CPSC 424 Assignment 4

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Due: October 28, 2022 in class

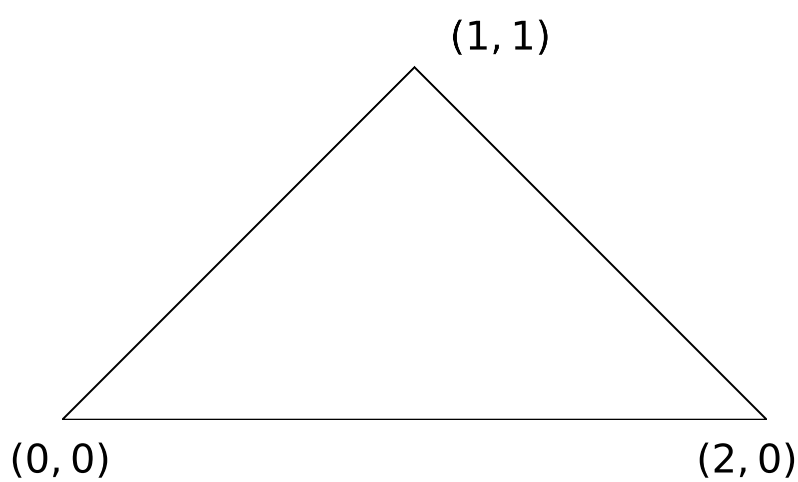
Assignment 4.1: Subdivision Curves (15 Points)

Given a closed polygon defined by the sequence of points $P_i = (p_1^i, p_2^i, ..., p_n^i)$, consider the following subdivision scheme that generates after one step of subdivision the polygon $P_{i+1} = (p_1^{i+1}, p_2^{i+1}, ..., p_{2n}^{i+1})$ using the following subdivision rules:

- $p_{2j-1}^{i+1} = \frac{1}{6}p_{j-1}^i + \frac{2}{3}p_j^i + \frac{1}{6}p_{j+1}^i$
- $p_{2j}^{i+1} = \frac{1}{2}p_j^i + \frac{1}{2}p_{j+1}^i$

(Note that if $j = 1$, then $j - 1$ becomes n, and if $j = n$, then $j + 1$ becomes 1)

a) Perform 2 levels of subdivision starting from the triangle in the figure below. **Be as accurate as possible and write down the exact positions of each point.**



b) Use the formulas above to define a subdivision matrix M relating $p^1_{2j-2}, p^1_{2j-1}, p^1_{2j}$ to $p^0_{j-1}, p^0_j, p^0_{j+1}$

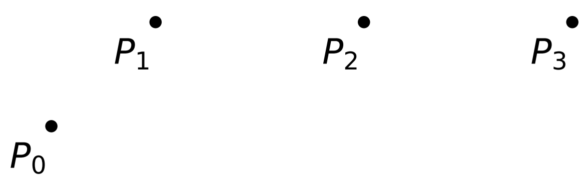
c) Analyze the matrix M (use a computer algebra system such as MATLAB if necessary) to determine whether the scheme converges.

d) Speculate what the positions and tangents will be at $\lim_{k\to\infty} p^k_{j\cdot 2^k}$ (limit control points corresponding to the original p_j). Use a computer algebra system if necessary. Sketch your results below - **be as accurate as possible**.

Assignment 4.2: Differential Geometry (10 Points)

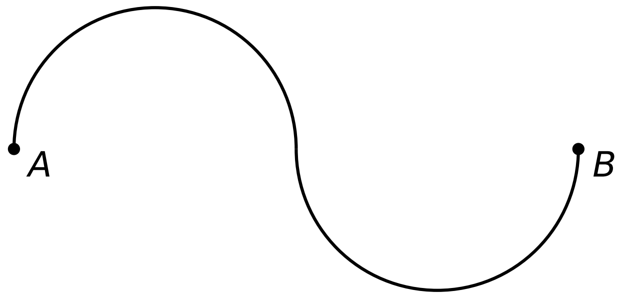
a) Sketch a 2D curve with constant non-zero curvature. Explain your drawing.

b) Sketch a smooth finite 2D curve which has regions of negative, positive, and zero curvature and interpolates the given points $\{P_0, P_1, P_2, P_3\}$. Annotate each region.



c) Provide a real life example of a 3D curve with non-zero torsion. Explain your choice.

d) Given the curve below, which starts at point A and ends at point B, has length l, and is parameterized using arc-length parameterization $s \in [0, l]$ mark the point on the curve at parameter value $s = 2l/3$. Explain your marking.



e) Prove that the curves $F(t) = (t, t^2 - 4)$ $t \in [0, 1]$ and $G(t) = (2t - 2, 4t^2 - 8t)$ $t \in [1, 1.5]$ are geometrically equivalent.

f) Given the curve $F(t) = (t, t)$ over the interval $t \in [0, s]$, write down the formula for the length of the curve as a function of s.

Assignment 4.3: BONUS: Continuity of Bézier Curves (5 Bonus Points)

Given the triangle with vertices $\mathbf{a} = (0, 0)$, $\mathbf{b} = (1, 0)$, $\mathbf{c} = (0, 2)$ as depicted in the figure below, we would like to find the control points of 3 cubic Bézier curves $F_1(t) : [0, 1] \to \mathcal{R}^2$, $F_2(t) : [1, 2] \to \mathcal{R}^2$, and $F_3(t) : [2, 3] \to \mathcal{R}^2$ with the following properties:

- $F_1(0) = F_3(3) = \mathbf{a}$, $F_1(1) = F_2(1) = \mathbf{b}$, and $F_2(2) = F_3(2) = \mathbf{c}$.
- the tangents at all the triangle vertices $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are parallel to the opposing edge of the triangle.
- the three curve segments are C^2 continuous at all three vertices

Note: we defined the C^2 continuity constraints only for adjacent parameter intervals $[a, b]$ and $[b, c]$. In the setup of this assignment, you will have to deal with vertex $\mathbf{a}$, where the intervals $[2, 3]$ and $[0, 1]$ are not directly adjacent. However, since the constraints really only use *ratios* of the interval lengths, it is possible to extend the continuity constraints from the lecture to this setting.

Find the position of the control points for all three segment. You may use a computer algebra system such as MATLAB to solve any system of equations that pop up…