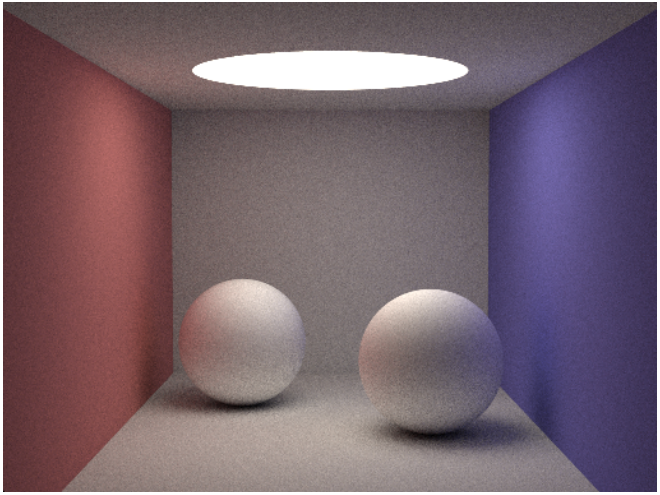
PHYSICALLY BASED RENDERING



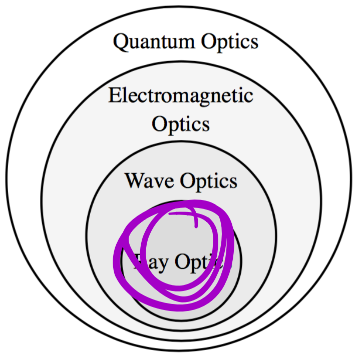


Figure 2.1: The theory of light is described by a series of increasingly complete optical models, where each successive model is able to account for more optical phenomena. In computer graphics and this dissertation, we will restrict ourselves to the simplest model, ray optics.

[https://cs.dartmouth.edu/wjarosz/publications/dissertation/chapter2.pdf]

RADIOMETRIC UNITS

Specification	Definition	Symbol	Unit	Notation
# photons, energy		Q_e	[J= Ws] Joule	radiant energy
power, flux	dQ/dt	Φ_e	[W= J/s]	radiant flux
flux density	dQ/dAdt	E_e	[W/m²]	Irradiance
flux density	dQ/dAdt	$M_e = B_e$	[W/m²]	Radiosity
	dQ/dA$^\Phi$dωdt	L_e	[W/m²/sr] $W/(m^2 \cdot sr)$	pixel values Radiance "brightness"
intensity	dQ/dωdt	I_e	[W/sr]	radiant intensity

$E_{photon} = hc/\lambda$

[https://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG11-HumanVision.pdf]

$$I = I_a k_a + \underbrace{I_L k_d (N \cdot L)}_{\text{diffuse}} + \underbrace{I_L k_s (R \cdot V)^n}_{\text{ambient}}$$

Photometry

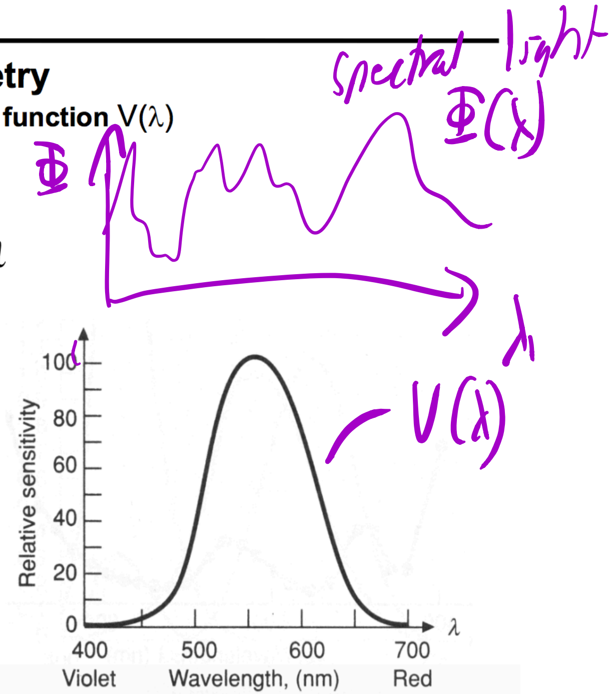
- **Equivalent units to radiometry**
  - Weight with **luminous efficiency function** V(λ) (luminous efficiency function)
  - Spectral or "total" units

$$\Phi_v = K_m \int_\lambda V(\lambda)\Phi_e(\lambda)\,d\lambda$$

$$K_m = 680\ lm/W$$

  - Distinction in English simple:
    - "rad": radiometric unit
    - "lum": photometric unit

