Scribe: Tyler House, Notes from Jan 4 and Jan 6

Computational problems involving geometry

- May be hidden
- Exposing them is the key point of designing and understanding relevant algorithms

Example 1: Salad Dressings

Imagine we are creating a salad dressing and have two bottles, each with their respective proportions of oil and vinegar as described below:

	Oil	Vinegar
a	15%	36%
b	9%	21%
c	10%	33%

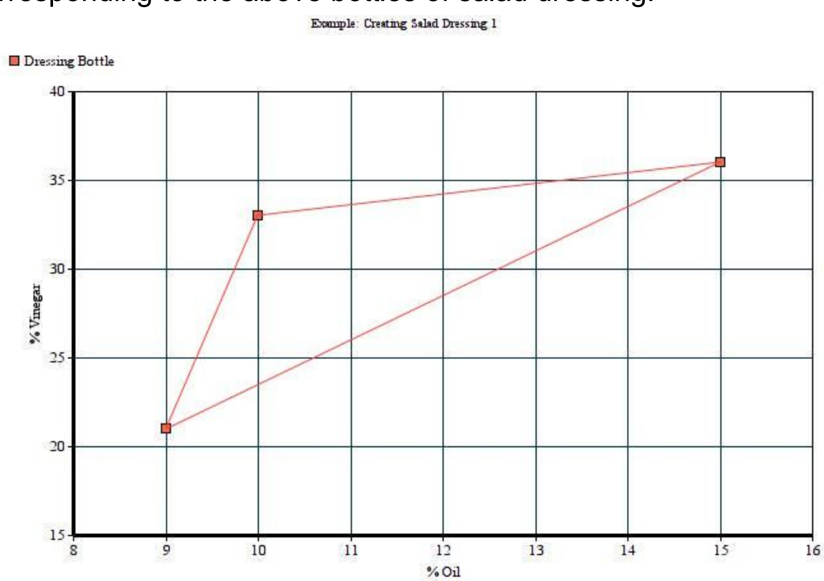
Problem: Can we mix bottles **a** and **b** to get 13% oil and 31% vinegar?

- Yes! 2 parts **a**, 1 part **b**

Problem: Can we mix bottles **a** and **b** to get 12% oil and 30% vinegar?

- No! No dressing for you.

This problem as described above is actually one of ***geometry***. Below is a plot containing points corresponding to the above bottles of salad dressing.



As shown above, these bottles of dressing can be modelled as points in a 2D plane, and any point inside the shape above can be created via some combination of the given points. These "mixtures" are **convex combinations** of the points representing contents of the bottles.

$$ConvexCombinationOfPoints(p1, p2, , pn) = \sum_{i=1}^{n} \alpha_i p_i$$

Where α_i = how much of bottle i, and $p_i = bottle_i$

Note:

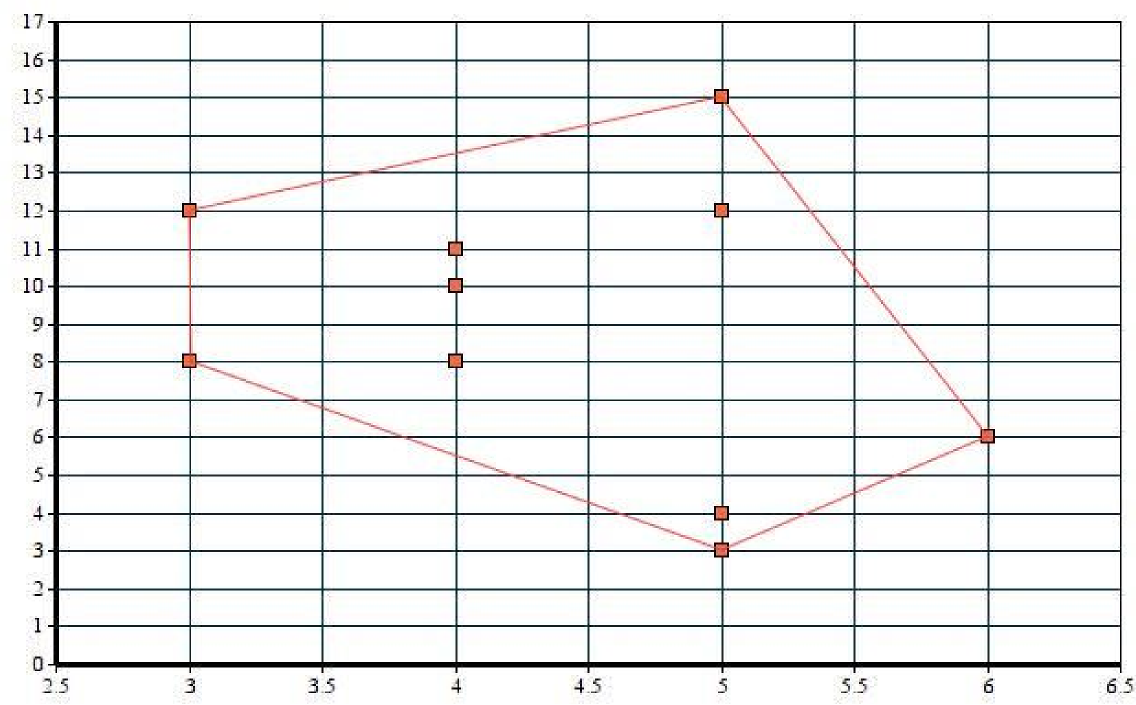
1) $\Sigma \alpha_i = 1$
2) $\forall i, \alpha_i \geq 0$

Convex Hulls

Given a spread of points P, a ***Convex Hull*** is the smallest convex set that contains P. Or in other words, a set T is convex iff $\forall a, b \in T, \overline{ab} \in T$

As an example:

Example: Convex Hull



If given a set of points P, how do we compute a convex hull?
Problem:

- Input → a set of points $P = \{p_1, p_2, , p_n\}$
- Output → Convex Hull of P (CH(P))

<u>Algorithm 1: Jarvis-March (Gift Wrapping Algorithm), 1973</u>

1. Find the point p_0 with the minimal y-coordinate
2. Set h = 0
3. <u>Repeat</u>

   Pick $q \in P \setminus \{p_h\}$

   For each $p \in P$

   If ∃ right turn (p_h, q, p)

   $q = p$

   $p_{h+1} = q$

   $h = h + 1$

   <u>Until</u> $p_h = p_0$

<u>Algorithm 2: Graham's Scan, 1972</u>

1. Find point $p_0 \in P$ with smallest y-coordinate
2. Sort remaining points by angle around p_0 in counter-clockwise (CCW) order
3. Calculate successively $CH(p_0, p_1, p_2), CH(p_1, p_2, p_3), etc$

More on Graham's Scan in the subsequent lecture.