CS420+500: Advanced Algorithm Design and Analysis

Lectures: Feb 8 + Feb 10, 2017

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In this lecture we:

- Continue the Pennant Race Problem
- Introduce the Open Pit Mining Problem

Miscellaneous:

- Midterms have been marked and can be viewed via Handback
- Midterm Average: 68.6%
- Additanl office hours with Will: Tuesday (Feb 14) 2pm-3pm

1 Pennant Race Problem

- $w = \#A$'s wins (assuming A wins all remaining games)
- $w_i = \#T_i$'s wins (assuming A wins all remaining games)
- $\{(T_i, T_j)\}$ = games remaining to be played

Assume $w_i \leqslant w$ for all i

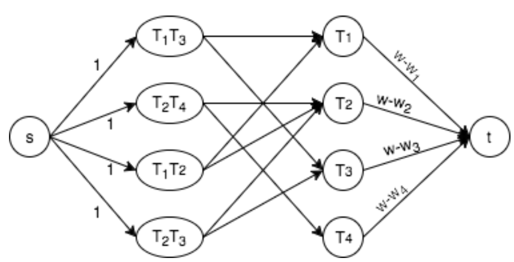

Figure 1: Flow graph of the Pennant Race example given on Feb 6

<u>Edges</u>

- (s, T_iT_j) with capacity 1
- $(T_iT_j, T_i)(T_iT_j, T_j)$ with capacity 1
- (T_i, t) with capacity $w - w_i$

If max flow = # games to play then A still has hope.

2 Open Pit Mining

What is Open Pit Mining? A mining technique where you attempt to dig to a location that would give a profit, but before you may do so you must remove a certain amount of dirt that lays above the location. Removing dirt has some cost associated with it. The goal is to achieve the maximum profit.

Input: Directed Acyclic Graph

- $G = (V, E)$ where V = set of tasks
- $E = \{(u, v) \mid$ u must be done before v $\}$
- A function $w(v)$ that specifies the profit from doing the task

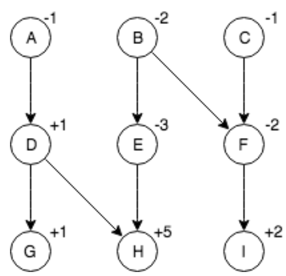

Figure 2: An example of a Directed Acyclic Graph

In this example:

- Both D and E must be done before H
- A must be done before D
- B must be done before E

Definition 1. *An initial set is a set of vertices that has no edge coming into it from the outside*

In example above:

- {D, G} is not an initial set
- {A,D,G} is an initial set

Convert the problem to a network flow problem so that

1. Any finite capacity cut corresponds to an initial set
2. A minimum capacity cut corresponds to max profit initial set

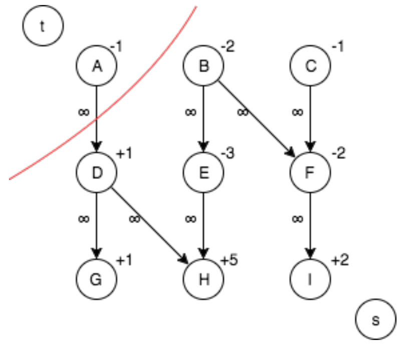

Figure 3: The example from Figure 2 with a finite capacity cut

2.1 Conversion Part 1: Finite Capacity Cut

Claim 2. *In this "network", any finite capacity cut (S,T) defines an initial set $T = \{t\}$*

Proof. If cut (S,T) has finite capacity then no original edge is directed into T from S thus T-{t} is an initial set. If set U is an initial set then $T = U \cup \{t\}$, $S = V - T$ is a cut with no original edge entering T thus it has finite capacity □

2.2 Conversion Part 2: Minimum Capacity Cut

Given a directed acyclic graph, we want to connect the vertices so that:

- If w(u) is positive, then (u) $\xrightarrow{w(u)}$ (t)
- If w(v) is negative, then (s) $\xrightarrow{-w(v)}$ (v)

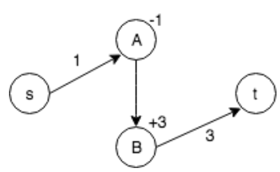

Figure 4: Simple example of only two vertices: A and B

Note that in Figure 4 we get $maxflow = mincut = 1$. Then the min cut gives us initial set {A,B}. But the max flow value does not correspond to the total profit from the task.