W → lumens



[https://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG11-HumanVision.pdf]

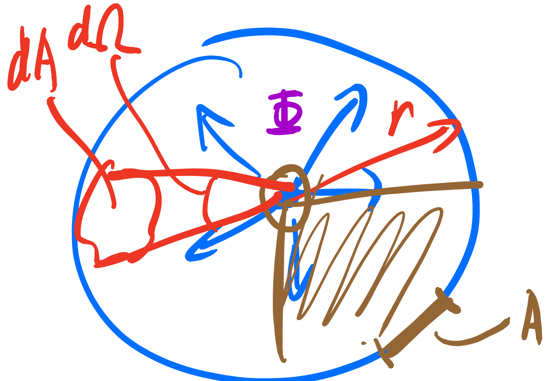
PHOTOMETRIC UNITS

W → lumens

Specification	Definition	Symbol	Units	Notation
energy		Q_v	[talbot]	luminous energy
power, flux	dQ/dt	Φ_v	[lm (Lumen) = talbot/s]	luminous flux
flux density	$dQ/dAdt$	E_v	[lux= lm/m^2]	Illuminance
flux density	$dQ/dAdt$	$[M_v=]\ B_v$	[lux]	Luminosity
	$dQ/dA^{\Phi}d\omega dt$	L_v	[lm/m^2/sr] [nit] (cd/m^2)	Luminance
intensity	$dQ/d\omega dt$	I_v	[cd (candela) = lm/sr]	radiant intensity

[https://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG11-HumanVision.pdf]

POINT LIGHT IN SPHERE



$d\Omega = \frac{dA}{r^2}$ [sr]

for this e.g. equally distributed in all dir

total radiant flux Φ [W]

