CPSC 424
Subdivision Curves

Syllabus

Curves in 2D and 3D

- Implicit vs. Explicit vs. Parametric curves
- Bézier curves
- Continuity
- B-Splines
- Rational curves
- **Subdivision curves**

Properties of Curves and Surfaces

- Differential Geometry

Surfaces

Subdivision Curves

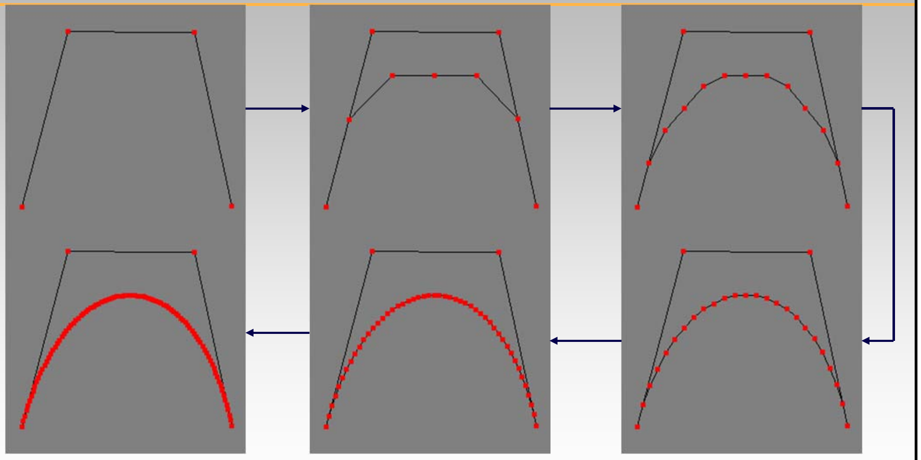
Represent smooth curve by approximating polyline

At the limit = curve

Each iteration

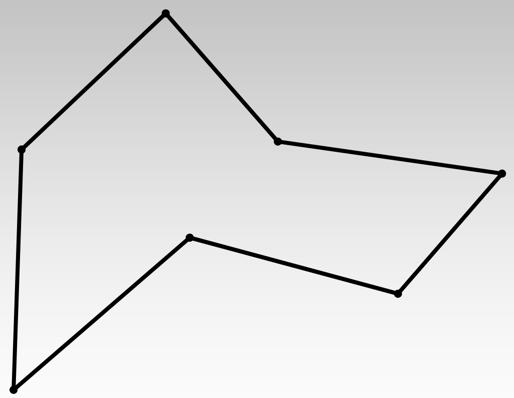
- Add new points (~double)
- Approximating - Move old points
- Interpolating – Keep old points

Bezier Subdivision



To create complex curve need to explicitly enforce continuity – not very useful

Corner Cutting



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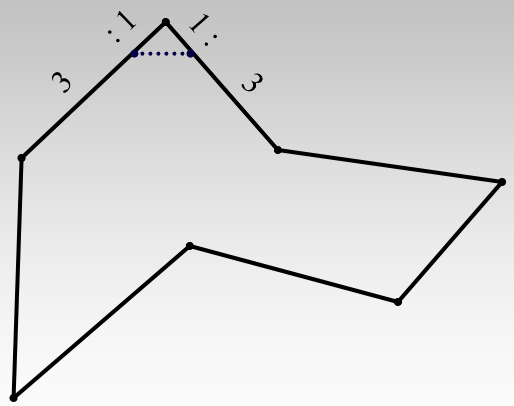

