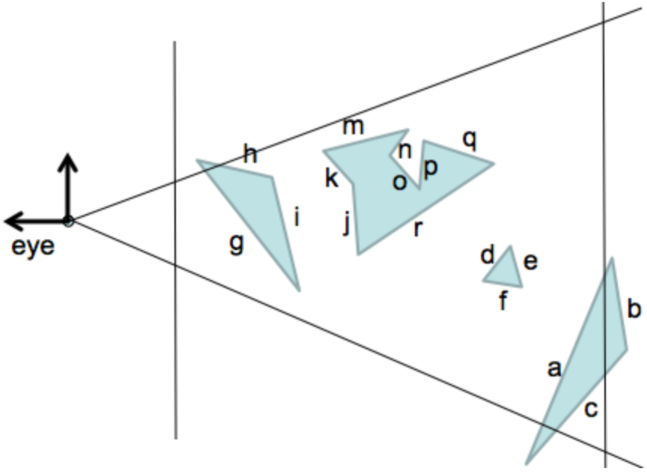
This midterm has 6 questions, for a total of 37 points.

1. Culling

   The solid objects below are depicted in VCS, and the labels correspond to faces.



   (a) (2 points) In alphabetical order, list the faces that would be removed by back-face culling. Do not take possible view-frustum culling or clipping into account.

   b, c, e, (f), h, i, (m), n, o, q

   (b) (2 points) In alphabetical order, list the faces that would be removed by view-frustum culling. Do not take possible back-face culling into account.

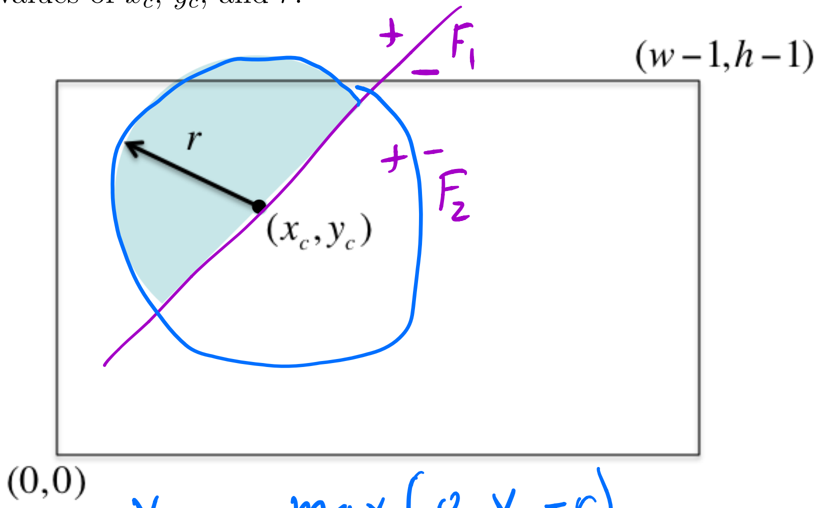
   b

   (c) (2 points) After view-frustum culling, clipping, and back-face culling, which remaining faces would be completely removed by z-buffer tests? List these in alphabetical order.

