Clicker Question

Do you have a clicker?

• A. Yes
• B. No
Clicker (real) question

Are the two curves $F(t)$ and $G(t)$ on the right $C^1$ continuous?
A. Yes
B. No
C. Not enough information

Syllabus

Curves in 2D and 3D
- Implicit vs. Explicit vs. Parametric curves
- Bézier curves
- Continuity
- B-Splines
- Rational curves
- Subdivision curves

Properties of Curves and Surfaces
- Differential Geometry

Surfaces
Subdivision Curves

Represent smooth curve by approximating polyline

At the limit = curve

Each iteration
• Add new points (~double)
• Approximating - Move old points
• Interpolating – Keep old points

Bezier Subdivision

To create complex curve need to explicitly enforce continuity – not very useful
Corner Cutting

Corner Cutting
Corner Cutting
Corner Cutting

Corner Cutting
Corner Cutting – Chaikin Algorithm

Limit – quadratic B-spline curve

Cubic B-Spline (corner cutting)
Cubic Corner Cutting

The 4-point scheme
The 4-point scheme

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The 4-point scheme

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The 4-point scheme
The 4-point scheme

- Control polygon
- Limit curve

Why does it work?

\[ C(t) = at^3 + bt^2 + ct + d, \]
\[ C(0) = d \]

- \( A = C(-3) = -27a + 9b - 3c + d \)
- \( B = C(-1) = -a + b - c + d \)
- \( D = C(1) = a + b + c + d \)
- \( E = C(3) = 27a + 9b + 3c + d \)
- \( d = \frac{M + (M - M')}{8}, \) with \( 2M = B + D \) and \( 2M' = A + E \)
- \( d = \frac{1}{8} \) of \( ||MM'|| \)
Subdivision curves

Non interpolatory subdivision schemes
  * Corner Cutting

Interpolatory subdivision schemes
  * The 4-point scheme

Proving scheme works

**Proving scheme works:**

- Convergence
  - *Will do on board & more details later*
- Degree of continuity
- Affine invariance
  - *As long as weights sum to 1*
  - *Proof on board*
Subdivision Matrix

On Board – for Chaikin subdivision

\[
\begin{bmatrix}
P_0^i \\
P_1^i \\
P_2^i \\
P_3^i
\end{bmatrix} = \begin{bmatrix}
1/4 & 3/4 & 0 & 0 \\
0 & 3/4 & 1/4 & 0 \\
0 & 1/4 & 3/4 & 0 \\
0 & 0 & 3/4 & 1/4
\end{bmatrix} \begin{bmatrix}
P_0^0 \\
P_1^0 \\
P_2^0 \\
P_3^0
\end{bmatrix}
\]

Eigen Decomposition

Diagonalize subdivision matrix

- eigenvectors \( x_i, i = 0..N \)
- eigenvalues \( \lambda_i, i = 0..N \)
- \( p \): vector of points in a neighborhood

\[
p = \sum_i a_i x_i
\]

(N+1)-vector of 3D points

3D vector
Eigenvectors

“Good” case:

- $\lambda_0 = 1$ & $|\lambda_i| < 1, i = 1, \ldots, n-1$

\[ S^m p = a_0 x_0 + \lambda_1^m a_1 x_1 + \lambda_2^m a_2 x_2 + \lambda_3^m a_3 x_3 + \ldots \]

- can make $a_0$ zero by moving control points (by affine invariance)

Subdominant Eigenvectors

Next higher order terms

- Assume $\lambda_i > \lambda_0, i = 2, \ldots, n-1$
- move control points so that $a_0 = 0$

\[ \frac{1}{\lambda^m} S^m p = a_1 x_1 + \left( \frac{\lambda_2}{\lambda} \right)^m x_2 + \ldots \]
Subdivision continuity

**Continuity of limit curve**

- Corner cutting (quadratic & cubic) – $C_{\text{inf}}$ nearly everywhere, $C_1/C_2$ at a finite number of points
  - *B-Spline continuity*

- Four-point scheme – $C_1$ everywhere
  - Proof later

**Upcoming Lectures**

*Friday, Feb 6:*
- Midterm