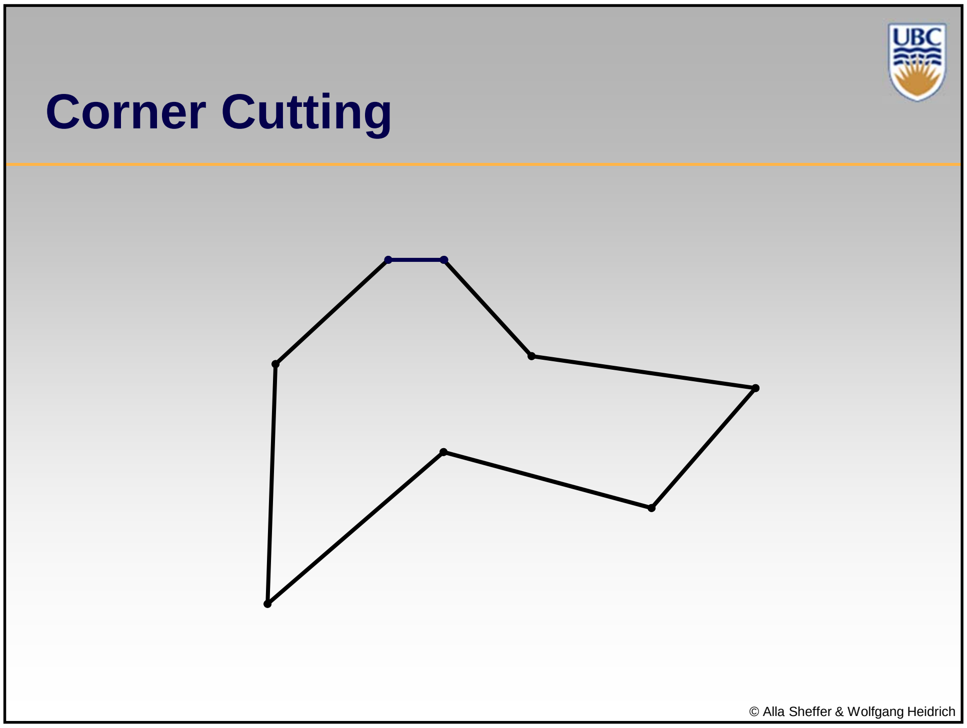

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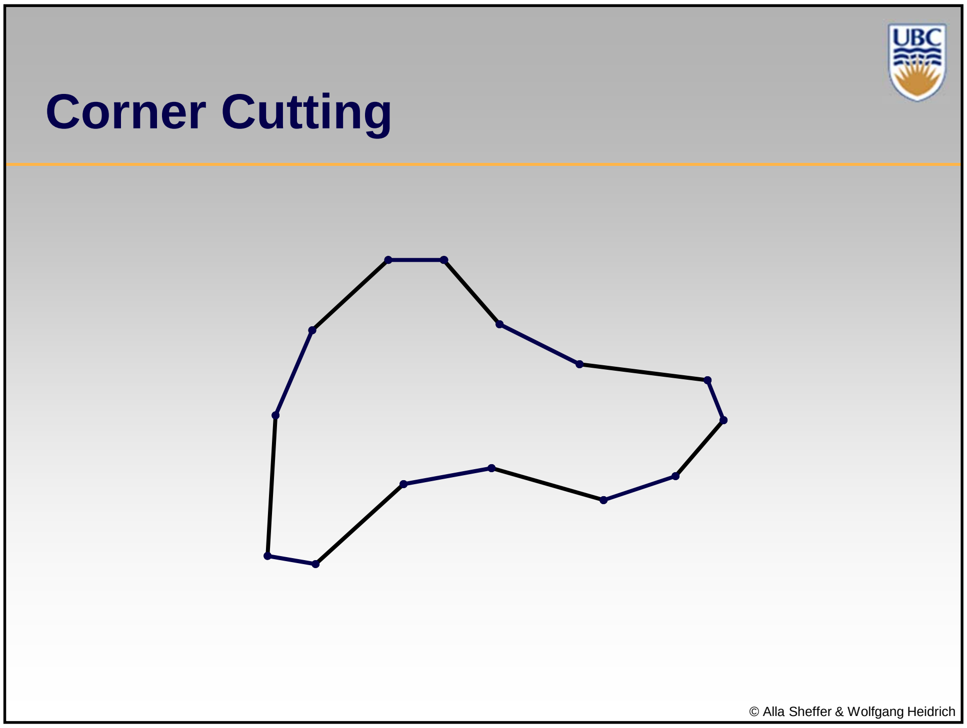

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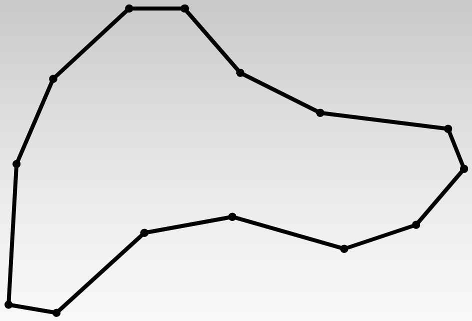

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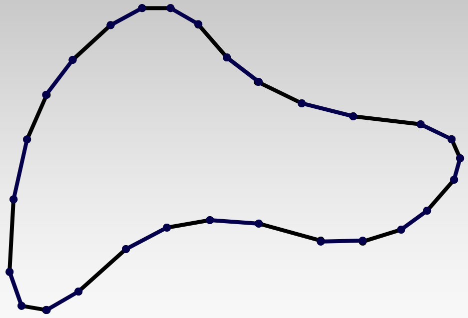

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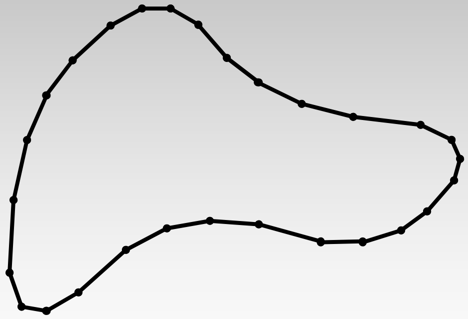

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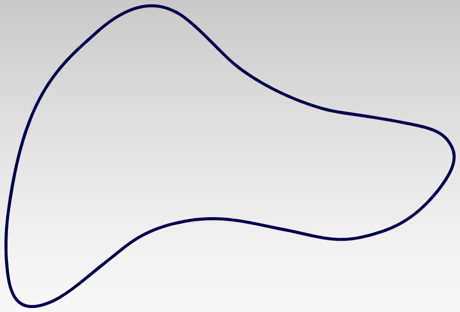
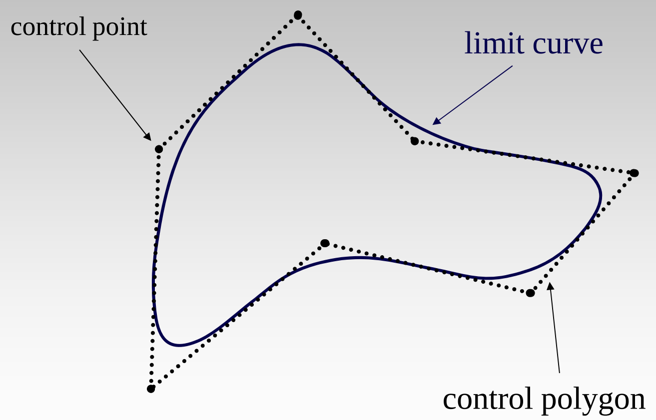
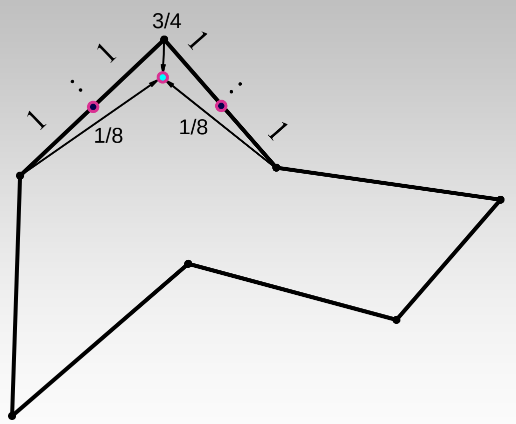