   d, (f), j, k, (m), p

2. (6 points) Scan Conversion

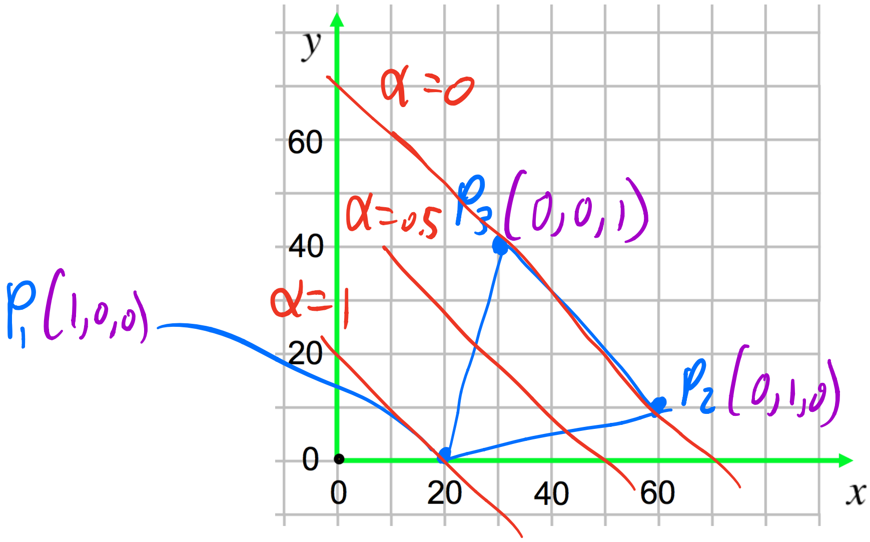
Write pseudocode for scan converting the shape illustrated below, i.e., a half circle, with circle center at (x_c, y_c), radius r, and with a bottom edge of slope 1, i.e., 45 degrees. Use implicit equations to develop your solution. Assume that a pixel `(x,y)` is set using `Image[x][y]=1`, and that the image spans from `(0,0)` in the bottom left to `(w-1,h-1)` in the top right, giving an image of $w \times h$ pixels. If part of the shape is located off-screen, the remainder should still be properly rendered. The result should be correct for any values of x_c, y_c, and r.



```
$X_{min} = max(0, x_c - r)$
$X_{max} = min(w-1, x_c + r/\sqrt{2})$
$y_{min} = max(0, y_c - r/\sqrt{2})$
$y_{max} = min(h-1, y_c + r)$
for ($x = x_{min}$; $x \leq X_{max}$; x++) {
    for ($y = y_{min}$; $y \leq y_{max}$; y++) {
        $F_1 = (y - y_c) - (x - x_c)$
        $F_2 = r^2 - (x - x_c)^2 - (y - y_c)^2$
        if ($F_1 \geq 0$ && $F_2 \geq 0$)
            Image[x][y] = 1
    }
}
```

3. Barycentric Coordinates

Assume that the barycentric coordinates are defined according to $P = \alpha P_1 + \beta P_2 + \gamma P_3$. A given triangle is defined by $P_1(20, 0)$, $P_2(60, 10)$, $P_3(30, 40)$.



(a) (2 points) On the diagram above, sketch the triangle and label the vertices with their barycentric coordinates, e.g., $P_1(\alpha, \beta, \gamma)$.

(b) (1 point) Sketch the three lines that correspond to $\alpha = 0, \alpha = 0.5, \alpha = 1$.

(c) (1 point) Give an explicit equation for the line that corresponds to $\alpha = 0$. Then give an implicit equation for the same line.

$\boxed{y = 70 - x}$

$\boxed{F_{23}(x,y) = 0 = 70 - x - y}$

slope $\overline{P_2P_3} = \frac{\Delta y}{\Delta x} = \frac{-30}{30} = -1$

$y = -x + b \Rightarrow b = y + x$
$= 60 + 10 = 70$

(d) (2 points) Develop an expression for *alpha*, i.e., $\alpha = f(x, y)$.

$$\alpha = \hat{F}_{23}(x,y) = \frac{F_{23}(x,y)}{F_{23}(x_1,y_1)} = \frac{70-x-y}{70-20-0} = \boxed{\frac{70-x-y}{50} = \alpha}$$

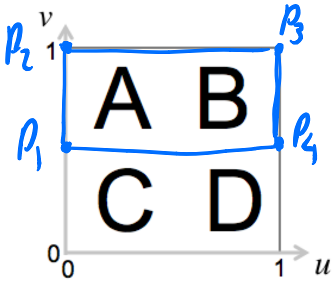
(e) (2 points) Use barycentric interpolation to compute a value v at a point defined by $\alpha = 0.5, \beta = 0.3$, if the value of this quantity at the three vertices is given by: $v_1 = 1$, $v_2 = 2$, $v_3 = 3$.

$\gamma = 1 - \alpha - \beta = 0.2$

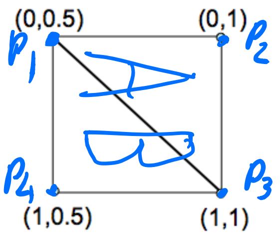
$$v = \alpha v_1 + \beta v_2 + \gamma v_3$$
$$= (0.5)(1) + (0.3)(2) + (0.2)(3) = \boxed{1.7}$$

4. Texture Mapping

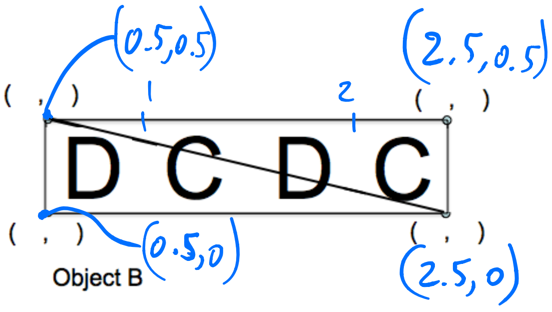
   (a) (4 points) Consider the texture map below, which is to be mapped to Objects A and B. Assume that the RepeatWrapping texture mode is being used.



