CPSC 420+500 Problem Set 4 22 Feb 2017
W. Evans Due: 6 Mar 2017 at 21.00 (9pm)

Submission Instructions

Handin your solutions using `handin`. You can write your solutions by hand and scan the pages or take pictures of them with your phone; or use a word processing package to typeset your solutions. Whatever you do, you should produce **pdf** files **assign4Q1.pdf**, **assign4Q2.pdf**, etc. containing your solutions for Question 1, Question 2, etc.

To handin: Copy your solution files to the directory `~/cs420/a4` in your home directory on a CS undergraduate machine. (You may have to create this directory using `mkdir ~/cs420/a4`.) Then run `handin cs420 a4` from your home directory.

If you do not have a CS department account, you can email your assign1.pdf to me `will@cs.ubc.ca`.

Late submissions are not accepted.

Grading Policy

We will grade a subset of these questions of size at least two. It's a good idea to do all of the questions because (1) you don't know which ones we'll grade and (2) answering these questions is good practice for the exams. We will email feedback to the email address associated with the account you used to handin the assignment.

Questions

Try to answer these on your own but if you work with someone or use an outside source you must acknowledge them in your write-up.

1. (from Problem 7.28 Algorithms by Dasgupta, Papadimitriou, Vazirani) *A linear program for shortest path.* Suppose we want to compute the shortest path from node s to node t in a directed graph with edge lengths $\ell_e > 0$.

   (a) Prove that this is equivalent to finding a flow f from s to t that minimizes $\sum_e \ell_e f_e$ subject to size$(f) = 1$. There are no capacity contraints.

   (b) Write the shortest path problem as a linear program.

   (c) Explain why the dual LP can be written as

$$\max x_s - x_t$$
$$x_u - x_v \leq \ell_{uv} \text{ for all } (u, v) \in E$$

2. (from Lecture 26 Problem 3 in Jeff Erickson's on-line notes)

   (a) Give a linear-programming formulation of the maximum bipartite matching problem. The input is a bipartite graph $G = (U \cup V, E)$, where $E \subseteq U \times V$; the output is the largest matching in G. Your linear program should have one variable for each edge.

   (b) Now dualize the linear program from part (a). What do the dual variables represent? What does the objective function represent? Optional (if you know): What problem is this?

3. Each of two players, *Row* and *Col*, hides a nickel (5 cents) or a dime (10 cents). If the two coins match then *Row* gets both; if they don't match then *Col* gets both. Show the payoff matrix. (The payoff is the *additional* money that player *Row* gets, which is negative if *Col* collects both coins.) Describe the optimal strategy for *Row* using a linear program. What is the optimal strategy? (If you need to, you may use any simplex implementation you like. There are several on the Web.) What is the optimal strategy if instead of a nickel or a dime the players hide a a-cent coin or a b-cent coin?

4. Players *Row* and *Col* each choose an integer x and y between 1 and n (inclusive). If $x < y - 1$ or $x = y + 1$, player *Row* wins. If $x > y + 1$ or $x = y - 1$, player *Col* wins. If $x = y$, no one wins. What is a *smallest* optimal strategy for player *Row* and why is it optimal for $n = 2$? for $n = 3$? for $n > 3$?

   A *smallest* optimal strategy is an optimal strategy that places positive probability on the fewest number of choices.