Corner Cutting – Chaikin Algorithm

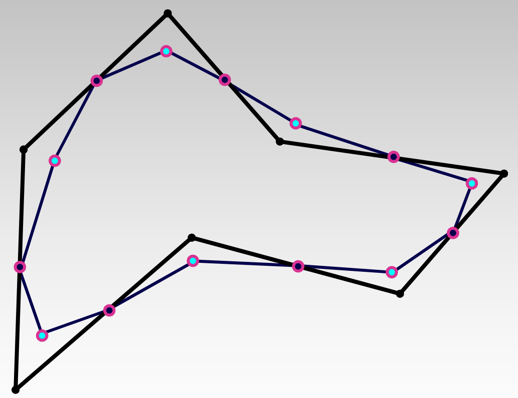


Limit – quadratic B-spline curve

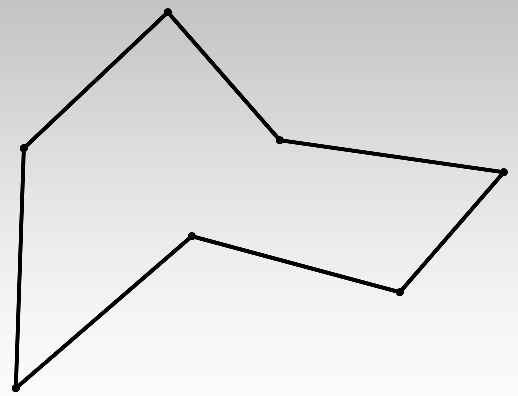
Cubic B-Spline (corner cutting)



Cubic Corner Cutting



The 4-point scheme



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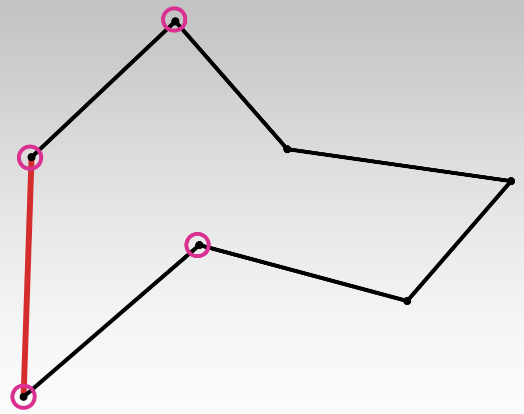

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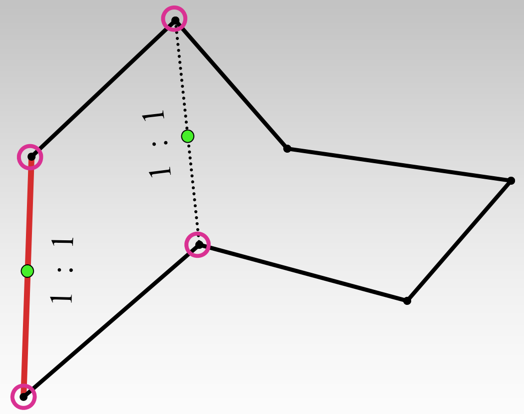

