THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 418: MIDTERM EXAMINATION – February 28, 2018

Full Name: _____

Signature: _____ UBC Student #: |_|_|_|_|_|_|_|_|

Important notes about this examination

1. You have 45 minutes to complete this exam.
2. Open book exam: Any printed material allowed. No sharing between students.
3. No communication devices allowed: no cell-phones, computers, tablets, etc.
4. A calculator is allowed, provided is has no communication function.
5. Answer all questions in the space provided. Give short but precise answers. Point form is fine.
6. Marks for each question are specified. Use this to manage you time.

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
   iii. purposely viewing the written papers of other examination candidates;
   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Please do not write in this space:

Question 1: |_|_|

Question 2: |_|_|

Question 3: |_|_|

Question 4: |_|_|

Question 5: |_|_|

Question 6: |_|_|

Question 7: |_|_|

Question 8: |_|_|

Question 9: |_|_|

Question 10: |_|_|

1 2 3 6 4 1 3 0 4 8 9 2 9

CpSc 418 **Midterm** February 28, 2017

Graded out of **83 points**.

Five questions.

1. **Erlang** (14 points)

   (a) (**6 points**) Let `V` be list of numbers – `V` represents a vector. Let `J` be an index for an element of `V`; in other words, `J` is an integer with `1 =< J` and `J =< length(V)`. Complete the implementation of function `q(J, V)` below that returns the square of the J^{th} element of `V` divided by the sum of the squares of all of the elements of `V`.

```
% q(J, V) -> (J^th element of V)^2 / sum_{J=1}^{length(V)} (J^th element of V)^2
% examples:
%   q(2, [1, 7, 4, 2]) -> 7*7/(1*1 + 7*7 + 4*4 + 2*2)
%                      -> 49/(1 + 49 + 16 + 4)
%                      -> 49/70
%                      -> 0.7
q(J, V) -> % write your answer below
```

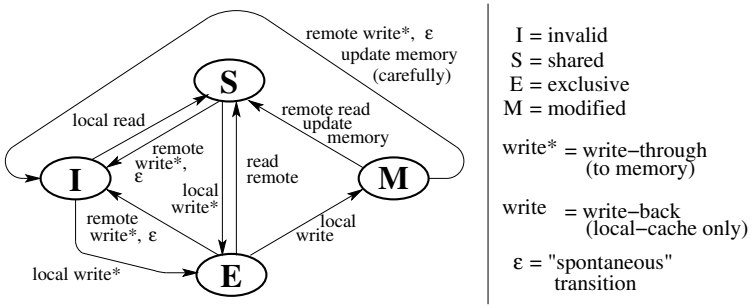
   Hint: To keep this simple for the exam, you don't need to add guards to check for invalid inputs.

   (b) (**6 points**) The function `lists:max(List)` in the Erlang standard library returns the largest element of `List`. Sometimes, we also want to know the position of this element. Complete the function `maxi` below:

```
% maxi(List) -> {Max, Index} where:
%   List is a list.
%   Max is the largest element of List
%   Index is the index of this largest element.
maxi([Hd | Tl]) -> maxi(Tl, {Hd, 1, 2}).
maxi([Hd | Tl], {Max, Index, CurPos}) when Hd > Max ->

  ____________________________________________.

maxi([Hd | Tl], {Max, Index, CurPos}) when Hd =< Max ->

  ____________________________________________.

maxi([], {Max, Index, CurPos}) ->

  ____________________________________________.
```

   (c) (**2 points**) Is the `maxi` function that you completed above head-recursive or tail-recursive?

   ☐ head-recursive ☐ tail-recursive



Action	Memory	Cache 0	Cache 1
	?	I	I
P0: write 3	3	E3	I
P0: write 5			
P1: read			
P0: read		S5	
	42	I	

Figure 1: MESI example for Question 2a

2. **Architecture** (14 points)

   (a) (**10 points**) Figure 1 shows the state-transition diagram for the MESI protocol. **Complete the table.**

   The table shows a sequence of reads and writes by two processors, P0 and P1. All reads and writes in the table are to the same address. Each row of the table shows the action taken, the value in main memory, and the value and state of the cache line in each cache. The notation "E3" means the cache line is in the E state and has the value 3. If the action is a write, the value that was written is given in the action. For example, "P0: write 5" means that processor P0 writes the value 5.

   Note that the last line shows the value in Memory and the state of Cache 0. For that line, you need to figure out what processor made what action and what the state is for Cache 1 (and the value if the state is not I).

   (b) (**2 points**) I'm planning to build a message-passing parallel computer with $2^{21} \approx 2$ million processors. I'm trying to decide whether to use a 3D-torus architecture or a hypercube for the interconnect between the processors. State **one** advantage of using a hypercube architecture. Answers which state more than one advantage will receive 0.

   (c) (**2 points**) For the same parallel computer with 2^{21} processors, state **one** advantage of using a 3D-torus architecture. Answers which state more than one advantage will receive 0.

