CS420+500: Advanced Algorithm Design and Analysis

Lecture: March 13th, 2017

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In this lecture we discussed:

- Witness for SAT;
- NP vs co-NP;
- Approximation Algorithms.

1 NP-Completeness Wrap-up

Witness example: SAT

$\phi = (x \vee y \vee z) \wedge (\overline{x} \vee \overline{y}) \wedge (x \vee \overline{z})$

$\phi \in SAT \equiv \phi$ has a satisfying truth assignment

A witness in this case is a truth assignment

<u>**Note**</u>: It may be impossible to find a small, easily veryfiable witness for some problems.
Ex: $\phi \in$ co-SAT $\equiv \phi$ has no satisfying assignment iff $\overline{\phi} \in$ SAT
A witness for co-SAT problem would have to show that ϕ is not satisfied by any assignment.

NP vs co-NP:

Example: True Quantified Boolean Formula

$$\forall x\, \exists y\, \forall x\, (x \vee y \vee z) \wedge (\overline{x} \vee \overline{y})$$

NP $\equiv \exists$ witness $\equiv \sum_1$
co-NP $\equiv \forall$ witness $\equiv \prod_1$

<u>**Note**</u>:
$\sum_2 \equiv \exists x \forall y$ (verifiable in polynomial time)
$\prod_2 \equiv \forall x \exists y$ (verifiable in polynomial time)

2 Approximation Algorithms

What to do when need to solve NP-hard optimization problem?

- **Heuristic**. Disadvantage: bad adaptability to new input.

- **Approximation**. Advantage: can prove how close is the approximated solution to the optimal solution for any input.

Definition: An algorithm A is a ρ-approximation if for every input I with optimal solution value OPT(I):

$$A(I) \approx OPT(I) \equiv \begin{cases} A(I) \leq \rho OPT(I) & \text{(minimization problem)} \\ \\ A(I) \geq \frac{OPT(I)}{\rho} & \text{(maximization problem)} \end{cases}$$