Q1. Give the symbol and units used for each of the following:

flux Φ, $[W]$

irradiance E, $\left[\frac{W}{m^2}\right]$

radiant intensity I, $\left[\frac{W}{sr}\right]$

radiance L, $\left[\frac{W}{m^2 \cdot sr}\right]$

radiosity B, $\left[\frac{W}{m^2}\right]$

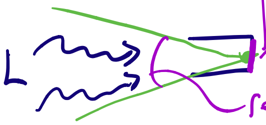
sensor value, S:

$S \propto$ # photons during exposure time

$\propto \Phi_S$

Q2. What are the physical units that correspond to observed brightness, ie, what a camera sensor or that cells in the eye ultimately measure?

$\frac{W}{m^2 \cdot sr}$ i.e., radiance



sensor, area dA

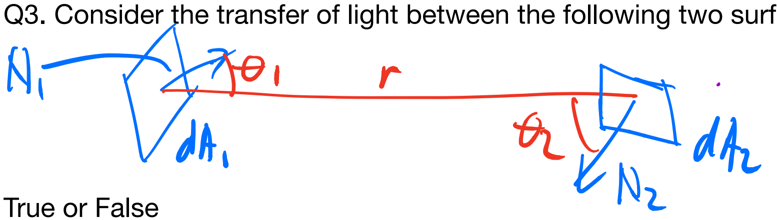
$\Phi_S = L \cdot \boxed{dA \cdot d\Omega} = L \cdot \boxed{k}$

sensor, angle $d\Omega$

some constant

Q3. Consider the transfer of light between the following two surfaces: specifically, consider

$dA_1 \rightarrow dA_2$



True or False

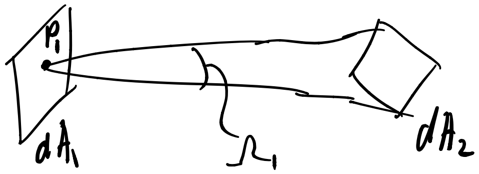
F irradiance is invariant with the distance r — $E \propto 1/r^2$

F flux transfer is invariant with the distance r — $\Phi \propto 1/r^2$

T radiance is invariant with the distance r — L

Q4. For the above scene, given the radiance, L, develop an expression for the flux transfer between the two surfaces.

$$d\Phi = L\, d\Omega\, dA_\perp$$



$$= L\Omega_1\, dA_{1\perp}$$

$$= L\left(\frac{dA_{2\perp}}{r^2}\right) dA_{1\perp}$$

$$= L\left(\frac{A_2 \cos\theta_2}{r^2}\right)\left(A_1 \cos\theta_1\right) = L\,\frac{A_1 A_2 \cos\theta_1 \cos\theta_2}{r^2}$$

(Note: we could also begin with a point P_2 on dA_2)

Q5. A 10 W point light distributes light equally in all directions, as shown below.

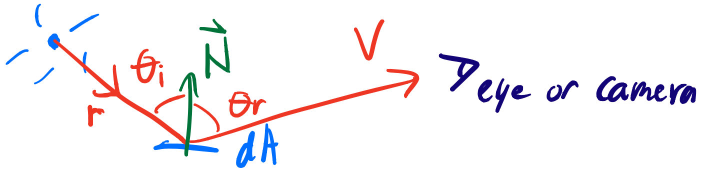
(a) What is the flux received by area dA?

(b) What is the irradiance received by a point on area dA?

(c) Suppose that the surface has a BRDF defined by $f_r = \pi\rho$ diffuse BRDF

$\uparrow$ albedo $\in [0,1]$

What is the observed radiance in the given viewing direction V?



(a) $$d\Phi_A = \Phi_{total} \cdot \underbrace{\frac{d\Omega_A}{\Omega_{total}}}_{\text{some fraction}} = \Phi_{total} \cdot \overbrace{\left(\frac{dA_\perp}{r^2}\right)}^{d\Omega_A} \cdot \left(\frac{1}{\underbrace{4\pi}_{\Omega_{total}}}\right)$$

$$d\Phi_A = \Phi_{total} \cdot \frac{dA\cos\theta_i}{4\pi r^2}$$ where $\Phi_{total} = 10\,W$

(b) $$E_{dA} = \frac{d\Phi_A}{dA} = \frac{\Phi_{total} \cdot \cos\theta_i}{4\pi r^2} \quad \left[\frac{W}{m^2}\right]$$

(c) $$L_r = f_r(\theta_i, \theta_r)\, E_i$$

$$= \pi\rho\frac{\Phi_{total}\ \cos\theta_i}{4\pi r^2} \quad \left[\frac{W}{m^2 \cdot sr}\right]$$

Note: observed brightness increases with:

- ρ : albedo, i.e, overall fraction of energy reflected by a diffuse surface
- Φ_{total} : power of light source
- $\cos\theta_i$: the smaller θ_i is, i.e, the more the surface directly faces the light source.

observed brightness decreases with:

- r^2 : doubling the distance decreases the brightness by a factor of four.