CPSC 424 Assignment 1

Term: September 2022, Instructor: Alla Sheffer, sheffa@cs.ubc.ca, http://www.ugrad.cs.ubc.ca/~cs424

Due: Sep 19/2022 in class

Assignment 1.1: Curve Representation: Lines (9 Points)

Given the points, $P_0 = (1, 0), P_1 = (5, 3), P_2 = (1, 3)$.
a) Find the *implicit* equation of an infinite straight line going through $\{P_0, P_1\}$.

b) Find the *implicit* equation of an infinite straight line going through $\{P_0, P_2\}$.

c) Convert the *implicit* equation in a) to *explicit* form, if it's possible. Show your work. If it's not possible to convert, explain your reasons.

d) Convert the *implicit* equation in b) to *explicit* form, if it's possible. Show your work. If it's not possible to convert, explain your reasons.

e) Find the parametric equation for the straight line passing through $\{P_0, P_1\}$.

Assignment 1.2: Curve Representation: Closed Curves (12 Points)

Implicit functions are often used to separate samples identified as being inside a given shape from those identified as being outside. For instance given a set of points I in 2D space identified as *inside*, and a set O identified as *outside*, one can define a closed curve separating the two by devising an implicit function F that is positive for all $p \in I$ ($F(p) > 0$) and negative for all $p \in O$ ($F(p) < 0$). The separating curve is then defined by all points p where $F(p) = 0$.

a) Given the following sets $I = \{(1,1)\}, O = \{(-1,0),(0,3)\}$ devise an implicit curve F that separates them. Write down the equation of F and sketch it below.

b) Is F unique, or are there other implicit functions that can be used to separate these sets?

c) Given the following sets $I = \{(0,0),(3,0),(6,0)\}, O = \{(-1,0),(1,0),(4,0),(7,0)\}$ devise an implicit curve F that separates them. Write down the equation of F and sketch it below.

d) Is F unique, or are there other implicit functions that can be used to separate these sets?

Assignment 1.3: Lagrange Polynomials (10 Points)

Given a set of $k + 1$ pairs of point positions $x_i \in R^N$ and corresponding parameter values t_i: $(t_0, x_0), (t_1, x_1), ..., (t_k, x_k)$, we can use the following polynomial function $F(t)$ of degree up to k to create a curve in R^N that interpolates them:

$$F(t) = \sum_{i=0}^{k} L_i(t) \cdot x_i, \tag{1}$$

and

$$L_i(t) = \prod_{0 \leq m \leq k, m \neq i} \frac{t - t_m}{t_i - t_m}. \tag{2}$$

a) Given two points:

- $t_0 = 0, x_0 = (3, 2)$,
- $t_1 = 2, x_1 = (6, 1)$,

write down the interpolating polynomial. Show your work. Sketch/Plot the resulting curve. Hint 1: plug in additional values of t to get the corresponding points. Hint 2: feel free to use software to plot it.

b) Given four points:

- $t_0 = 0, x_0 = (3, 2)$,
- $t_1 = 1, x_1 = (6, 1)$,
- $t_2 = 2, x_2 = (6, 6)$,
- $t_3 = 3, x_3 = (3, 10)$,

write down the interpolating polynomial. Show your work. Sketch/Plot the resulting curve. Hint 1: plug in additional values of t to get the corresponding points. Hint 2: feel free to use software to plot it.