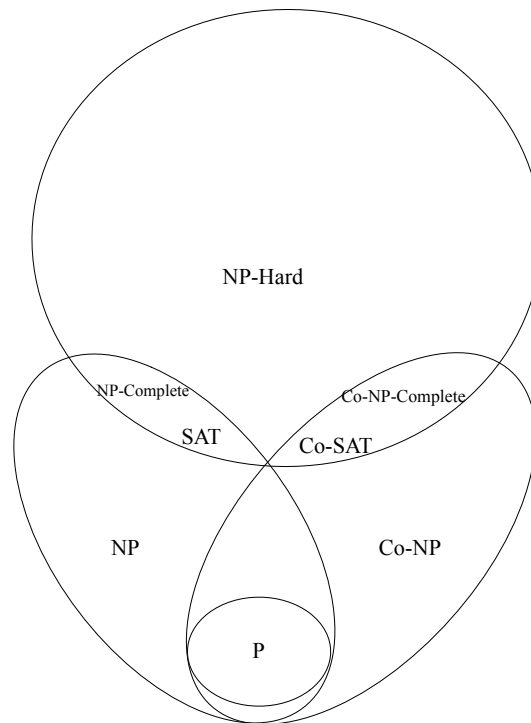


## 1 Review: SAT and Witnesses

Conjunctive normal form:  $\phi = (x \vee y \vee z) \wedge (\bar{x} \vee \bar{t}) \wedge (x \vee \bar{z})$ . It's the "AND" (the conjunction) of a set of "OR" clauses.

- $\phi \in \text{SAT} \equiv \phi$  has a satisfying (true) assignment.
- A witness is a true assignment.
- $\phi \in \text{Co-SAT} \equiv \phi$  has no satisfying true assignment if and only if  $\bar{\phi} \in \text{SAT}$ .
- $L$  is NP-complete  $\equiv L$  is in NP and  $L$  is NP-hard.

Briefly reviewed the relationship between complexity classes:



One question led to another and we started talking about *polynomial hierarchy*, which was interesting (but doubtful as an examinable topic). A clear notes corresponding to this discussion were not provided so perhaps refer to Wikipedia entry ([https://en.wikipedia.org/wiki/Polynomial\\_hierarchy](https://en.wikipedia.org/wiki/Polynomial_hierarchy)) as a starting point if interested in the details.

## 2 Approximation Algorithms for Optimization Problems

An algorithm  $A$  is a  $\rho$ -approx algo if for every input  $I$  with optimal solution value  $\text{OPT}(I)$ :

$$\frac{A(I)}{\text{OPT}(I)} \leq \rho.$$

Note: the above definition assumes that the optimization problem is to minimize some objective. If the objective is to maximize, then we say that  $A$  is a  $\rho$ -approx algo if for every input  $I$  with optimal solution value  $\text{OPT}(I)$ :

$$\frac{\text{OPT}(I)}{A(I)} \leq \rho.$$