CPSC 424
Meshes

Syllabus

Curves in 2D and 3D
Properties of Curves and Surfaces
Surfaces

- Tensor-product surfaces
- Bézier triangles
- Differential Geometry
- Polygonal meshes
- Subdivision Surfaces
Meshes

Standard Graph Definitions

\[ G = \langle V, E \rangle \]
\[ V = \text{vertices} = \{A,B,C,D,E,F,G,H,I,J,K,L\} \]
\[ E = \text{edges} = \{(A,B),(B,C),(C,D),(D,E),(E,F),(F,G),
(G,H),(H,A),(A,J),(A,G),(B,J),(K,F),
(C,L),(C,I),(D,I),(D,F),(F,I),(G,K),
(J,L),(J,K),(K,L),(L,I)\} \]

**Vertex degree (valence)** = number of edges incident on vertex

\[ \text{deg}(J) = 4, \text{deg}(H) = 2 \]

**Face**: cycle of vertices/edges which cannot be shortened

(C,D,I),
(D,E,F),(D,I,F),(L,I,F,K),(L,J,K),(K,F,G)\} \]
Connectivity

Graph is **connected** if there is a path of edges connecting every two vertices.

Graph \( G' = \langle V', E' \rangle \) is a **subgraph** of graph \( G = \langle V, E \rangle \) if \( V' \) is a subset of \( V \) and \( E' \) is the subset of \( E \) incident on \( V' \).

**Connected component** of a graph: maximal connected subgraph.

How do we find if a graph is connected?
Graph Embedding

Graph is **embedded** in $\mathbb{R}^d$ if each vertex is assigned a position in $\mathbb{R}^d$ and the edges are represented by straight lines.

Embedding in $\mathbb{R}^2$

Embedding in $\mathbb{R}^3$

Meshes

**Mesh**: graph embedded in $\mathbb{R}^3$

**Boundary edge**: adjacent to exactly one face

**Regular edge**: adjacent to exactly two faces

**Singular edge**: adjacent to more than two faces

**Closed mesh**: mesh with no boundary edges

**Manifold mesh**: mesh with no singular edges

Non-Manifold  Closed Manifold  Open Manifold
Mesches

Triangular mesh – mesh all of whose faces have three vertices

We will discuss connected (3-connected) manifold triangular meshes (closed or open)

Clicker question

Is the graph to the left connected? Is it manifold?

A. Yes & Yes
B. Yes & No
C. No & Yes
D. No & No
E. Not enough information
Topology

Genus of graph: half of maximal number of closed paths that do not disconnect the graph (number of "holes")

Genus(sphere) = 0
Genus(torus) = 1

What is the genus of a teapot?

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
Topology Quiz

What is the genus of a teapot?

A. 0  
B. 2  
C. 4  
D. 6  
E. None of the above

Topology

Euler-Poincare Formula

\[ v + f - e = 2(c - g) - b \]

v = # vertices  
c = # conn. comp  
f = # faces  
g = genus  
e = # edges  
b = # boundaries
How to find number of boundaries?

How to find number of connected components?
Topological Invariants

**Euler-Poincare Formula**

\[ v + f - e = 2(c - g) - b \]

- \( v \) = number of vertices
- \( f \) = number of faces
- \( e \) = number of edges
- \( c \) = number of connected components
- \( g \) = genus
- \( b \) = number of boundaries

**Exercises**

**Theorem:** Average vertex degree in closed manifold triangle mesh is \( \sim 6 \)

**Proof:** In such a mesh, \( f = \frac{2e}{3} \)

By Euler's formula: \( v + \frac{2e}{3} - e = 2 - 2g \)  

hence \( e = 3(v - 2 + 2g) \) and \( f = 2(v - 2 + 2g) \)

So Average(degree) = \( \frac{2e}{v} = \frac{6(v - 2 + 2g)}{v} \)  

\( \sim 6 \) for large \( v \)
Exercises

Corollary: Only toroidal (g=1) closed manifold triangle mesh can be regular (all vertex degrees are 6)

Proof: In regular mesh average degree is exactly 6
Can happen only if g=1

Theorem: In closed manifold mesh:
\[ 2e \geq 3f \] (equality for triangle mesh),
\[ 2e \geq 3v \]
Corollary: No closed manifold triangle mesh can have 7 edges
Corollary: \[ 2f-4 \geq v \]
Orientability

Orientation of a face is clockwise or anticlockwise order in which its vertices and edges are listed.

This defines the direction of face normal.

Straight line graph is orientable if orientations of its faces can be chosen so that each edge is oriented in both directions.

Moebius & Klein

Moebius strip or Klein bottle - not orientable.