Assignment 5.1: B-Splines (10 Points)

Draw/sketch a cubic B-spline curve (with uniformly distributed knots) defined by the points on the picture (you can draw freehand or use software),

a) assuming the first and the last points are replicated 4 times, and all the others are not replicated.

\[ P_0 = P_1 = P_2 = P_3 \]
\[ P_4 \]
\[ P_5 \]
\[ P_6 = P_7 = P_8 = P_9 = P_{10} \]

b) assuming no points are replicated.

\[ P_1 \]
\[ P_2 \]
\[ P_3 \]
\[ P_4 \]
Assignment 5.2: Curve equivalence (10 Points)

Given the following 3 curves \( F_i(t) \), sketch each curve, and for each \( F_i(t) \) write down the equations for an equivalent curve \( G_i(t) \) (check course notes for the definition of equivalence). For each one of the equivalent curves, include the corresponding parameter domain.

a) (2D) \( F_0(t) = \begin{pmatrix} t \\ t \end{pmatrix} \), \( t \in [0, 1] \)

b) (2D) \( F_2(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix} \), \( t \in [0, 2] \)

c) (3D) \( F_1(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \\ t \end{pmatrix} \), \( t \in [0, \pi] \)
Assignment 5.3: Surface types (10 Points)

For each of the following, describe/sketch an everyday object defined by this type of surface. Explain.

a) Extrusion surface

b) Surface of revolution

c) Ruled surface

d) Describe/sketch an everyday surface that can’t be described via the definitions above.

Assignment 5.4: [BONUS] Differential geometry (10 Points)

Find the equation of a 2D curve $F(t), t \in [0, 1]$ with the following unit tangent vector:

a) $\tau = \left( \frac{t}{\sqrt{1-t^2}} \right), t \in [0; 1]$

b) $\tau = \left( \frac{-\sin \alpha}{\cos \alpha} \right), \alpha \in [0; 1]$