irradiance (incoming) E $[W/m^2]$ $E = \frac{\Phi}{A} = \frac{10W}{4\pi r^2}$

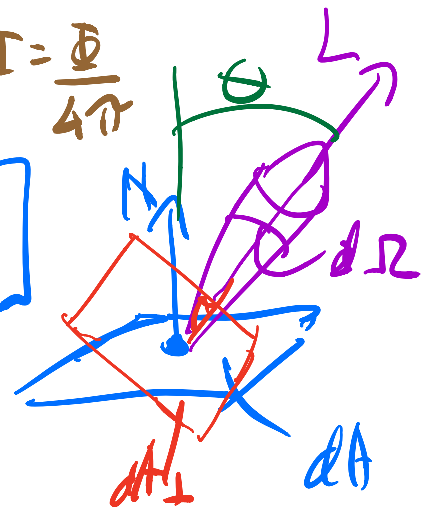
radiosity B $[W/m^2]$

radiant intensity I $[W/sr]$ $I = \frac{\Phi}{4\pi}$

radiance (visible intensity) L $\left[\frac{W}{m^2 \cdot sr}\right]$

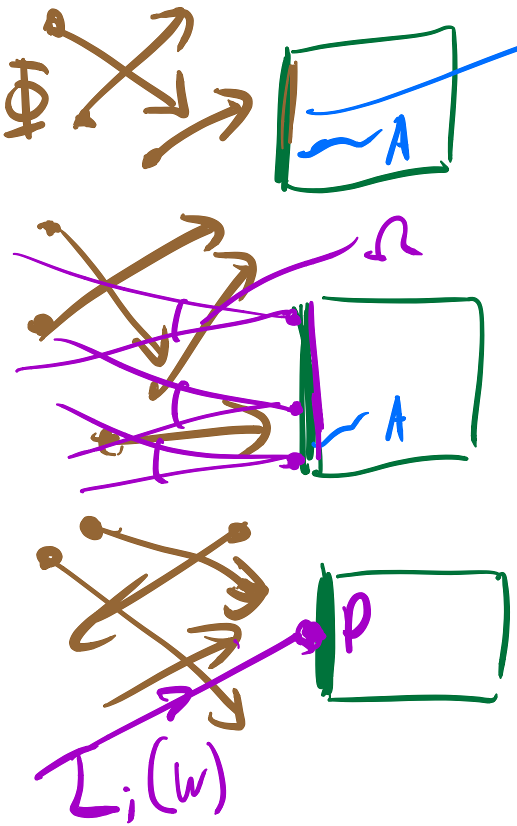
$\Phi = L \cdot dA_\perp \cdot d\Omega$

$dA_\perp = dA \cdot \cos\theta$



PHOTONS IN SPACE

Sunlight 1353 W/m^2



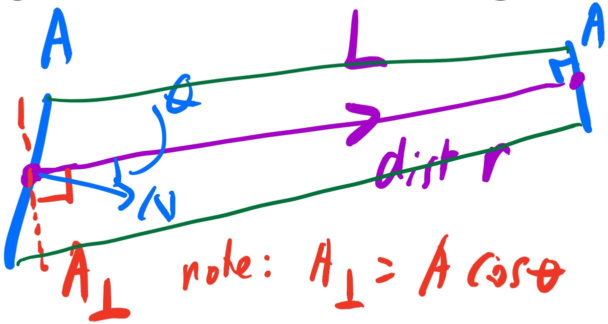
$$E = \frac{\Phi}{A}$$

$$L_i = \frac{\Phi}{\Omega \cdot A} \left[\frac{W}{m^2 \cdot sr}\right]$$

spatial bucket area A
angular bucket Ω

$$L_i(p, \omega) = \lim_{\substack{\Omega \to 0 \\ A \to 0}} \frac{\Phi}{\Omega \cdot A} = \frac{d^2 \Phi}{d\Omega \, dA}$$