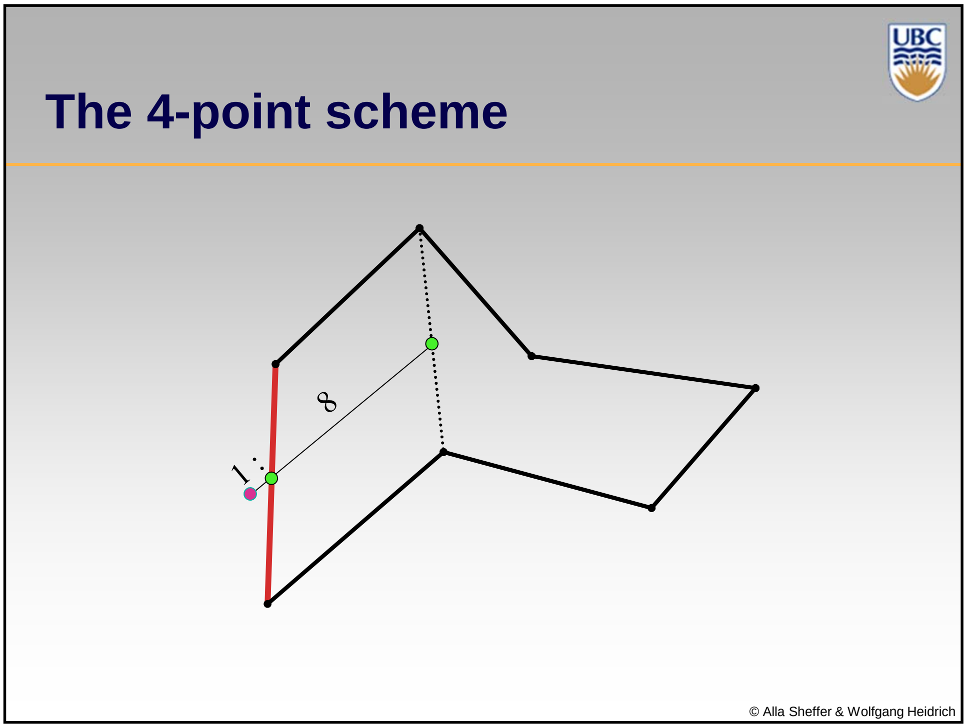

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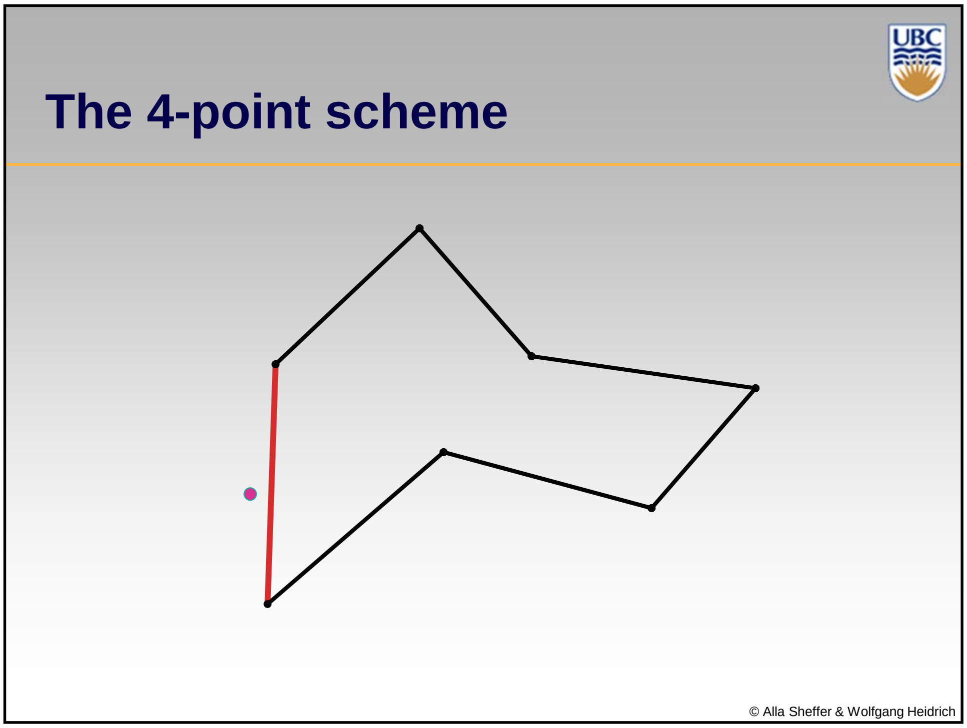

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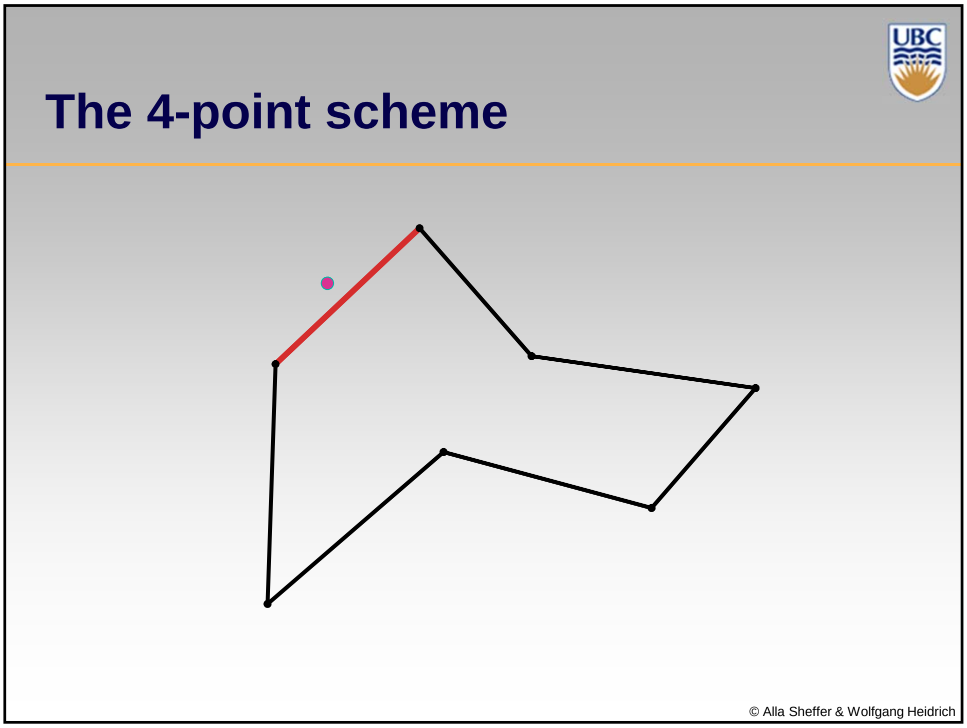

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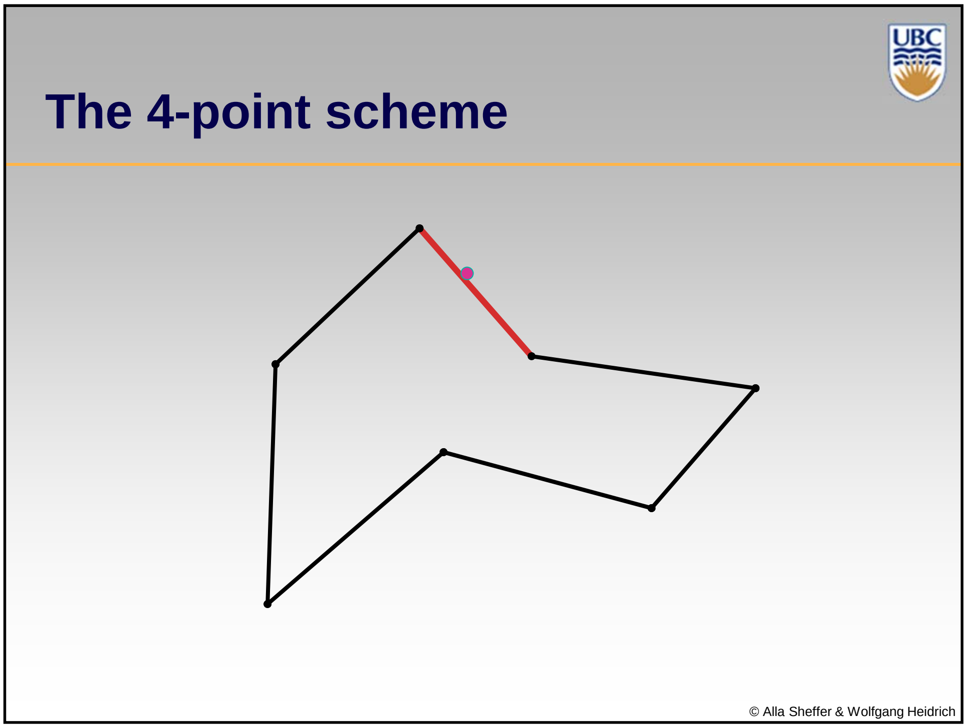

