CPSC 424 Handout

Theorem: The Bernstein polynomials $B_i^m(t)$ of degree m can be expressed as an affine combination of two Bernstein polynomials of degree $m-1$ as follows:

$$B_i^m(t) = t \cdot B_{i-1}^{m-1}(t) + (1-t) \cdot B_i^{m-1}(t)$$

Proof:

$$\begin{aligned}
B_i^m(t) =& \binom{m}{i} \cdot t^i \cdot (1-t)^{m-i} && (1)\\
=& \frac{m!}{i! \cdot (m-i)!} \cdot t^i \cdot (1-t)^{m-i} \cdot \left(\frac{i}{m} + \frac{m-i}{m}\right) && (2)\\
=& \frac{(m-1)!}{(i-1)! \cdot (m-i)!} \cdot t^i \cdot (1-t)^{m-i} + \frac{(m-1)!}{i! \cdot (m-1-i)!} \cdot t^i \cdot (1-t)^{m-i} && (3)\\
=& t \cdot \binom{m-1}{i-1} \cdot t^{i-1} \cdot (1-t)^{m-i} + (1-t) \cdot \binom{m-1}{i} \cdot t^i \cdot (1-t)^{m-1-i} && (4)\\
=& t \cdot B_{i-1}^{m-1}(t) + (1-t) \cdot B_i^{m-1}(t) \quad (Q.E.D.) && (5)
\end{aligned}$$