1. (1 pt) Write down the 4x4 matrix for rotating an object by 90° around the z axis.

\[
\begin{pmatrix}
\cos 90° & -\sin 90° & 0 & 0 \\
\sin 90° & \cos 90° & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} =
\begin{pmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

2. (2 pts) Describe in words what this matrix does (be specific about the order of operations)

\[
\begin{pmatrix}
.707 & 0 & .707 & 0 \\
0 & 2 & 0 & 0 \\
-.707 & 0 & .707 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} =
\begin{pmatrix}
.707 & 0 & .707 & 0 \\
0 & 1 & 0 & 0 \\
-.707 & 0 & .707 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

OR \[
\begin{pmatrix}
1 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

scale in y by 2, then rotate (45°, 90°)

rotate (45°, 90°), then scale in y by 2

3. (1 pt) Draw a picture of the object below transformed by the above matrix

![Transformed Object Diagram]

4. (1 pt) Give sequence of OpenGL commands necessary to implement the above transformation.

\[
glRotatef(45, 0, 1, 0) \\
glScalef(1, 2, 1)
\]
5. (2 pts) Draw houseA and houseB, transformed by the appropriate OpenGL commands. The untransformed house is below.

```c
glIdentity();
glTranslatef(1, 0, 0);
glRotate(90, 0, 0, 1);
glPushMatrix();
glTranslatef(0, 2, 0);
drawHouseA();
glPopMatrix();
glTranslatef(-1, 0, 0);
drawHouseB();
```

6. (1 pt) Give the series of affine transformations (assuming postmultiplying) needed to create the picture below, assuming the house started from the position shown in the above questions.

```c
glTranslatef(3, 2, 0)
glRotate(45, 0, 0, 1)
glScale(\sqrt{2}, \sqrt{2}, 1)
```
7. (1 pt) The point coordinate \( P \), as shown below to the right, can be thought of as \( P = 1*i + 3*j \), where \( i \) and \( j \) are basis vectors of unit length along the \( x \) and \( y \) axes, respectively. In effect, a coordinate system is defined by the location of its origin, and its basis vectors. Describe the point \( P \) in terms of the 3 other coordinate systems given below.

\[
\begin{align*}
P &= -2A_i + 2A_j \\
P &= -1B_i + 1.5B_j \\
P &= 0.5C_i + 1.25C_j - 1/2
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2} &= a(0.5) + b(-1) + c(2) \\
1 &= 0.5a - b + 2 \\
3 &= -a - 2b + 6 \\
0.5a - b &= -1 \\
a - 2b &= -3 \\
-a - 2b &= -3 \\
2a &= 1 \\
a &= \frac{1}{2} \\
b &= \frac{5}{2} \\
l &= -\frac{1}{2} - 2b = -3
\end{align*}
\]

8. (1 pt) Normalize the homogenous point \((2,4,6,2)\).

\[
(1,2,3,1) = \left( \frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{2}{2} \right)
\]

9. (1 pt) Draw the cavalier projection of a box of size \( x=4 \), \( y=2 \), \( z=6 \). Use a \( 20^\circ \) projection (that is, the \( z \) axis in the scene should make a \( 20^\circ \) angle with the \( x \) axis in the projection). The drawing doesn’t have to be exactly to scale, but you must label the point locations.
10. (2 pts) Derive a 4x4 matrix that when applied to the point \((x, y, z, 1)^T\) would result in the projection in the picture below. Show your work.