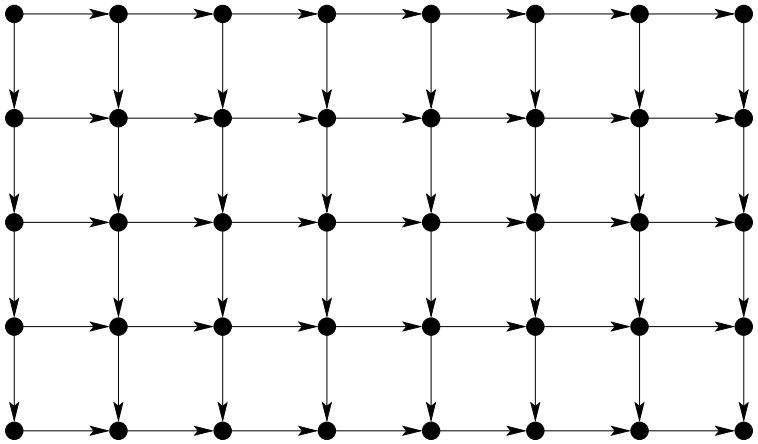


Figure 2: Work-Span Graph for Question 3

3. **Performance** (19 points)
Figure 2 shows a directed, acyclic graph for the work-span model. Vertices represent tasks, assume that all tasks execute in unit time. Edges represent dependencies – a task can execute once all of the tasks for its incoming edges have completed.

   (a) (**2 points**) Circle the task that must be performed first, and write "first" next to the circle.

   (b) (**2 points**) Circle the task that must be performed last, and write "last" next to the circle.

   (c) (**2 points**) What is the "work" for the computation depicted by the graph in Figure 2? Your answer should be a positive integer.

   _____

   (d) (**2 points**) What is the "span" for the computation depicted by the graph in Figure 2? Your answer should be a positive integer.

   _____

   (e) (**3 points**) What is the maximum possible speed-up for the computation depicted by the graph in Figure 2?

   _____

   (f) (**2 points**) Can the work-span model be used to derive upper bounds for speed-up (yes or no)?
   ☐ yes ☐ no

   (g) (**2 points**) Can the work-span model be used to derive lower bounds for speed-up (yes or no)?
   ☐ yes ☐ no

   (h) (**2 points**) Does the work-span model account for communication delays (yes or no)?
   ☐ yes ☐ no

   (i) (**2 points**) Which of the following is most often associated with Gordon Moore (check one)?
   ☐ Amdahl's Law ☐ Denard Scaling ☐ the golden rule
   ☐ Gustsfson's Law ☐ Little's formula ☐ Moore's Law
   ☐ the work-span model

4. **Sorting** (16 points)
In class, Mark described a "N-merge" that has inputs `x` and `y` and outputs `lo` and `hi`. Each of these is a sorted list (or array) of N elements. The merge element merges the lists `x` and `y` to produce a list, `z` of length $2N$. The smallest N elements of `z` are output as `lo`, and the greatest N elements are output as `hi`.

(a) (**2 points**) How long does it take to perform a N-merge on lists of arbitrary integers (e.g. a C `int`) using a sequential computer such as an x86? Your answer should be something like $\mathcal{O}(1)$, $\mathcal{O}(\log N)$, $\mathcal{O}(N)$, or $\mathcal{O}(N^2)$.

(b) (**3 points**) **Fill in the upper right part of the Figure 3** with the values for the lists `lo` and `hi`. We provided the upper-left part of the figure as an example. To get you started, we've provided the values for the smallest value of `lo` and the largest value of `hi`.

For questions 4c and 4d we consider N-merge elements where the inputs and outputs are lists consisting only of 0s and 1s. With this assumption, we can represent each input or output of a N-merge with an integer k, where k is the number of 1s in the list. When we use integers instead of lists, we call the operation a N-CountMerge. The lower left part of Figure 3 shows the example from the upper left, where the lists are represented as integers.

(c) (**3 points**) **Fill in the lower right part of Figure 3** with the integer values corresponding to the 6-Merge from the upper right part of the figure (i.e. from Question 4b). To get you started, we've provided the value for `y`.

(d) (**8 points**) In class, we showed that the outputs of a N-CountMerge are given by:

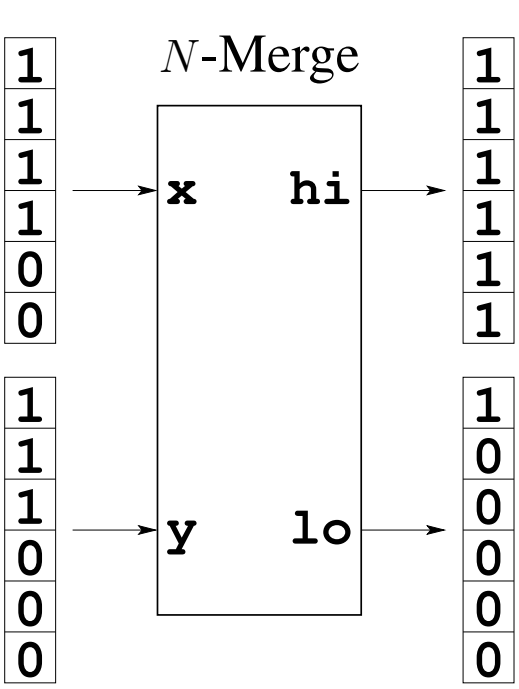
$$\begin{aligned} \texttt{hi} &= \min(\texttt{x} + \texttt{y}, N) \\ \texttt{lo} &= \max((\texttt{x} + \texttt{y}) - N, 0) \end{aligned}$$

We say that an input or output of a N-CountMerge is "clean" if its value is either 0 or N. Conversely, the value is "dirty" if it is greater than 0 and less than N. Prove that for any inputs `x` and `y` to a N-CountMerge, at least one of the outputs, `lo` or `hi` must be clean. My proof has two cases. I've provided the first case; you need to complete the second case.

