CS420+500: Advanced Algorithm Design and Analysis

Lectures: Feb 8 + 10, 2017

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In this lecture we discussed two problems that can be solved by network flows:

- the pennant race problem;
- the open pit mining.

1 Pennant Race Problem

Assume A wins all the remaining games. Let

- w= the number of A's wins after A wins all the remaining games,
- w_i= the number of wins of Team i (T_i),
- $\{(T_i, T_j)\}$= games remaining to be played.

If $w < w_i$ for some i, then A has no hope. [Done]

So assume $w_i \leq w$ for all i. We solve this problem through the network flow (see Figure 1).

Edges:
$(s, (T_iT_j))$ with capacity 1,
$((T_iT_j), T_i), ((T_iT_j), T_j)$ with capacity 1,
(T_i, t) with capacity $w - w_i$.

If the max flow size equals to the number of games to play, then A still has hope; otherwise, A has no hope.

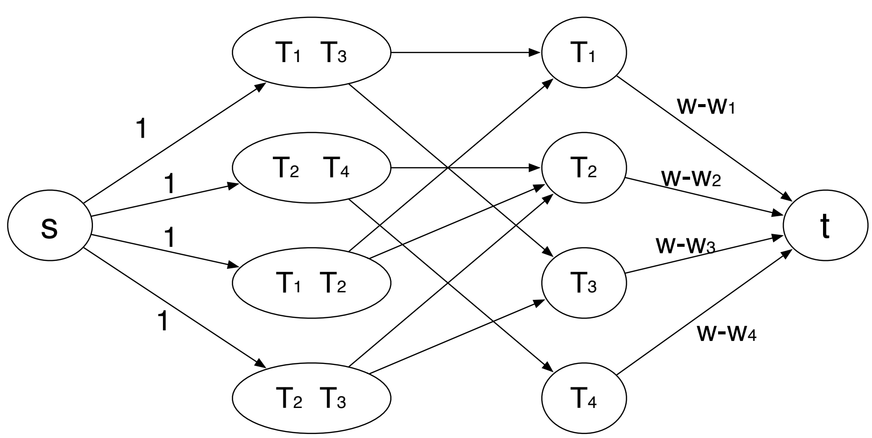

Figure 1: Network flow for the pennant race problem.

2 Open Pit Mining

Input: a directed acyclic graph $G = (V, E)$, where V is a set of tasks and $E = \{(u, v)|u$ must be done before v (called *precedence constraint*) $\}$, and a function $w(v)$ that specifies the profit from doing task $v \in V$.

Find: the most profitable set of tasks to perform subject to precedence constraints.

An input example is given in Figure 2, where $V = \{A, B, C, \cdots, I\}$, $E = \{(A, B), (D, G), \cdots, (F, I)\}$, $w(A) = -1, w(B) = -2, \cdots, w(I) = +2$.

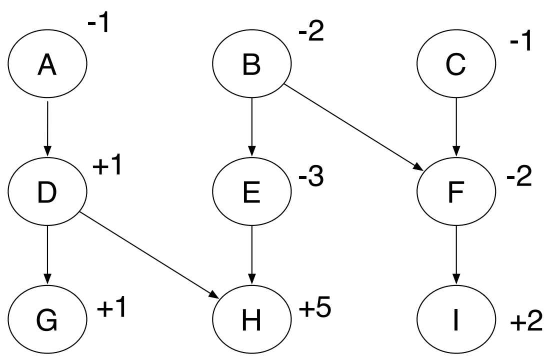

Figure 2: An input example for the open pit mining problem.

An initial set is a set of vertices that has no edge coming into it from outside. For example, in Figure 2, $\{D, G\}$ is not an initial set, $\{A, D, G\}$ is.

To solve this problem, we need to convert it to a network flow problem so that

- any finite capacity cut corresponds to an initial set and
- a minimum capacity cut corresponds to max profit initial set.

So the first thing we do is to add infinite capacity to all the original edges in E.

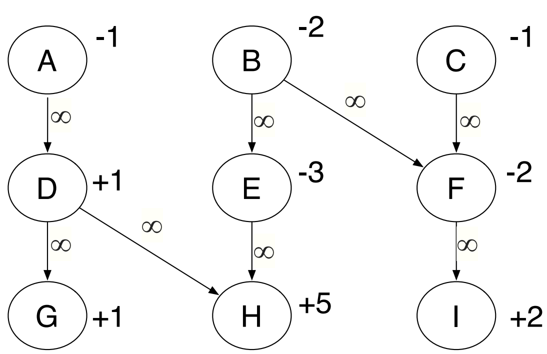

Figure 3: Adding infinite capacity to $e \in E$.

Claim: In this "network", any finite capacity cut (S, T) defines an initial set $T - \{t\}$.

Proof:

- If cut (S, T) has finite capacity, then no original edge is directed into T from S. Therefore, $T - \{t\}$ is an initial set.
- If set U is an initial set, then $T = U \cup \{t\}$, $S = V - T$ is a cut with no original edge entering T. Therefore, the cut (S, T) has a finite capacity.

How to convert to network flow?

- If $w(u)$ is positive, add an edge $u \to t$ with capacity $w(u)$,
- If $w(v)$ is negative, add an edge $s \to v$ with capacity $-w(v)$,
- If $w(u) = 0$, do nothing.

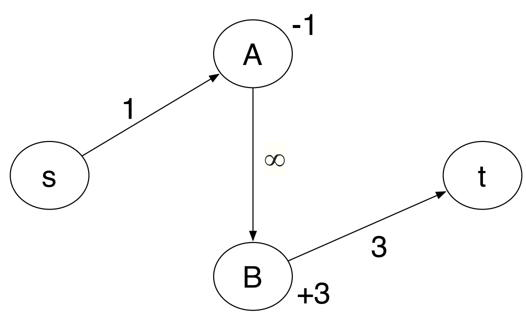

Figure 4: An example of converting to network flow.