CPSC 424
Meshes

Syllabus

Curves in 2D and 3D
Properties of Curves and Surfaces
Surfaces
• Tensor-product surfaces
• Bézier triangles
• Differential Geometry
• Polygonal meshes
• Subdivision Surfaces
**Meshes**

![Heart and Sculpture Image]

**Standard Graph Definitions**

\[ G = (V, E) \]

- **V** = vertices = \{A, B, C, D, E, F, G, H, I, J, K, L\}
- **E** = edges = \{(A, B), (B, C), (C, D), (D, E), (E, F), (F, G), (G, H), (H, A), (A, J), (A, G), (B, J), (K, F), (C, L), (C, I), (D, I), (D, F), (F, I), (G, K), (J, L), (J, K), (K, L), (L, I)\}

**Vertex degree (valence)** = number of edges incident on vertex
- \( \text{deg}(J) = 4 \)
- \( \text{deg}(H) = 2 \)

**Face**: cycle of vertices/edges which cannot be shortened
- **F** = faces = \{(A, H, G), (A, J, K, G), (B, A, J), (B, C, L, J), (C, I, L), (C, D, I), (D, E, F), (D, I, F), (L, I, F, K), (L, J, K), (K, F, G)\}
**Connectivity**

Graph is *connected* if there is a path of edges connecting every two vertices.

Graph $G' = \langle V', E' \rangle$ is a *subgraph* of graph $G = \langle V, E \rangle$ if $V'$ is a subset of $V$ and $E'$ is the subset of $E$ incident on $V'$.

*Connected component* of a graph: maximal connected subgraph.

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**Graph Embedding**

Graph is *embedded* in $\mathbb{R}^d$ if each vertex is assigned a position in $\mathbb{R}^d$ and the edges are represented by straight lines.

Embedding in $\mathbb{R}^2$

Embedding in $\mathbb{R}^3$
Meshes

Mesh: graph embedded in $\mathbb{R}^3$

Boundary edge: adjacent to exactly one face
Regular edge: adjacent to exactly two faces
Singular edge: adjacent to more than two faces

Closed mesh: mesh with no boundary edges
Manifold mesh: mesh with no singular edges

Meshes

Triangular mesh – mesh all of whose faces have three vertices

We will discuss connected (3-connected) manifold triangular meshes (closed or open)
Clicker question

Is the graph to the left connected? Is it manifold?

A. Yes & Yes
B. Yes & No
C. No & Yes
D. No & No
E. Not enough information

Topology

Euler-Poincare Formula

\[ v + f - e = 2(c - g) - b \]

- \( v \) = # vertices
- \( c \) = # conn. comp
- \( f \) = # faces
- \( g \) = genus
- \( e \) = # edges
- \( b \) = # boundaries

Genus of graph: half of maximal number of closed paths that do not disconnect the graph (number of "holes")

- Genus(sphere) = 0
- Genus(torus) = 1
**Topology Quiz**

What can you say about the genus of these meshes?

**Exercises**

**Theorem:** Average vertex degree in closed manifold triangle mesh is ~6

**Proof:** In such a mesh, \( f = \frac{2e}{3} \)

By Euler’s formula: \( v + 2e/3 - e = 2 - 2g \)

hence \( e = 3(v-2+2g) \) and \( f = 2(v-2+2g) \)

So \( \text{Average(deg)} = \frac{2e}{v} = \frac{6(v-2+2g)}{v} \)

\(~ 6 \) for large \( v \)

**Corollary:** Only toroidal \((g=1)\) closed manifold triangle mesh can be regular (all vertex degrees are 6)

**Proof:** In regular mesh average degree is exactly 6

Can happen only if \( g=1 \)

**Theorem:** In closed manifold mesh:

\( 2e \geq 3f \) (equality for triangle mesh),

\( 2e \geq 3v \)

**Corollary:** No closed manifold triangle mesh can have 7 edges

**Corollary:** \( 2f-4 \geq v \)
Orientability

Orientability of a face is clockwise or anticlockwise order in which its vertices and edges are listed.

This defines the direction of face normal.

Straight line graph is orientable if orientations of its faces can be chosen so that each edge is oriented in both directions.

Oriented

Not Oriented

Moebius & Klein

Moebius strip or Klein bottle - not orientable