CPSC 424 Assignment 5

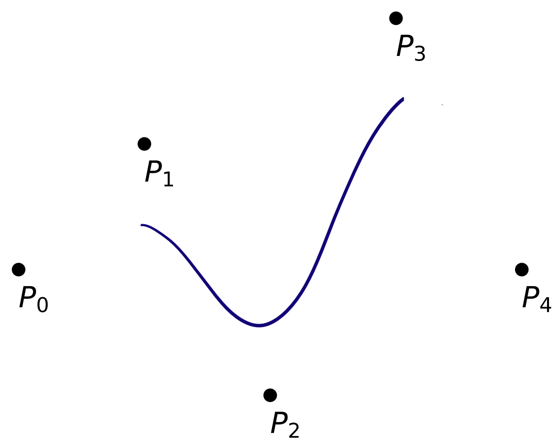
Term: September 2022, Instructor: Alla Sheffer, sheffa@cs.ubc.ca
https://www.students.cs.ubc.ca/ cs-424

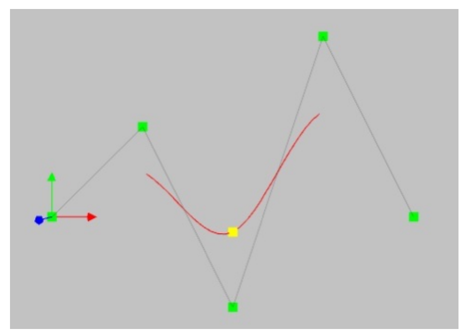
Due: November 14, 2022 in class

Assignment 5.1: B-Splines (10 Points)

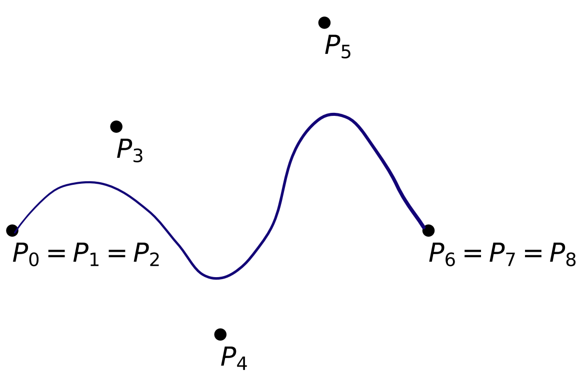
Draw/sketch a cubic B-spline curve (with uniformly distributed knots) defined by the points on the picture (you can draw freehand or use software),

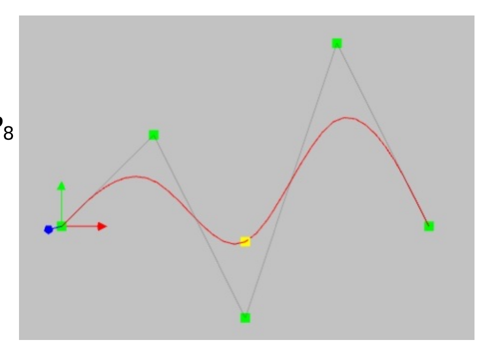
a) assuming no points are replicated.





b) assuming the first and the last points are replicated 3 times, and all the others are not replicated.





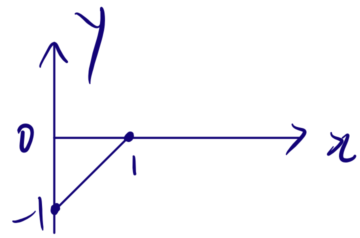
Assignment 5.2: Curve equivalence (10 Points)

Given the following 3 curves $F_i(t)$, **sketch each curve**, and for each $F_i(t)$ write down the equations for an equivalent curve $G_i(t)$ (check course notes for the definition of equivalence). For each one of the equivalent curves, include the corresponding parameter domain.

a) (2D) $F_0(t) = \begin{pmatrix} t \\ t-1 \end{pmatrix}, t \in [0,1]$

$$G_0(t) = \begin{pmatrix} \frac{t}{2} \\ \frac{t}{2} - 1 \end{pmatrix} t \in [0,2]$$