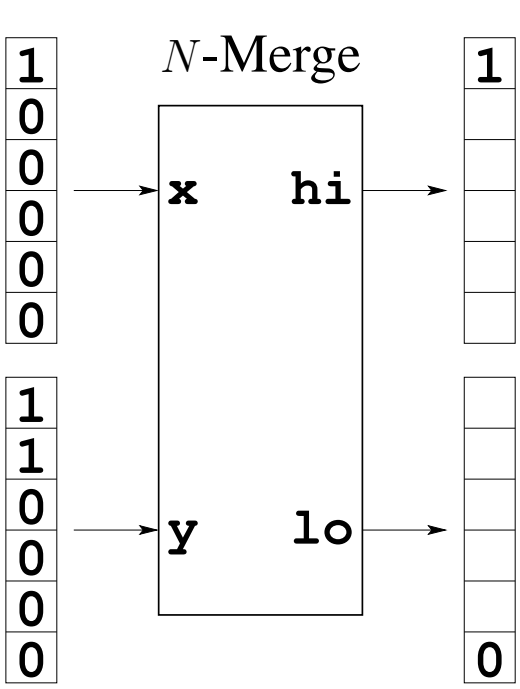
Proof:

case $\texttt{x} + \texttt{y} \leq N$:
Subtracting N from both sides, we get $(\texttt{x} + \texttt{y}) - N \leq 0$.
Thus, $\texttt{lo} = \max((\texttt{x} + \texttt{y}) - N, 0) = 0$.
For this case, `lo` is clean.

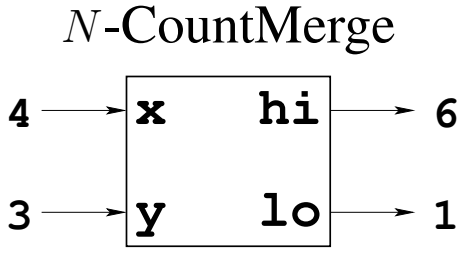
case $\texttt{x} + \texttt{y} > N$:



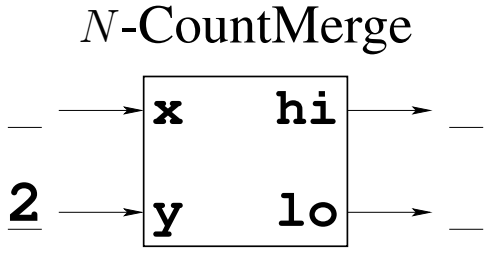
Example for Question 4b



Question 4b



Example for Question 4c



Question 4c

Figure 3: Merge Networks

5. **Reduce** (20 points)
   Implement a parallel version of `maxi` using `wtree:reduce`. There are four parts to the problem:

   (a) (**2 points**) Choose a data structure to be returned by the leaf function and used by the combine and root functions. Your choices are:

   ☐ `Max`, the largest value in the segment of the list for the subtree rooted at this node.

   ☐ {`Max, Index`}, where `Max` is the largest value in the segment, and `Index` is the index of the largest value.

   ☐ {`Left, Max, Index, Right`}, where `Max` and `Index` are as defined for options i and ii; `Left` is the first element of the segment; and `Right` is the last element of the segment.

   ☐ {`Max, Index, Length`}, where `Max` and `Index` are as defined for options i and ii, and `Length` is the length of the segment.

   (b) (**8 points**) Implement the function `maxi_leaf` in Figure 4 below. You may use the `maxi` function as specified in Question 1b.

   (c) (**8 points**) Implement the `maxi_combine` function in Figure 5 on page 8.

   (d) (**2 points**) Implement the `maxi_root` function in Figure 5 on page 8.

```
maxi_par(W, Key) ->
  wtree:reduce(W,
    fun(ProcState) -> maxi_leaf(workers:get(ProcState, Key)) end, % Leaf
    fun(Left, Right) -> maxi_combine(Left, Right) end,            % Combine
    fun(RootTally) -> maxi_root(RootTally) end,                   % Root
  ).

% Question 5b (8 points)
%     Complete the function maxi_leaf below.
%     For simplicity, you may assume that List is a non-empty list.
maxi_leaf(List) ->
```

Figure 4: Code for Question 5 (part 1)

```
% Question 5c (8 points)
%     Write the function maxi_combine.
```

```
% Question 5d (2 points)
%     Write the function maxi_root.
```

Figure 5: Code for Question 5 (part 2)