

[12] 1. Short Answers

- [3] a. Mr. Isulo, the well-known alien computer scientist, draws a decision tree for the `MarvinSort` algorithm for  $n = 10$  and shows it to you. What information can you infer from this tree (even though you have never heard of `MarvinSort`)?
- [3] b. Is the Gale-Shapley algorithm greedy? Explain why or why not.
- [3] c. Is  $n/100 \in o(n)$ ? Explain why or why not.

[5] 2. A computer scientist who believes both genders are created equal wants to modify the Gale-Shapley algorithm as follows:

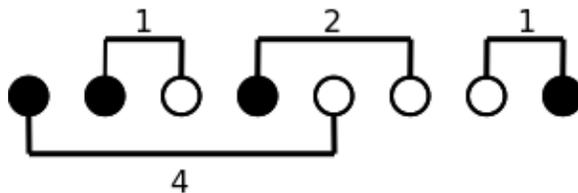
- In each iteration, either a free man or a free woman proposes to the first person on his/her preference list that he/she has not yet proposed to.
- The algorithm terminates when no free man or woman remains.

Give an example that proves that the matching produced by this modified version of the algorithm is not necessarily stable.

[11] 3. Consider the following problem: there are  $2n$  dots that are equally spaced on a line. Each dot is either white or black, and there are  $n$  dots of each color. The dots of a given color are not necessarily together; colors are interleaved arbitrarily. For instance:



We want to match each black dot with a white dot, while minimizing the sum of the distances between the elements of each pair. For instance, if we pair the dots of the previous figure as follows:



then the sum of the distances is  $4 + 1 + 2 + 1 = 8$ .

- [8] a. Describe a greedy algorithm to find the matching that minimizes the sum of the distances between the elements of each pair. The input to your algorithm should be an array  $C$  of colors; for the example we would have  $C[0] = \text{black}$ ,  $C[1] = \text{black}$ ,  $C[2] = \text{white}$ , etc.
- [3] b. Analyze the time complexity of your algorithm from part (a).