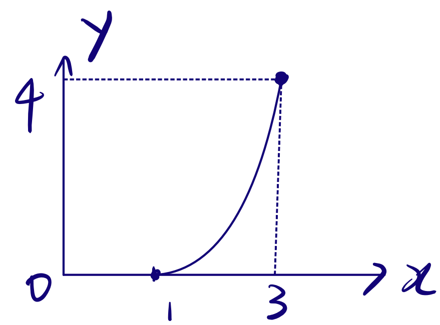
$$\phi(t) = 2t$$



b) (2D) $F_2(t) = \begin{pmatrix} t+1 \\ t^2 \end{pmatrix}, t \in [0,2]$

$$G_2(t) = \begin{pmatrix} 2t+1 \\ 4t^2 \end{pmatrix}, t = [0,1]$$

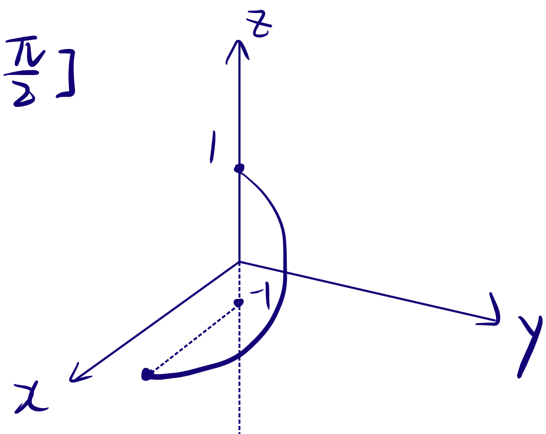
$$\phi(t) = \frac{1}{2}t$$



c) (3D) $F_1(t) = \begin{pmatrix} t \\ sin(t) \\ cos(t) \end{pmatrix}, t \in [0,\pi]$

$$G_1(t) = \begin{pmatrix} 2t \\ \sin(2t) \\ \cos(2t) \end{pmatrix}, t \in [0, \frac{\pi}{2}]$$

$$\phi(t) = \frac{t}{2}$$



Assignment 5.3: Surface types (10 Points)

For each of the following, describe/sketch/provide an image of an everyday object defined by this type of surface. **Explain your choice**.

a) Extrusion surface

A corrugated iron sheet: its shape is exactly taking a wavy curve and extruding along the line

b) Surface of revolution

Any symmetric vase: every horizontal slice would be a circle, surface is obtained by taking a curve (profile of vase) and rotate.

c) Ruled surface

Cone (without bottom surface). Whatever point on the cone you take, there is a straight line (connecting this point with cone's tip) that lies on the surface.

d) Describe/sketch/provide a photo of an everyday surface that can't be described via the definitions above **but** can be described using Coons or Bezier patches.

A simple car hood can be described by a Bezier patch.

Assignment 5.4: [BONUS] Differential geometry (8 Points)

Find the equation of a 2D curve $F(t), t \in [0,1]$ with the following unit tangent vector:

a) $\tau = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}, t \in [0;1]$

$F(t) = \begin{pmatrix} \frac{\sqrt{2}}{2}t \\ \frac{\sqrt{2}}{2}t \end{pmatrix}, t \in [0,1]$

b) $\tau = \begin{pmatrix} -cos(t) \\ sin(t) \end{pmatrix}, t \in [0;1]$

$F(t) = \begin{pmatrix} -\sin(t) \\ -\cos(t) \end{pmatrix}, t \in [0,1]$

a) As $|T(t)|=1$, t is an arclength parameterization.

To get the original curve, integrate:

$$F(t) = \int T(t)\,dt = \int \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} dt$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2}t \\ \frac{\sqrt{2}}{2}t \end{pmatrix}, \quad t \in [0,1]$$

The parameter interval stays the same.

b) Similarly, $|T(t)|=1$, t is an arclength parameterization, integrate:

$$F(t) = \int \begin{pmatrix} -\cos t \\ \sin t \end{pmatrix} dt = \begin{pmatrix} -\sin t \\ -\cos t \end{pmatrix} dt$$

$$t \in [0,1]$$