CPSC 424
Triangles & Differential Surface Properties

Syllabus

Curves in 2D and 3D

Properties of Curves

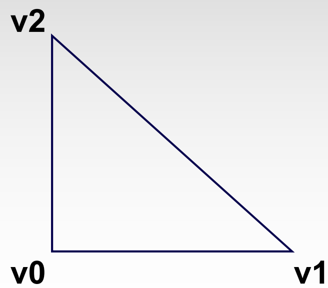
Surfaces

- Sweeping/extrusion, surfaces of revolution,etc..
- Tensor-product surfaces
- **Triangles**
- **Differential Geometry of Surfaces**
- Polygonal meshes, mesh data structures
- Subdivision Surfaces
- Implicits

Triangles

Parameterization

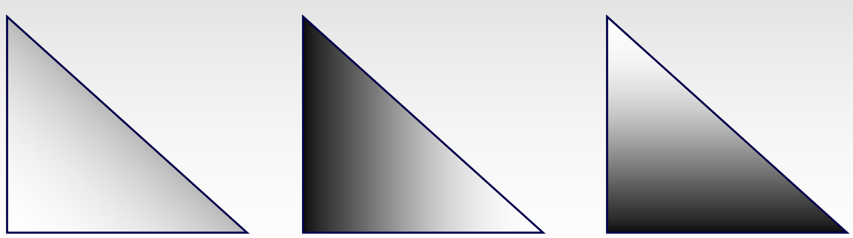
- Searching for
  - *Coordinate system over triangle*
  - *Should be symmetric in all vertices*



Triangles

Barycentric Coordinates:

$$\mathbf{p} = \alpha \mathbf{v}_0 + \beta \mathbf{v}_1 + \gamma \mathbf{v}_2 ; \alpha + \beta + \gamma = 1$$



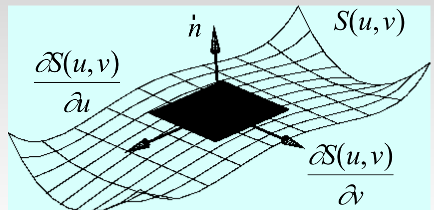
Surfaces – differential geometry

***Tangent plane to surface* S(u,v) *is spanned by two partials of* S:**

$$\frac{\partial S(u,v)}{\partial u} \quad \frac{\partial S(u,v)}{\partial v}$$

Normal *to surface*

$$\vec{n} = \frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}$$



- perpendicular to tangent plane

***Any vector in tangent plane is tangential to* S(u,v)**

Curvature

Normal curvature *of surface is defined for each tangential direction*

Principal curvatures Kmin & Kmax*: maximum and minimum of normal curvature*

- Correspond to two **orthogonal** tangent directions
  - *Principal directions*
- Not necessarily partial derivative directions
- Independent of parameterization

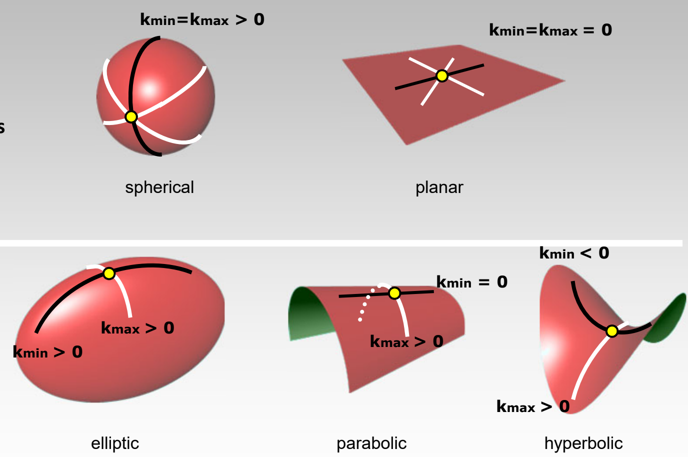
3D Curvature

Isotropic

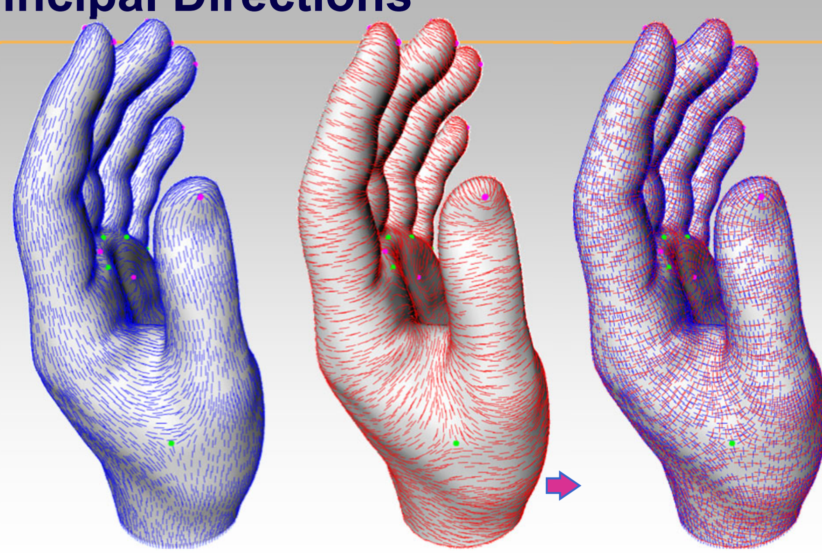
Equal in all directions

Anisotropic

2 distinct principal directions



Principal Directions



min curvature max curvature

Curvature

Typical measures:

- ***Gaussian*** curvature

$$K = k_{\min} k_{\max}$$

- ***Mean*** curvature

$$H = \frac{k_{\min} + k_{\max}}{2}$$

Developable Surfaces

Surfaces with Gaussian curvature K= 0