. 0
B. 1
C. 2
D. 3
E. 4
F. 5



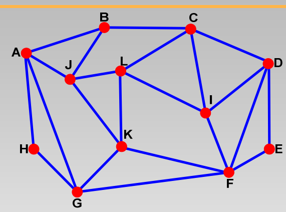
Topology Quiz

What is the genus of this shape ?

A. 0
B. 2
C. 4
D. 6
E. None of the above



Topology



v =12
f = 11
e = 22
c = 1
g = 0
b = 1

Euler-Poincare Formula

v+f-e = 2(c-g)-b

v = # vertices c = # conn. comp
f = # faces g = genus
e = # edges b = # boundaries

How to find number of boundaries?

How to find number of connected components?

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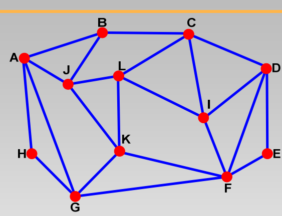
Topology



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Euler-Poincare Formula

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v = # vertices c = # conn. comp
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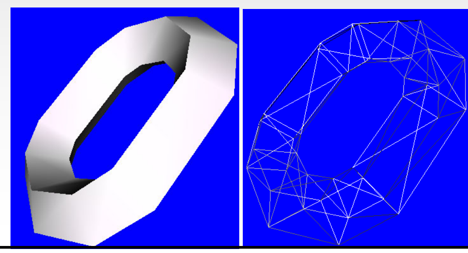
Exercises

Theorem: Average vertex degree in closed manifold triangle mesh is ~6

Proof: In such a mesh, f = 2e/3
By Euler's formula: v+2e/3-e = 2-2g
hence e = 3(v-2+2g) and f = 2(v-2+2g)

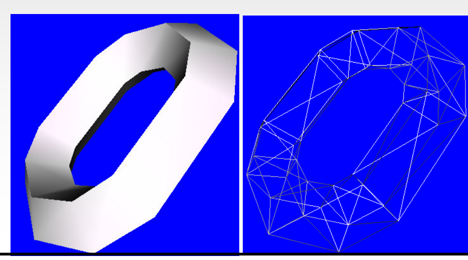
So Average(deg) = 2e/v = 6(v-2+2g)/v
~ 6 for large v



Exercises

Corollary: Only toroidal (g=1) closed manifold triangle mesh can be regular (all vertex degrees are 6)

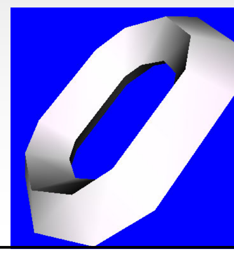
Proof: In regular mesh average degree is *exactly* 6
Can happen only if g=1

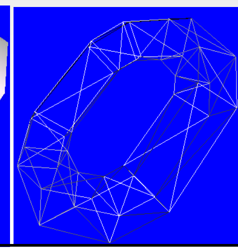


Exercises

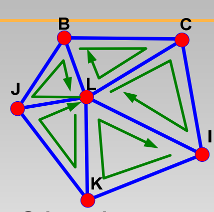
Theorem: In closed manifold mesh:
$2e \geq 3f$ (equality for triangle mesh),
$2e \geq 3v$
Corollary: No closed manifold triangle mesh can have 7 edges

Corollary: $2f-4 \geq v$





Orientability

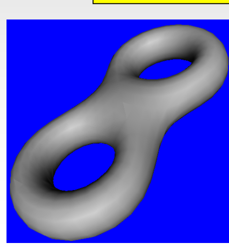


Oriented
F={(L,J,B),(B,C,L),(L,C,I),(I,K,L),(L,K,J)}
Not Oriented
F={(B,J,L),(B,C,L),(L,C,I),(L,I,K),(L,K,J)}

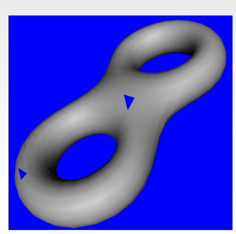
Orientation of a face is clockwise or anticlockwise order in which its vertices and edges are listed

This defines the direction of face ***normal***

Straight line graph is ***orientable*** if orientations of its faces can be chosen so that each edge is oriented in *both* directions

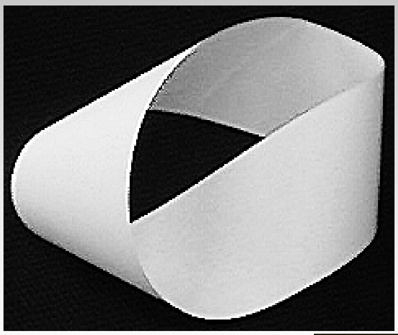


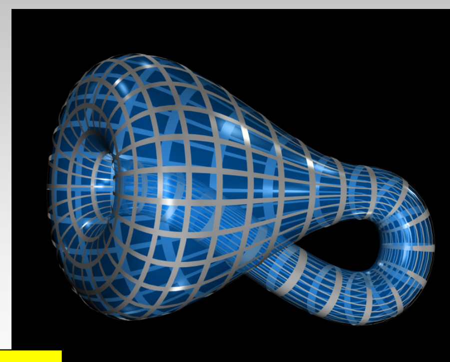
Not Backface Culled



Backface Culled

Moebius & Klein





Moebius strip or
Klein bottle
- not orientable