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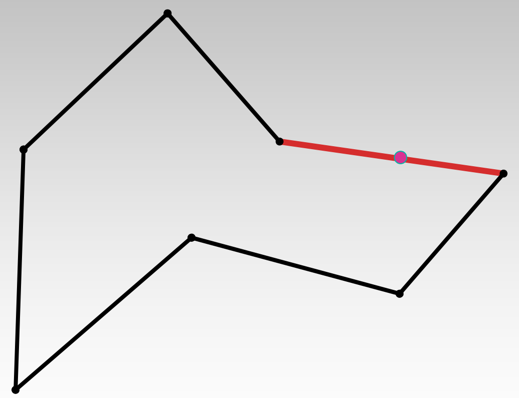

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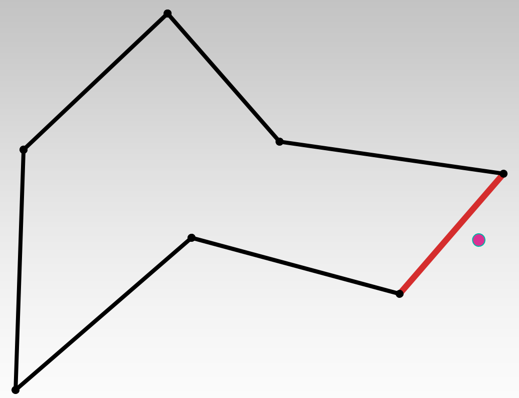

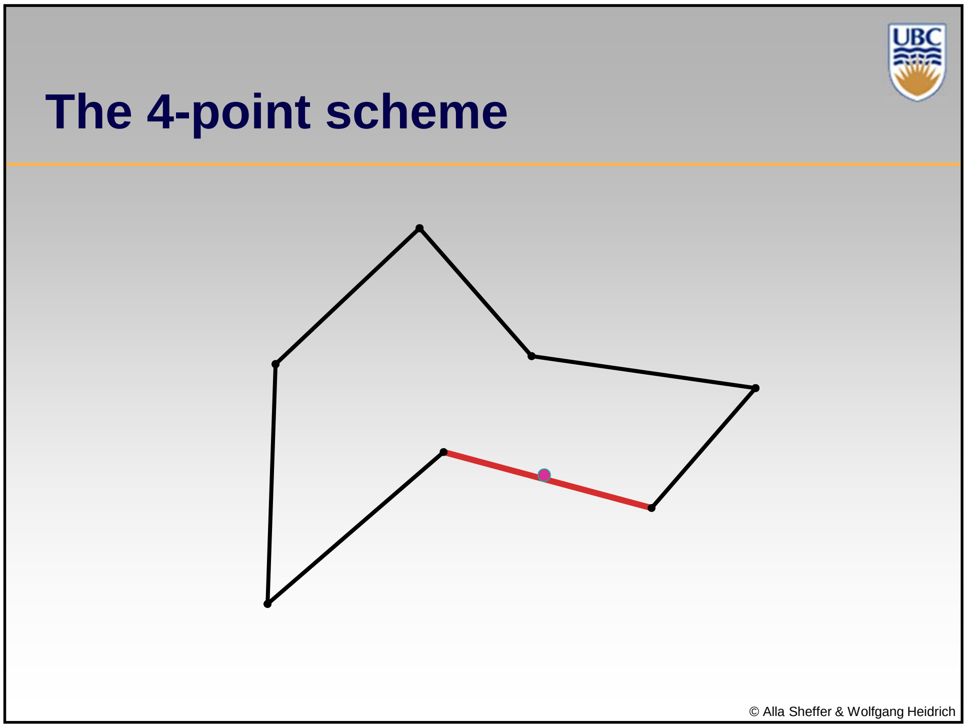

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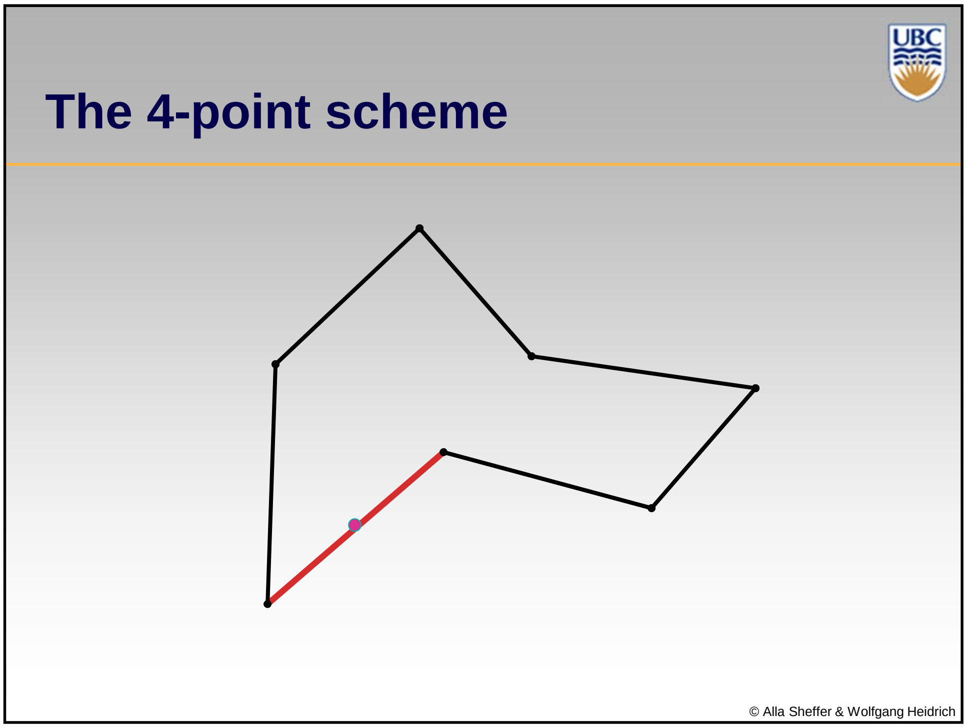

