CPSC 420+500 | Problem Set 1 | 6 Jan 2017

W. Evans | Due: ~~13~~ 16 Jan 2017 at 21.00 (9pm)

Submission Instructions

Handin your solutions using `handin`. You can write your solutions by hand and scan the pages or take pictures of them with your phone; or use a word processing package to typeset your solutions. Whatever you do, you should produce **pdf** files **assign1Q1.pdf**, **assign1Q2.pdf**, etc. containing your solutions for Question 1, Question 2, etc.

To handin: Copy your solution files to the directory ∼`/cs420/a1` in your home directory on a CS undergraduate machine. (You may have to create this directory using `mkdir` ∼`/cs420/a1`.) Then run $\boxed{\texttt{handin cs420 a1}}$ from your home directory.

If you do not have a CS department account, you can email your assign1.pdf to me `will@cs.ubc.ca`.

Late submissions are not accepted.

Grading Policy

We will grade a subset of these questions of size at least two. It's a good idea to do all of the questions because (1) you don't know which ones we'll grade and (2) answering these questions is good practice for the exams. We will email feedback to the email address associated with the account you used to handin the assignment.

Questions

Try to answer these on your own but if you work with someone or use an outside source you must acknowledge them in your write-up.

1. I have a jar that contains one liter of oil and another jar with one liter of vinegar. I take one tablespoon of oil from the oil jar and add it to the vinegar jar. (Don't worry. Nothing spills.) I then mix the oil with the vinegar (I might not do a very good mixing job) and transfer one tablespoon of the mixture back to the oil jar. What is true about the fraction of oil in the oil jar compared to the fraction of vinegar in the vinegar jar? and why?

2. (from Problem 4 in 3.2.3. Exercises from "Computational Geometry in C" by J. O'Rourke) Affine hulls. An affine combination of points $p_1, \ldots, p_n$ is a sum of the form $\alpha_1 p_1 + \cdots + \alpha_n p_n$, with $\alpha_1 + \cdots + \alpha_n = 1$. Note this differs from the definition of a convex combination in that the condition $\alpha_i \geq 0$ is dropped. What is the affine hull (the set of possible affine combinations) of two points in the plane? Three points? $n > 3$ points? What is the affine hull of two points in three dimensional space? Three points? Four points? $n > 4$ points?

3. (from Problem 3 in 3.2.3. Exercises from "Computational Geometry in C" by J. O'Rourke) Min supporting line. Design an algorithm that, given a set of points $P = \{p_1, p_2, \ldots, p_n\}$ (where $p_i = (x_i, y_i)$ is a point in the plane), finds a line L that
   (a) has all the points of P on one side, and
   (b) minimizes the sum of the distances of the points in P to L.
   (The distance of a point p_i to L is the length of the line segment, perpendicular to L,

from p_i to L. If $ax + by + c = 0$ is the line equation for L. Then

$$d(p_i, L) = \frac{|ax_i + by_i + c|}{\sqrt{a^2 + b^2}}$$

is the distance from p_i to L.)

You may assume that you are given the convex hull of P for free (as a circular doubly-linked list of its boundary vertices). Your algorithm should run in time $O(n)$. Hint: Consider the centroid $\bar{p}$ of P, which is

$$\bar{p} = \left(\frac{1}{n} \sum_{i=1}^{n} x_i, \frac{1}{n} \sum_{i=1}^{n} y_i \right).$$

4. Let P and Q be two convex hulls (represented, say, as circular, doubly-linked lists of their vertices in counter-clockwise order) with m and n vertices, respectively. Design an $O(m + n)$ time algorithm for constructing the convex hull of $P \cup Q$ (the union of the vertices in P and the vertices in Q). Does your algorithm handle degenerate cases?

5. The diameter of a set of points $P = \{p_1, p_2, \ldots, p_n\}$ is the largest distance between any two points in the set. Prove that the diameter of P is achieved by two points on the boundary of the convex hull of P.