CS420+500: Advanced Algorithm Design and Analysis

Lectures: March 17th, 2017

Prof. Will Evans — *Scribe: Jennifer (Xin Bei) She*

In this lecture we discussed examples of approximation algorithms including

- Greedy and matching vertex cover algorithms for minimum vertex cover
- Greedy approximation for list scheduling

Handouts (posted on webpage): none

Reading: Jeff Erickson's notes on approximation algorithms

1 Minimum Vertex Cover

Given an undirected graph $G = (V, E)$, find the smallest set of vertices $S \subset V$ such that all edges in G have at least 1 endpoint in S.

Algorithm 1 Greedy

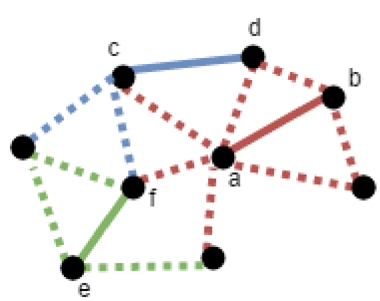
repeat
  add vertex $v \in G$ with highest degree to S
  remove v and all incident edges of v from G
until no vertices left in G

Guarantee: size of greedy vertex cover $\leq log(n) \times$OPTVC for all graphs

Algorithm 2 Matching Vertex Cover

$S = \emptyset$
repeat
  pick edge $(u, v) \in G$ (any edge)
  remove (u, v) from G and all edges adjacent to u or v
  add u and v to S
until no edges left in G

Example: $S = \{a, b, c, d, e, f\}$



Claim: MVC is a 2-approximation for vertex cover
Proof: we don't know how big OPTVC(G) is but we can lower bound its size

- $|\text{OPTVC}(G)| \geq$ number of edges picked by MVC (because edges selected by MVC form a matching and no vertex covers more than 1 edge in a matching)
- number of vertices picked by MVC is 2×number of edges picked

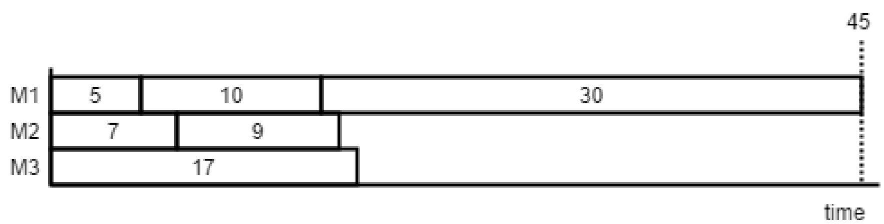
$\Rightarrow |\text{MVC}(G)| \leq 2 \times |\text{OPTVC}(G)|$

2 List Scheduling (Graham 1966)

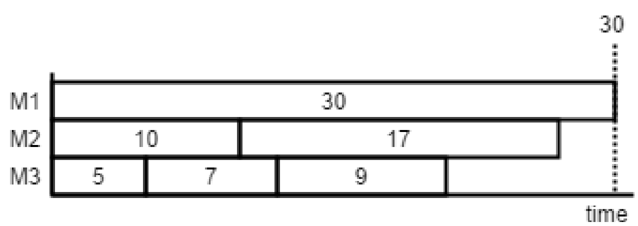
Given n jobs and m (identical) machines, where job i must execute uninterruptedly for p_i time units and each machine can work on 1 job at a time, find a schedule of jobs that minimizes completion time.

Greedy algorithm: whenever a job becomes idle, assign the next job to it.

Example: p_i's $= 5, 7, 17, 10, 9, 30$
Greedy solution:



Optimal solution:



One benefit of this algorithm is that it can be an online algorithm, whereas an algorithm that requires sorting the jobs can only be performed offline.

Claim: $|\text{GREEDY}(p_i, p_2, ..., p_n)| \leq (2 - \frac{1}{m})|\text{OPT}(p1, ..., pn)|$
Proof:

- $\text{OPT} \geq p_i, \forall i$
- $\text{OPT} \geq \frac{\sum p_i}{m}$

let k be the last job to finish and s_k be the start time of job k

$$\begin{aligned}
p_k &\leq \text{OPT} \\
s_k &\leq \frac{\sum_{i \neq k} p_i}{m} \\
s_k + p_k &\leq \frac{1}{m} \sum_{i \neq k} p_i + p_k \\
&= \frac{1}{m} \sum pi + (1 - \frac{1}{m}) p_k \\
&\leq \text{OPT} + (1 - \frac{1}{m}) \times \text{OPT} \\
&= (2 - \frac{1}{m}) \times \text{OPT}
\end{aligned}$$