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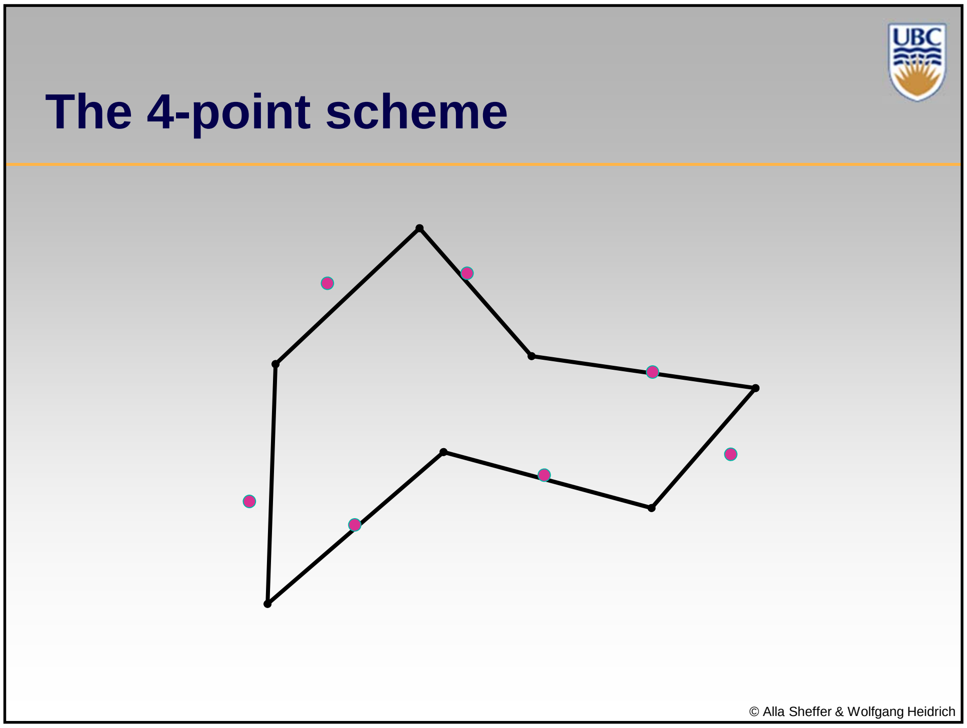

The 4-point scheme

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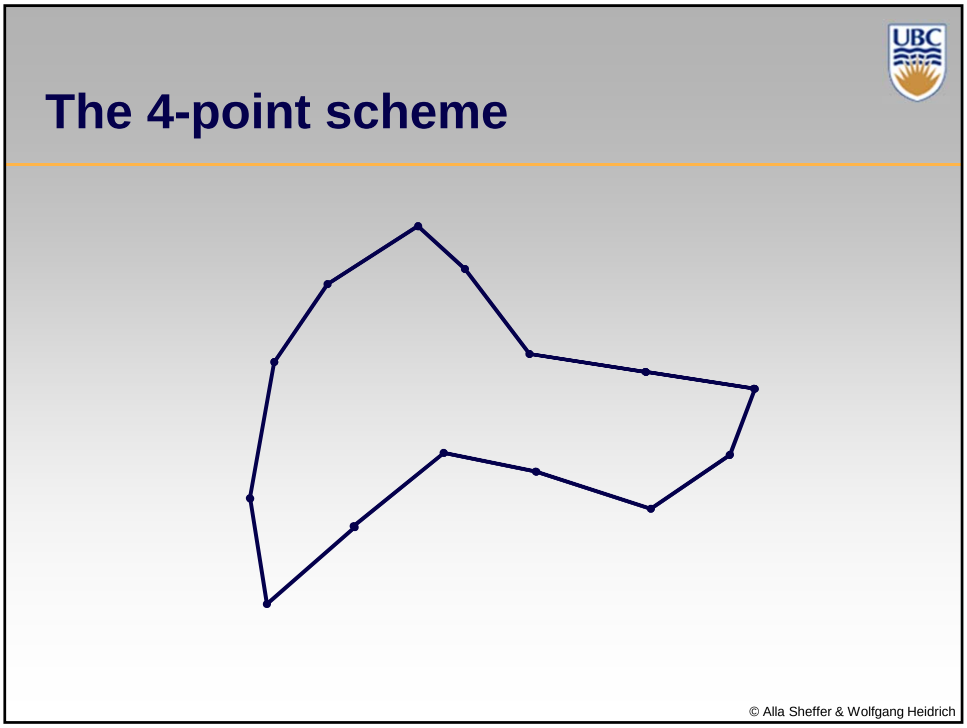

