CS420+500: Advanced Algorithm Design and Analysis

Lectures: January 4 + 6, 2017

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In this lecture we discussed:

- Convex Hulls
- Jarvis March (gift-wrapping algorithm)
- Introduction to Graham's Scan algorithm

Handouts (posted on webpage): No handouts.

Reading: No assigned readings.

1 Convex Hulls

Motivating Example: Salad Dressing

Consider 2 bottles both containing oil and vinegar.

	Oil	Vinegar
Bottle a	15%	36%
Bottle b	9%	21%

- Can we mix the 2 bottles to get 13% oil and 31% vinegar?

  Yes $\rightarrow$ 2 parts Bottle a, 1 part Bottle b.

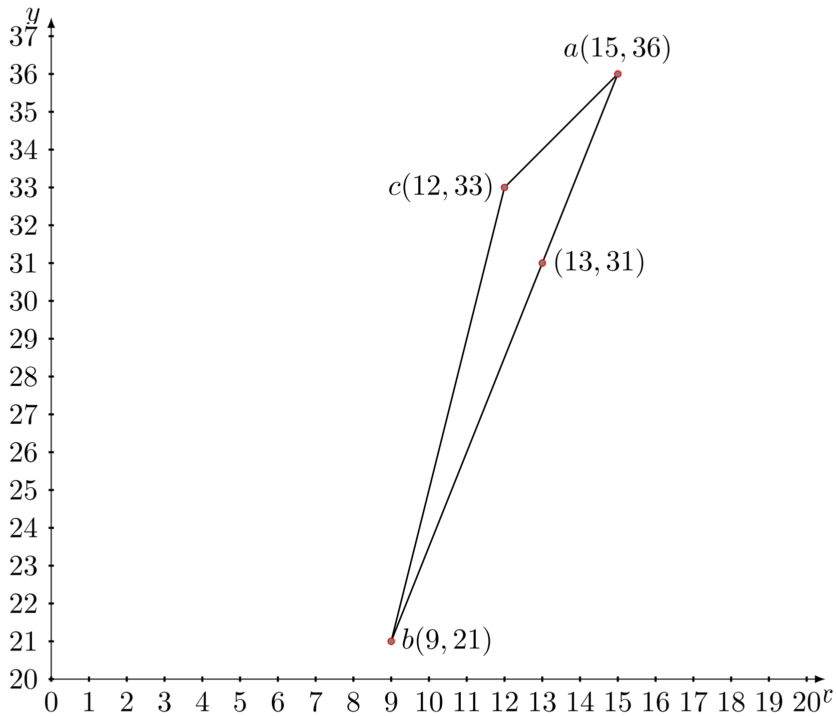
- What about 12% oil and 30% vinegar?

  No.

Now consider the case with 3 bottles.

	Oil	Vinegar
Bottle a	15%	36%
Bottle b	9%	21%
Bottle c	12%	33%

Notice we can translate this example onto the Cartesian Plane:



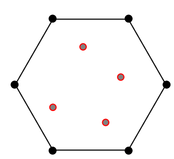
- A **mixture** is a convex combination of points representing the contents of bottles.
- A **convex combination** of points $p_1,\ p_2,\ \ldots,\ p_n$ is

$$\sum_{i=1}^{n} \alpha_i p_i$$

where

$$\sum_{i=1}^{n} \alpha_i = 1 \quad and \quad \alpha_i \geq 0$$

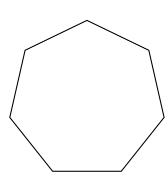
Ex. If P is a set of points, then the (closed) interior of the outline is the set of all convex combinations of P



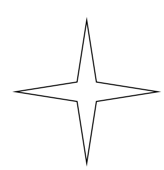
Definition: *Convex Hull of P.* Let P be a set of points.

CH(P) ≡ the smallest convex set that contains P.

- A set T is **convex** if for all a, b ∈ T, segment $\overline{\text{ab}}$ is in T.



Convex



Not Convex

2 Jarvis March (Gift-Wrapping Algorithm) 1973

Problem. Given a set of points P = p_1, p_2, ..., p_n, find the convex hull of P.

Algorithm.

1. Find point p_0 in P with minimum y-coordinate O(n)
2. Set h=0
3. Repeat O(n^2)

   Pick q $\in$ P $\setminus$ p_h

   For each p $\in$ P

   If rightturn(p_h, q, p)

   q=p

   p_{h+1} = q

   h = h+1

   Until

   p_h = p_0

Worst Case: $\Theta(n^2)$

3 Introduction to Graham's Scan 1972

Algorithm.

1. Find point p_0 in P with minimum y-coordinate O(n)
2. Sort remaining points by angle around P (counter clockwise)
3. Calculate successively:

   - CH(p_0, p_1, p_2)
   - CH(p_0, ..., p_3)
   - CH(p_0, ..., p_4)
   - ...