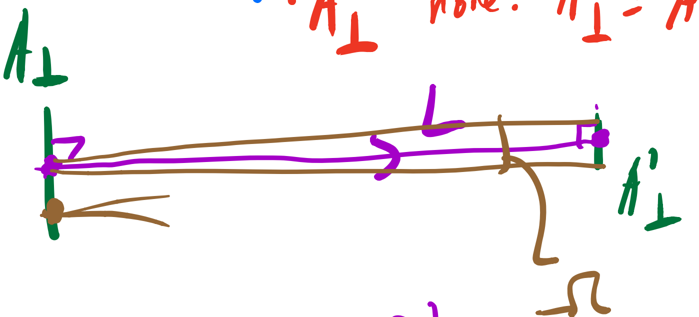
TWO ELEMENT FLUX TRANSFER



note: $A_\perp = A\cos\theta$

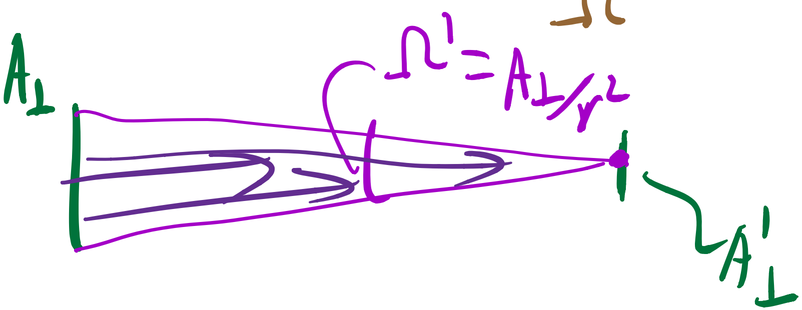
$\theta' = \phi$

$A'_\perp = A' \cos\theta'$



$$\Phi_{out} = \int_{A_\perp} L\,\Omega\, dA_\perp = L\cdot\Omega\cdot A_\perp$$

$$\boxed{\Phi = L\,\frac{A'_\perp}{r^2}\,A_\perp}$$

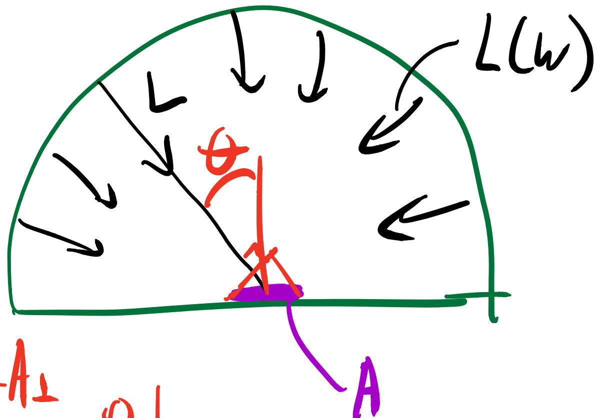


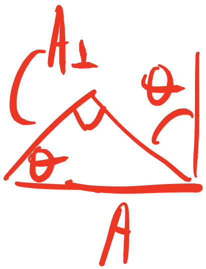
$$\Phi_{in} = \int_{A'_\perp} L\,\Omega'\, dA'_\perp = L\,\Omega' A'_\perp$$

$$\boxed{\Phi = L\,\frac{A_\perp}{r^2}\,A'_\perp}$$

Same!

TOTAL IRRADIANCE FROM RADIANCE





$A_\perp = A\cos\theta$

$$\Phi_i = \int_{w_i} L \cdot A_\perp \cdot \overbrace{dw_i}^{d\Omega_i}$$

$$= \int_{w_i} L \cdot A \cdot \cos\theta \cdot dw_i$$

$$\boxed{E_i = \frac{\Phi_i}{A} = \int_{w_i} L \cdot \underbrace{\cos\theta}_{N \cdot L} \cdot dw_i}$$

BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTION (BRDF)

units: $\left[\frac{1}{sr}\right]$

Physically realistic BRDFs have additional properties,[2] including,

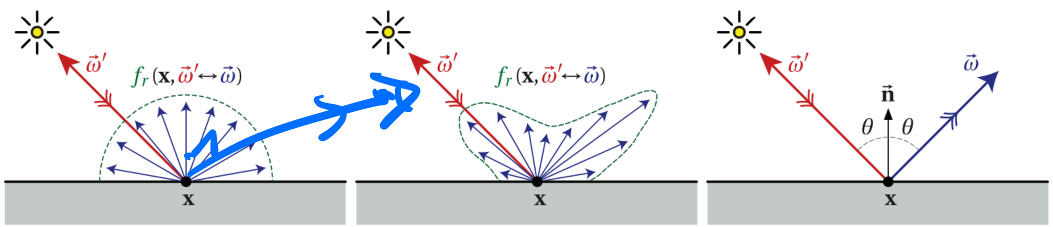
- positivity: $f_r(\omega_i, \omega_r) \geq 0$
- obeying Helmholtz reciprocity: $f_r(\omega_i, \omega_r) = f_r(\omega_r, \omega_i)$
- conserving energy: $\forall \omega_i, \int_\Omega f_r(\omega_i, \omega_r) \cos\theta_r d\omega_r \leq 1$

outgoing radiance

$$\boxed{f_r(\omega_i, \omega_r)} = \frac{\boxed{d\,L_r(\omega_r)}}{d\,E_i(\omega_i)} = \frac{d\,L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\ d\,\omega_i}$$

incoming irradiance

[https://en.wikipedia.org/wiki/Bidirectional_reflectance_distribution_function]



[https://cs.dartmouth.edu/wjarosz/publications/dissertation/chapter2.pdf]

(A) $L_r(w_r) = f_r(w_i, w_r)\, E_i(w_i)$

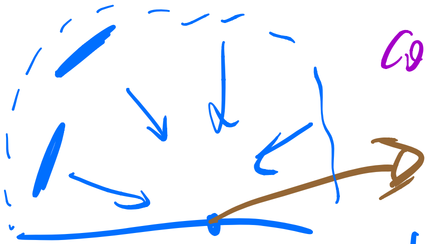
also

(B) $L_r(w_r) = f_r(w_i, w_r)\, L_i(w_i) \cos\theta_i\, \underline{dw_i}$

Point Light Source.

$$E_i = \frac{\Phi_{total}}{4\pi r^2} \cos\theta_i$$

from ① $$L_r = \sum_{lights} f_r(\omega_i, \omega_r) \frac{\Phi_{total}}{4\pi r^2} \cos\theta_i$$



Compare: $$I = \sum_{light} \frac{I_L}{r^2} k_d (N \cdot L)$$

$$L_r(\omega_r) = \int_\Omega f_r(\omega_i, \omega_r) L_i(\omega_i) \cos\theta_i \, d\omega_i$$