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
d
\end{pmatrix} =
\begin{pmatrix}
x' \\
y' \\
\frac{z}{d} \\
1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\frac{x'd}{z} \\
\frac{y'd}{z} \\
\frac{z}{d} \\
1
\end{pmatrix} =
\begin{pmatrix}
x' \\
y' \\
1 \\
1
\end{pmatrix}
\]

11. (1 pt) Sketch a side view (yz plane) of the perspective view frustum, in VCS, that is specified by the following parameters:
accer = 3, top = 2, right = 1, far = 5, bot = -1, left = -1

12. (1 pt) Write out the OpenGL perspective transformation matrix for the above configuration.

\[
\begin{pmatrix}
\frac{6}{2} & 0 & 0 & 0 \\
0 & \frac{6}{3} & 1 & 0 \\
0 & 0 & -\frac{8}{2} & -3c \\
0 & 0 & -1 & 0
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 2 & 1 & 0 \\
0 & 0 & -4 & -16 \\
0 & 0 & -1 & 0
\end{pmatrix}
\]
13. (1 pt) Briefly describe how to implement per-object picking using the back buffer

STORE A UNIQUE COLOR FOR EACH OBJECT (PICKABLE) IN SCENE IN A TABLE. RENDER SCENE TO BACK BUFFER WITH SHADING TURNED OFF. READ BACK PIXEL AT CURSOR LOCATION AND CHECK AGAINST TABLE.

14. (1 pt) A point in a triangle can be expressed using barycentric coordinates as follows: $P = \alpha P_1 + \beta P_2 + \gamma P_3$, where $0 \leq \alpha, \beta, \gamma \leq 1$ and $-\alpha + \beta + \gamma = 1$. Draw the line corresponding to $\alpha = .6$ on the following triangle which sits in the $xy$-plane.

![Triangle with barycentric coordinates](image)

15. (1 pt) Briefly describe how to use parity when scan converting a general polygon.

**Parity Test:**

SCAN ALONG EACH SCANLINE. ON ODD NUMBER OF EDGE CROSSINGS, RASTERIZE PIXELS. ON EVEN EDGE CROSSINGS STOP RASTERIZING.

**Special Cases:**

(i) COUNT CONCAVE (SPLIT) VERTICES TWICE (E.G., X)

(ii) DON'T RASTERIZE BOTTOM HORIZONTAL EDGE.
In the problems below, use the Phong illumination model given by
\[ I = I_a k_a + I_d l_d(N \cdot L) + I_v l_v(R \cdot V)^n \]
with parameters \( I_a = .8, I_d = 1.0, k_a = .2, k_d = .9, k_v = .5, n = 30 \).

\[ l_p = \left( -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]
\[ L = (1,2,0) \quad (5,2,0) \quad E \]
\[ l_c = \left( -\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]
\[ n_A = \left( -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right) \]
\[ u_A = \left( \frac{3}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]
\[ n_c = \left( \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right) \]
\[ u_c = \left( \frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]

16. (2 pts) Give the specular component of B, using the Gouraud shading model.
\[ R_A = 2 \times n_A \times (n_A \cdot l_A) - l_A = \left( -\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \qquad \because R_A \cdot l_A = 0 \]
\[ \therefore \text{SPECULAR}_A = k_A \times l_A \times (R_A \cdot l_A)^n = 0 \]
\[ R_c = 2 \times n_c \times (n_c \cdot l_c) - l_c = -l_c \left[ n_c \cdot l_c = 1 \right] \qquad \because R_c \cdot l_c = 0 \]
\[ \therefore \text{SPECULAR}_C = k_A \times l_A \times (R_c \cdot l_c)^n = 0 \]
\[ \therefore \text{SPECULAR}_B = \text{SPECULAR}_A + \text{SPECULAR}_C = 0 \]

17. (2 pts) Give the specular component of B, using the Phong shading model.
\[ n_B = \frac{n_A + n_c}{|n_A + n_c|} = (0, 1, 0) \]
\[ l_B = \left( -\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]
\[ u_B = \left( \frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]
\[ R_B = 2 \times n_B \times (n_B \cdot l_B) - l_B = \left( \frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right) \]
\[ \therefore R_B \cdot u_B = \left( \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{5}} + 0 \right) = 1.0 \]
\[ \therefore \text{SPECULAR}_B = 0.5 \times 1.0 \times 1.0 = 0.5 \]