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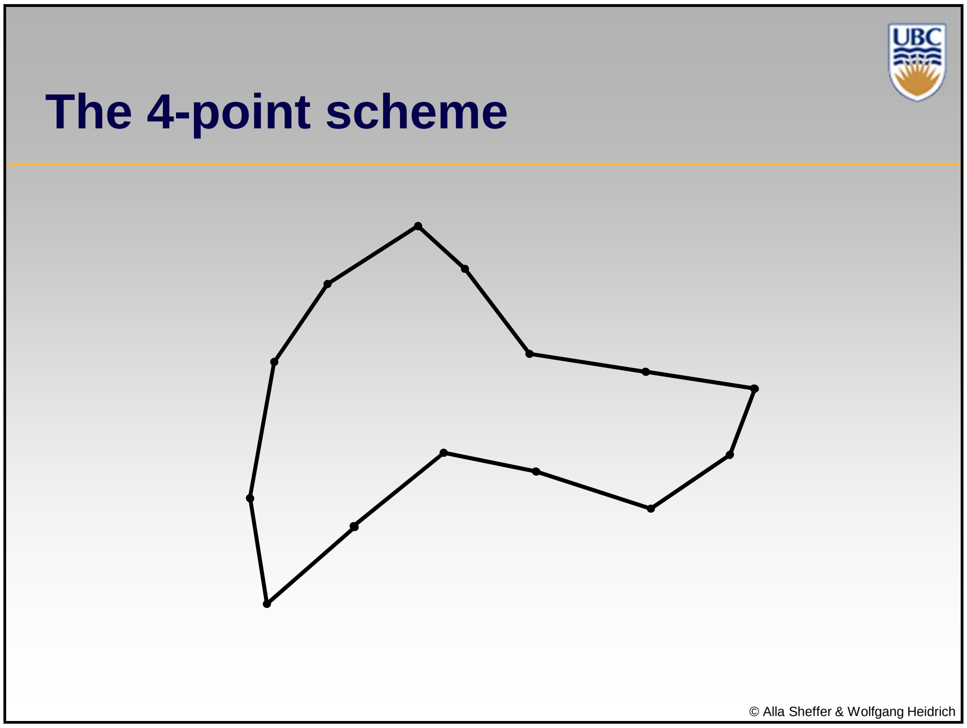

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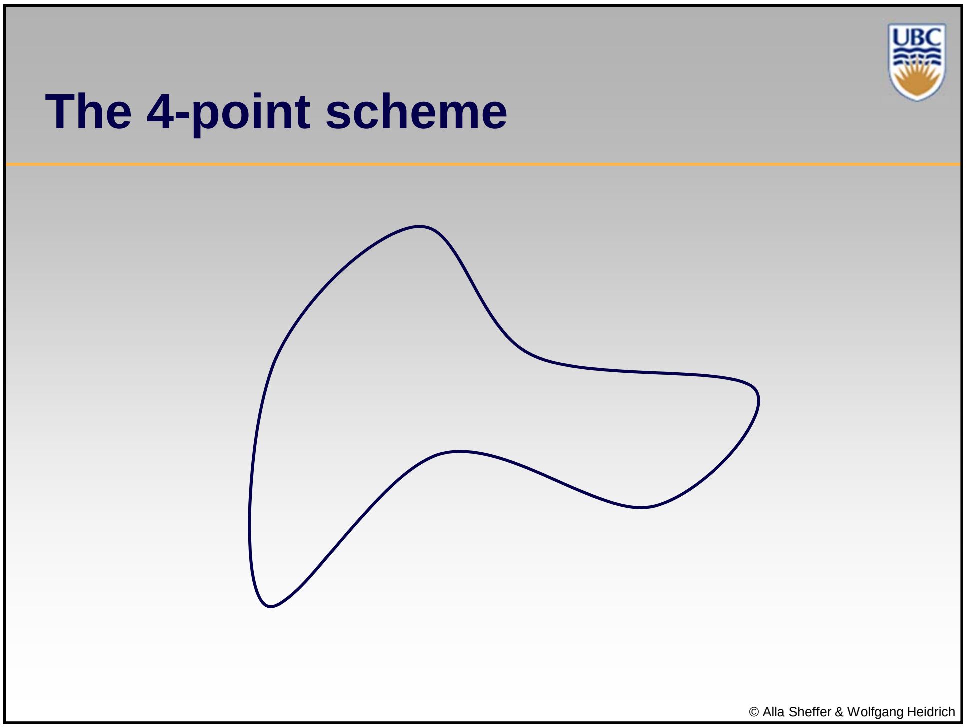

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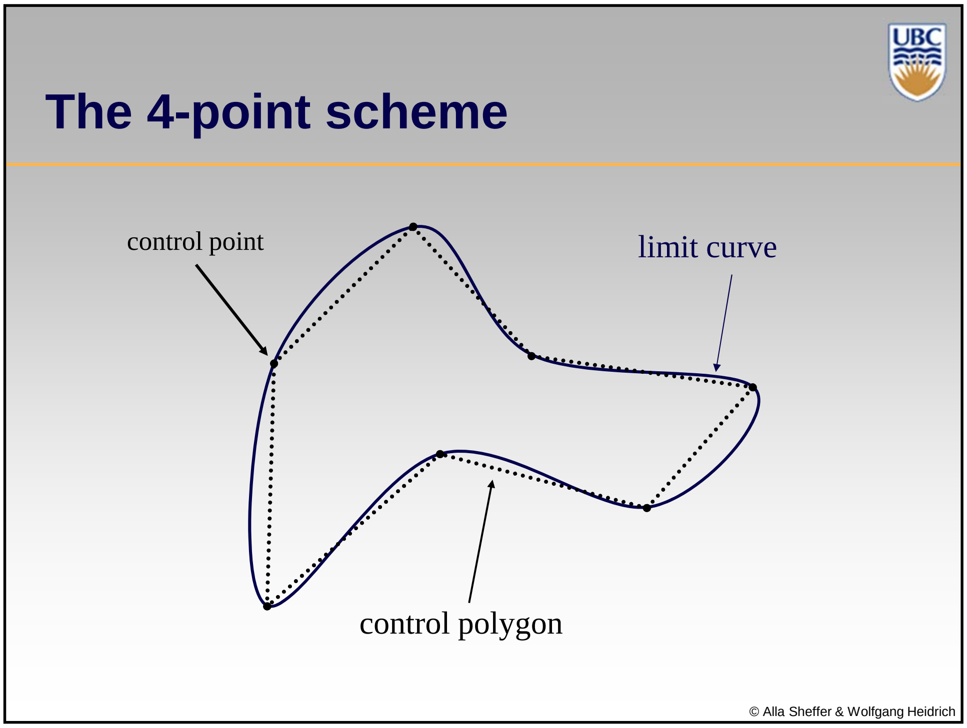

