Assignment 2.1: De Casteljau Algorithm (6 Points)

For each of the following three cubic control polygons:

a) draw the control polygon, and perform the de Casteljau algorithm to evaluate the Bézier curve at parameter values $t = 1/2$, $t = 1/4$, and $t = 1/3$.

b) Use the properties of Bézier curves and the specific control polygons to determine the location of the points for $t = 0$, $t = 2/3$, $t = 3/4$, and $t = 1$.

c) Then sketch the curve itself. (Note: use a thin pencil for the de Casteljau construction, otherwise the diagram will be cluttered quickly).

\begin{align*}
a) & \quad b_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
b) & \quad b_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
c) & \quad b_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\end{align*}
**Assignment 2.2: Quadratic Approximation of a Quarter Circle (4 Points)**

In this assignment, we would like to approximate a quarter circle over the first quadrant (see figure) using a Bézier curve. In particular, we want to define a quadratic Bézier curve that shares the same endpoints as the quarter circle (i.e. $(1, 0)^T$ and $(0, 1)^T$). Moreover, we want our Bézier curve to be tangential to the quarter circle in these points.

![Diagram of a quarter circle and Bézier curve](image)

**a)** Where do the control points of the quadratic Bézier curve have to be located to meet the above conditions?

**b)** Edit the provided MATLAB script (a2.m) to plot both the quarter circle and the Bézier curve into a single plot. Are the two curves the same?

MATLAB is installed on the computers in the undergrad lab. To run the script, just open it in MATLAB, click "Run" (F5), in the appeared dialog - "Add to the path".