CS420+500: Advanced Algorithm Design and Analysis

Lectures: Jan 4 + Jan 6, 2017

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In this lecture we:

- Discussed THE COURSE SYLLABUS;
- TOOK A SHORT QUIZ (not graded, solutions posted to Piazza);
- AND started studying CONVEX HULLS - Jarvis March

Handouts (posted on webpage):

- CS 420+500 Syllabus (aka the webpage)
- quiz

Reading: NO ASSIGNED READING(S) THIS WEEK.

1 CONVEX HULLS

Suppose we're in charge of making the salad dressing. We have bottles of oil and vinegar in the following ratios:

Table 1: Ingredients in unsatisfactory premixed salad dressing

bottle	**oil**	**vinegar**
A	15%	36%
B	9%	21%

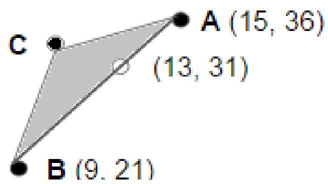
Q: Can we mix bottles A and B to get 13% oil and 31% vinegar?
A: Yes. We use 2 parts A and 1 part B.

Table 2: New salad dressing

bottle	**oil**	**vinegar**	proportion	oil'	vinegar'
A	15%	36%	2/3	10%	24%
B	9%	21%	1/3	3%	7%
		A + B	1	13%	31%

Q: Can we create a mixture of 12% oil and 30% vinegar?
A: No.

How can we tell which mixtures are achievable?



We can make any dressing with ratios lying on the line connecting $\overline{AB}$.

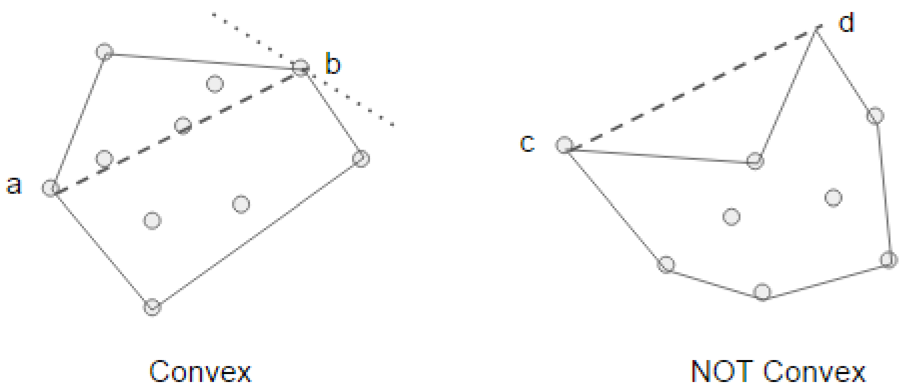
If we have a new bottle C, then we can make any ratio within the area of the connected shape $\triangle ABC$.

A *mixture* is a *convex combination* of points $P = \{p_1, p_2, ...p_n\}$ representing the contents of bottles or $\sum_{i=1}^{n} \alpha_i p_i$ where $\sum_{i=1}^{n} \alpha_i = 1$, $\alpha_i \geq 0$ for all i.

[def] The *convex hull* of P or CH(P) is the smallest convex set containing P.

[def] A set T is *convex* if for all $a, b \in T$, $\overline{ab}$ is in T.

[def] A *supporting line* is a line going through a *boundary point* $b \in T$ such that all points in T fall on one side of that line.



Problem: *Input* set of points $P = \{p_1, p_2, ..., p_n\}$
Output convex hull of P

1.1 Jarvix March (Gift wrapping) 1973

ALGORITHM: *Imagine hammering nails (representing P) into a board, then wrapping a rubber band around the outer nails.*

(1) Find point p_0 that is guaranteed to be in the convex hull of P such as the point with the minimum y-coordinate.
(2) set $h = 0$
(3) repeat
    pick $q \in P \setminus \{p_h\}$
    for each $p \in P$
        if rightturn(p_h, q, p)
            $q = p$
    $p_{h+1} = q$
    $h = h + 1$
  until
    $p_h = p_0$

RUNTIME Now we consider the running time of Jarvis March.

Step(1) $\in O(n)$
Step(2) $\in O(1)$
Step(3) $\in \Theta(n^2)$

[THINK] worst case: each right turn check $\in O(n)$ and we must do this for each point n times $\therefore O(n^2)$ but since we must go through all elements, run time is also $\Omega(n^2)$ thus overall, we have $\Theta(n^2)$

The CON of Jarvis March is that we end up checking the same points multiple times, repeating similar work. What if we do a sort by angle first?

1.2 NEXT LECTURE: Graham's Scan 1972