1. Culling

The solid objects below are depicted in VCS, and the labels correspond to faces.

(a) (2 points) In alphabetical order, list the faces that would be removed by back-face culling. Do not take possible view-frustum culling or clipping into account.

\[ b, c, e, f, h, i, m, n, o, q \]

(b) (2 points) In alphabetical order, list the faces that would be removed by view-frustum culling. Do not take possible back-face culling into account.

\[ b \]

(c) (2 points) After view-frustum culling, clipping, and back-face culling, which remaining faces would be completely removed by z-buffer tests? List these in alphabetical order.

\[ d, f, j, k, l, p \]
2. (6 points) Scan Conversion

Write pseudocode for scan converting the shape illustrated below, i.e., a half circle, with circle center at \((x_c, y_c)\), radius \(r\), and with a bottom edge of slope 1, i.e., 45 degrees. Use implicit equations to develop your solution. Assume that a pixel \((x, y)\) is set using \(\text{Image}[x][y]=1\), and that the image spans from \((0,0)\) in the bottom left to \((w-1,h-1)\) in the top right, giving an image of \(w \times h\) pixels. If part of the shape is located off-screen, the remainder should still be properly rendered. The result should be correct for any values of \(x_c, y_c,\) and \(r\).

\[
\begin{align*}
x_{\min} &= \max(0, x_c - r) \\
x_{\max} &= \min(w-1, x_c + r\sqrt{2}) \\
y_{\min} &= \max(0, y_c - r\sqrt{2}) \\
y_{\max} &= \min(h-1, y_c + r) \\
\text{for} \ (x = x_{\min} ; x \leq x_{\max} ; x++) \{ \\
\quad \text{for} \ (y = y_{\min} ; y \leq y_{\max} ; y++) \{ \\
\quad \quad F_1 = (y-y_c) - (x-x_c) \\
\quad \quad F_2 = r^2 - (x-x_c)^2 - (y-y_c)^2 \\
\quad \quad \text{if} \ (F_1 \geq 0 \And F_2 \geq 0) \\
\quad \quad \quad \text{Image}[x][y] = 1
\end{align*}
\]
3. Barycentric Coordinates

Assume that the barycentric coordinates are defined according to 
\[ P = \alpha P_1 + \beta P_2 + \gamma P_3. \]
A given triangle is defined by \( P_1(20, 0), P_2(60, 10), P_3(30, 40) \).

(a) (2 points) On the diagram above, sketch the triangle and label the vertices with their barycentric coordinates, e.g., \( P_1(\alpha, \beta, \gamma) \).

(b) (1 point) Sketch the three lines that correspond to \( \alpha = 0, \alpha = 0.5, \alpha = 1 \).

(c) (1 point) Give an explicit equation for the line that corresponds to \( \alpha = 0 \). Then give an implicit equation for the same line.

\[ \frac{y - 70}{-x} = \frac{60 - 10}{10} \]

(d) (2 points) Develop an expression for alpha, i.e., \( \alpha = f(x, y) \).

\[ \alpha = \frac{F_{23}(x, y)}{F_{23}(x_1, y_1)} = \frac{70 - x - y}{70 - 20 - 0} = \frac{70 - x - y}{50} \]

(e) (2 points) Use barycentric interpolation to compute a value \( v \) at a point defined by \( \alpha = 0.5, \beta = 0.3, \) if the value of this quantity at the three vertices is given by:

\[ v_1 = 1, v_2 = 2, v_3 = 3. \]

\[ y = 1 - \alpha - \beta = 0.2 \]

\[ v = \alpha v_1 + \beta v_2 + \gamma v_3 = (0.5)(1) + (0.3)(2) + (0.2)(3) = 1.7 \]
4. Texture Mapping

(a) (4 points) Consider the texture map below, which is to be mapped to Objects A and B. Assume that the RepeatWrapping texture mode is being used.

(i) In the Object A diagram above, sketch the image that would appear for Object A for the assigned texture coordinates.

(ii) In the Object B diagram above, assign texture coordinates to Object B so that it would yield the given image.

(b) (2 points) A square is texture mapped with a checkerboard texture, and rendered using two triangles. Sketch what the texture would look like for (i) linearly-interpolated texture coordinates, and (ii) perspective-correct interpolated texture coordinates.
5. Short Answer

(a) (1 point) **True** 
Back-face culling can be computed in NDCS.

(b) (1 point) **True** 
Clipping can be computed in NDCS.

(c) (1 point) **False** 
Barycentric coordinates can be interpreted as fractional areas.

(d) (1 point) **Varying** variables are used to interpolate values from the vertices that are then passed onto the fragment shader.

(e) (1 point) The output of the vertex shader is in the **clipping** coordinate system.

(f) (1 point) Guest lecture: What are examples of artifacts are created by the basic “linear blend skinning” method? What types of effects can more advanced skinning methods achieve?

   artifacts: collapsing joints, no muscle bulging, interpenetrating joints
   better methods: sliding skin, muscle shapes, skin folds

6. (5 points) Line-Plane Intersection

Describe how to compute the intersection between a line defined by two points \( P_A P_B \) and a plane that embeds a triangle \( P_1 P_2 P_3 \). Then compute the point where the line \( P_A(0, 1, 2) P_B(6, 7, 8) \) intersects the plane that embeds the triangle \( P_1(5, 0, 0) P_2(0, 5, 0) P_3(0, 0, 5) \).

**Line:** \( \mathbf{P}(t) = (1-t) \mathbf{P}_A + t \mathbf{P}_B = \mathbf{P}_A + t(\mathbf{P}_B - \mathbf{P}_A) \)

**Plane:**
\[
\mathbf{N} = (\mathbf{P}_3 - \mathbf{P}_1) \times (\mathbf{P}_2 - \mathbf{P}_1) \\
0 = \mathbf{N} \cdot \mathbf{P} + \mathbf{D} = -\mathbf{N} \cdot \mathbf{P}_1
\]

Implicit plane equation:
\[
\mathcal{F}(\mathbf{P}) = \mathbf{N} \cdot \mathbf{P} + \mathbf{D} = 0
\]

One possibility among several:
\[
\mathbf{N} = <1, 1, 1> \\
\mathbf{D} = -\mathbf{N} \cdot (5, 0, 0) = -5
\]

\[
\mathcal{F}(\mathbf{P}) = x + y + z - 5
\]

\[
d_A = a + b + 2 - 5 = -2 \\
d_B = 6 + 7 + 8 - 5 = 16
\]

\[
t = \frac{2}{18} = \frac{1}{9}
\]

\[
\mathbf{P} = \left[ \begin{array}{c} 0 \\ \frac{1}{9} \\ \frac{2}{9} \end{array} \right] + \frac{1}{9} \left[ \begin{array}{c} 6 \\ 15 \\ 2 \end{array} \right] = \left[ \begin{array}{c} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{array} \right]
\]