CS420+500: Advanced Algorithm Design and Analysis

Lectures: Mar13,2017 + Mar15,2017

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1 NP wrap up

Student question: How do we come up with a witness? Example for the satisfiability question: CNF(Conjunctive normal form):

$\Phi = (x \vee y \vee z)(\bar{x} \vee \bar{y})(\bar{x} \vee \bar{z})$

- $\phi \in$ SAT $\equiv \phi$ has a satisfying true assignment. A witness is a true assignment.
- $\phi \in$ CO−SAT $\equiv \phi$ has no satisfying true assignment iff $\bar{\phi} \in$ SAT.
- L is NP−complete $\equiv$ L is in NP and L is NP-hard.
- NP is the set of decision problems with the following property: If the answer is Yes, then there is a proof of this fact that can be checked in polynomial time. Intuitively, NP is the set of decision problems where we can verify a Yes answer quickly if we have the solution in front of us. Or you could say: NP is the class of problems that have polynomial time verifiers.
- To show a problem is in NP, we follow this routine: Given a input $->$ Find a witness $->$ Verify in Polynomial time. If all the tests passed, then accept; otherwise reject. A more formal description could be found at chapter 7 of *Introduction to the Theory of Computation* by Michael Sipser.
- CO-NP is essentially the opposite of NP. If the answer to a problem in co-NP is No, then there is a proof of this fact that can be checked in polynomial time.
- Relationship between complexity classes

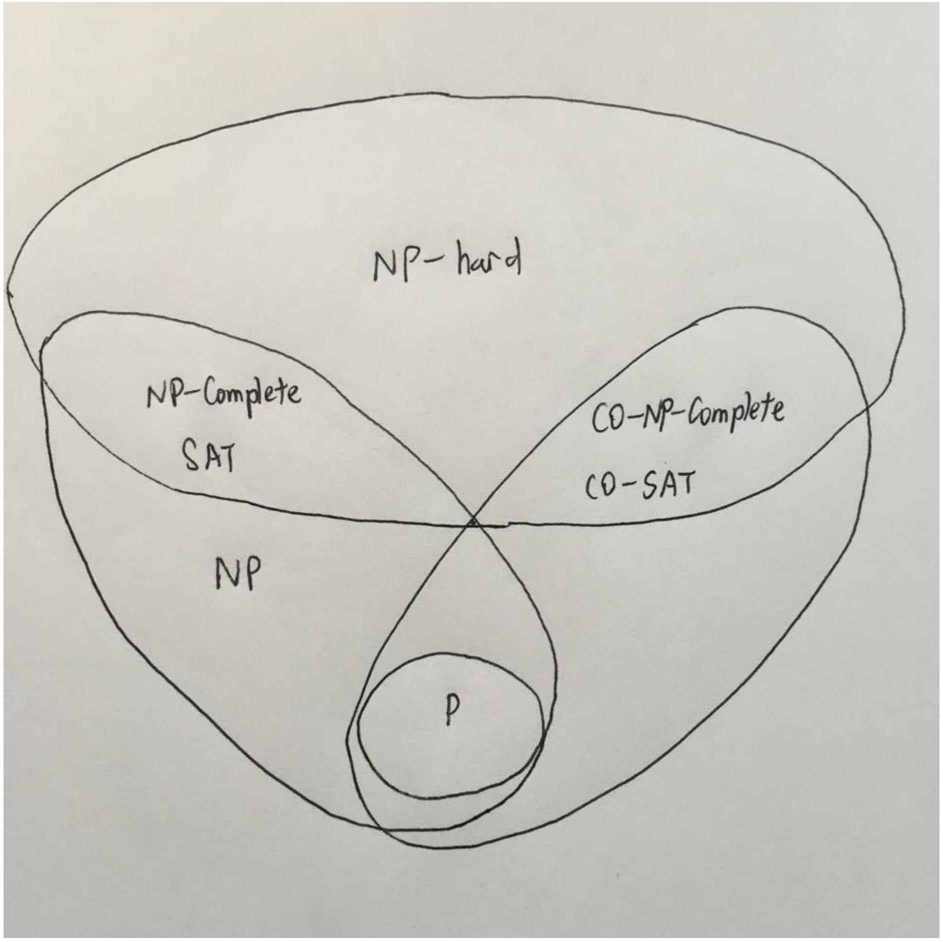

Figure 1: Complexity Classes

2 Approximation Algorithms(For Optimization Problems)

Consider an optimization problem. OPT(X) denote the value of the optimal solution for a given input X, and let A(X) denote the value computed by algorithm A given the same input X. We say that A is an ρ approximation algorithm iff $\frac{OPT(X)}{A(X)} \leq \rho$ for maximization problem and $\frac{A(X)}{OPT(X)}$ $\leq \rho$ for minimization problem. ρ is called approximation factor.