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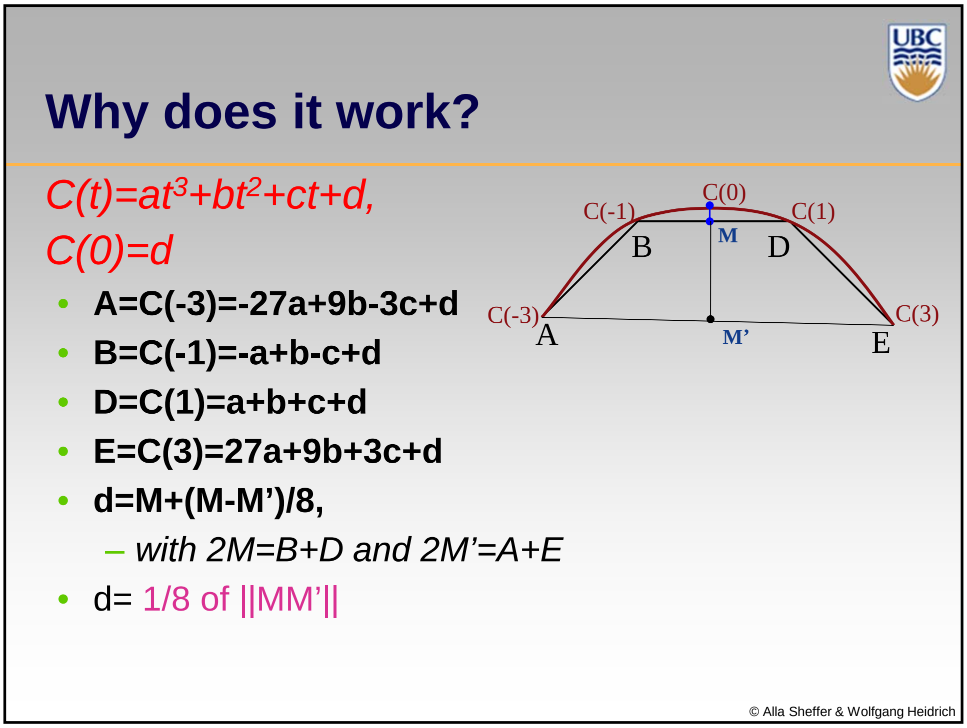


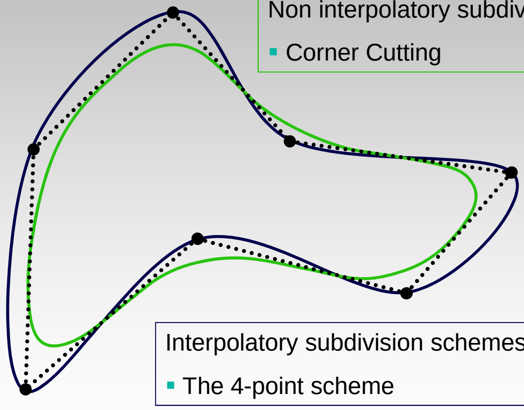
Why does it work?

$C(t)=at^3+bt^2+ct+d$,

$C(0)=d$

- **$A=C(-3)=-27a+9b-3c+d$**
- **$B=C(-1)=-a+b-c+d$**
- **$D=C(1)=a+b+c+d$**
- **$E=C(3)=27a+9b+3c+d$**
- **$d=M+(M-M')/8$,**
  - *with $2M=B+D$ and $2M'=A+E$*
- d= 1/8 of $||MM'||$

Subdivision curves



Non interpolatory subdivision schemes

- Corner Cutting

Interpolatory subdivision schemes

- The 4-point scheme

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Proving scheme works

Proving scheme works:

- **Convergence**
  - ***Will do on board & more details later***
- Degree of continuity
- Affine invariance
  - *As long as weights sum to 1*
  - *Proof on board*

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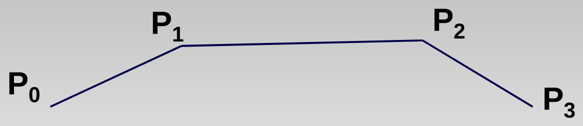
Subdivision Matrix

On Board – for Chaikin subdivision



$$\begin{pmatrix} P_0^i \\ P_1^i \\ P_2^i \\ P_3^i \end{pmatrix} = \begin{pmatrix} 1/4 & 3/4 & 0 & 0 \\ 0 & 3/4 & 1/4 & 0 \\ 0 & 1/4 & 3/4 & 0 \\ 0 & 0 & 3/4 & 1/4 \end{pmatrix}^i \begin{pmatrix} P_0^0 \\ P_1^0 \\ P_2^0 \\ P_3^0 \end{pmatrix}$$

Eigen Decomposition

Diagonalize subdivision matrix

- eigenvectors $x_i,\ i = 0..N$
- eigenvalues $\lambda_i,\ i = 0..N$
- p: vector of points in a neighborhood

$$p = \sum_i a_i x_i$$

(N+1)-vector of 3D points

3D vector

Eigenvectors

"Good" case:

- $\lambda_0 = 1$ & $|\lambda_i| < 1, i = 1, \ldots, n-1$

$$S^m p = a_0 x_0 + \lambda_1^m a_1 x_1 + \lambda_2^m a_2 x_2 + \lambda_3^m a_3 x_3 + \ldots$$

limit position

tangent vector

- can make a_0 zero by moving control points (by affine invariance)

Subdominant Eigenvectors

Next higher order terms

- Assume $\lambda_1 > \lambda_i, i = 2, \ldots, n-1$
- move control points so that $a_0 = 0$

$$\frac{1}{\lambda^m} S^m p = a_1 x_1 + \left(\frac{\lambda_2}{\lambda}\right)^m x_2 + \ldots$$

subdominant eigenvector

vanishes

Subdivision continuity

Continuity of limit curve

- Corner cutting (quadratic & cubic) – C_{inf} nearly everywhere, C_1/C_2 at a finite number of points
  - *B-Spline continuity*
- Four-point scheme – C_1 everywhere
  - **Proof later**

Upcoming Lectures

Wednesday, Feb 13:

- Midterm