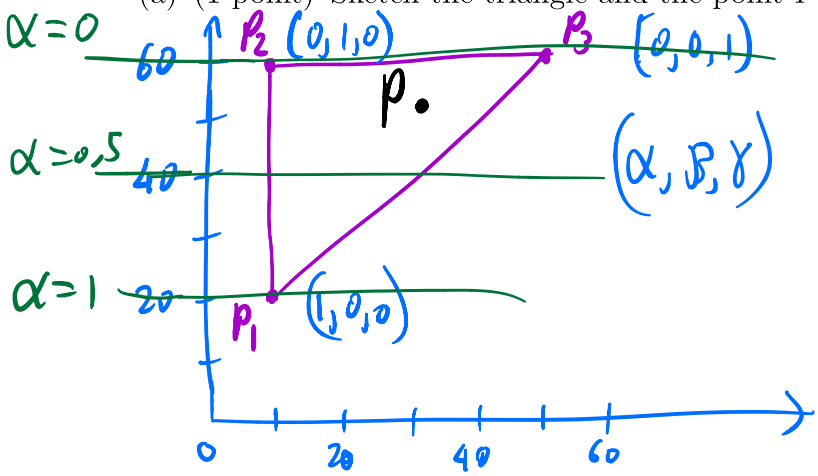
2. Barycentric coordinates and interpolation

A triangle has device coordinates $P_1(10, 20)$, $P_2(10, 60)$, $P_3(50, 60)$. We wish to be able to interpolate a value v for an arbitrary point $P(x, y)$ in the triangle, given the values at the vertices.

(a) (1 point) Sketch the triangle and the point $P(30, 50)$.



(b) (3 points) Write an explicit line equation for each of the three edges, i.e., $y = f(x)$ or $x = f(y)$.

$\overline{P_1P_2}$: $x = 10$

$\overline{P_2P_3}$: $y = 60$

$\overline{P_1P_3}$: $y = x + 10$

(c) (3 points) Rearrange the terms of each of these equations to trivially transform these into implicit line equations where $F(x, y) = 0$ for points on the line. Label these with the vertices that they pass through, i.e., $F_{12}(x, y)$, $F_{23}(x, y)$, and $F_{13}(x, y)$.

$F_{12}(x,y) = 0 = x - 10$ or $10 - x$

$F_{23}(x,y) = 0 = y - 60$ or $60 - y$

$F_{13}(x,y) = 0 = y - x - 10$ or $-y + x + 10$

(d) (3 points) Give scaled implicit line equations, e.g., $\hat{F_{12}}(x, y)$, for each edge such that $\hat{F}(x, y) = 1$ at the third vertex, i.e., the vertex not on the line segment. Relate these expressions to the barycentric coordinates (α, β, γ) of a point $P(x, y)$, where $P = \alpha P_1 + \beta P_2 + \gamma P_3$.

$$\gamma = \hat{F}_{12}(x,y) = (x-10)/(x_3-10) = \left(\frac{1}{40}\right)x - \frac{1}{4}$$

$$\alpha = \hat{F}_{23}(x,y) = (y-60)/(y_1-60) = \left(\frac{-1}{40}\right)y + \frac{3}{2}$$

$$\beta = \hat{F}_{13}(x,y) = (y-x-10)/(y_2-x_2-10) = \left(-\frac{1}{40}\right)x + \left(\frac{1}{40}\right)y - \frac{1}{4}$$

(e) (1 point) Verify that $\alpha + \beta + \gamma = 1$ holds true for any point $P(x, y)$ by using your expressions.

$$\begin{array}{l|c|c|c}
\gamma: & \frac{1}{40}x & +0y & -\frac{1}{4} \\
\alpha: & 0x & -\frac{1}{40}y & +\frac{3}{2} \\
\beta: & -\frac{1}{40}x & +\frac{1}{40}y & -\frac{1}{4} \\
\hline
 & 0 & 0 & 1
\end{array}$$

$\gamma + \alpha + \beta = 1$

(f) (2 points) On your diagram above, label each of the vertices with their (α, β, γ) values.

(g) (2 points) Sketch and label lines corresponding to $\alpha = 0, \alpha = 0.5, \alpha = 1$ in the above diagram.

(h) (2 points) Compute the barycentric coordinates for $P(30, 50)$. Then use them to interpolate v for that point, given the following known values for v at the vertices: $v_1 = 10, v_2 = 20, v_3 = 60$.

$$\gamma = \frac{30}{40} - \frac{10}{40} = \frac{1}{2}$$

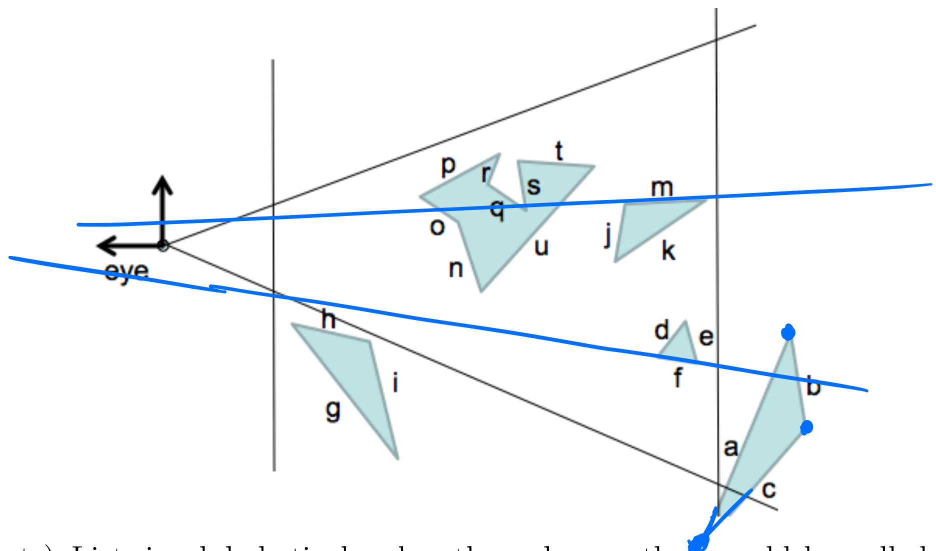
$$\alpha = 0 - \frac{50}{40} + \frac{60}{40} = \frac{1}{4}$$

$$\beta = -\frac{30}{40} + \frac{50}{40} - \frac{10}{40} = \frac{1}{4}$$

$$v = \alpha v_1 + \beta v_2 + \gamma v_3$$

$$= \frac{10}{4} + \frac{20}{4} + \frac{60}{2} = \frac{10 + 20 + 120}{4} = \frac{150}{4} = 37.5$$

3. Visibility and Culling
Consider the scene below, shown as a side-view of VCS, together with view frustum. Assume that all the objects shown are solid, and that the labelled lines represent polygonal faces of the objects.



(a) (2 points) List, in alphabetical order, the polygons that would be culled by view frustum culling.

b, g, h, i

(b) (2 points) List, in alphabetical order, the polygons that would be culled by back-face culling. Note: consider both types of culling independently of each other.

b, c, e, i, k, m, q, r, t, v

(c) (2 points) After view-frustum culling and back-face culling, list in alphabetical order the remaining faces that would be completely removed by z-buffer tests.

j, s