texture map



Object A,
with given (u,v) coords

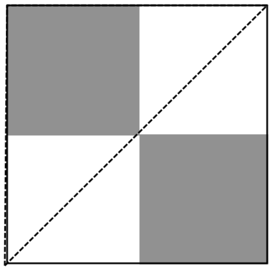


Object B

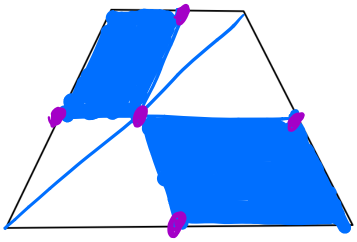
   (i) In the Object A diagram above, sketch the image that would appear for Object A for the assigned texture coordinates.
   (ii) In the Object B diagram above, assign texture coordinates to Object B so that it would yield the given image.

   (b) (2 points) A square is texture mapped with a checkerboard texture, and rendered using two triangles. Sketch what the texure would look like for (i) linearly-interpolated texture coordinates, and (ii) perspective-correct interpolated texture coordinates.

view from above



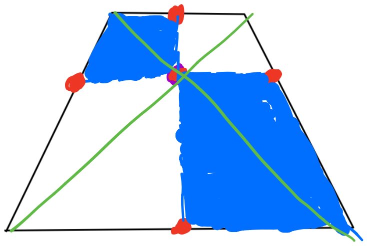
perspective view



linear interpolation

• midpoints in image space

perspective view



perspective-correct interpolation

• midpoints in world space

5. Short Answer

   (a) (1 point) [✓ True] False Back-face culling can be computed in NDCS.

   (b) (1 point) True [✓ False] Clipping can be computed in NDCS.

   (c) (1 point) [✓ True] False Barycentric coordinates can be interpreted as fractional areas.

   (d) (1 point) Varying variables are used to interpolate values from the vertices that are then passed onto the fragment shader.

   (e) (1 point) The output of the vertex shader is in the Clipping (CCS) coordinate system.

   (f) (1 point) Guest lecture: What are examples of artifacts are created by the basic "linear blend skinning" method? What types of effects can more advanced skinning methods achieve?

   artifacts: collapsing joints, no muscle bulging, interpenetrating joints

   better methods: sliding skin, muscle shapes, skin folds

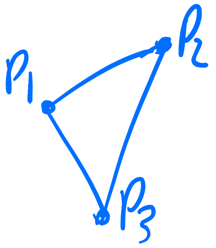
6. (5 points) Line-Plane Intersection

   Describe how to compute the intersection between a line defined by two points $P_A P_B$ and a plane that embeds a triangle $P_1 P_2 P_3$. Then compute the point where the line $P_A(0,1,2)P_B(6,7,8)$ intersects the plane that embeds the triangle $P_1(5,0,0)P_2(0,5,0)P_3(0,0,5)$.

Line: $P(t) = (1-t)P_A + tP_B = P_A + t(P_B - P_A)$

Plane: $N = (P_3 - P_1) \times (P_2 - P_1)$

one possibility among several

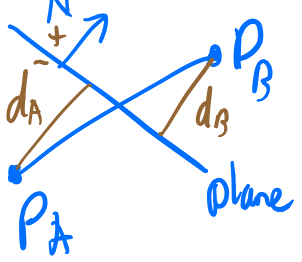


$0 = N \cdot P + D \Rightarrow D = -N \cdot P_1$

Implicit plane eq'n:

$F(P) = N \cdot P + D = d$

$t = \frac{-d_A}{-d_A + d_B}$ where $d_A = F(P_A)$, $d_B = F(P_B)$



$P = P_A + t(P_B - P_A)$

$N = \langle 1,1,1 \rangle$

$D = -N \cdot (5,0,0) = -5$

$F(P) = x + y + z - 5$

$d_A = 0 + 1 + 2 - 5 = -2$

$d_B = 6 + 7 + 8 - 5 = 16$

$t = \frac{2}{18} = \frac{1}{9}$

$$P = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + \frac{1}{9}\begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} = \boxed{\begin{bmatrix} 2/3 \\ 1\,2/3 \\ 2\,2/3 \end{bmatrix}}$$