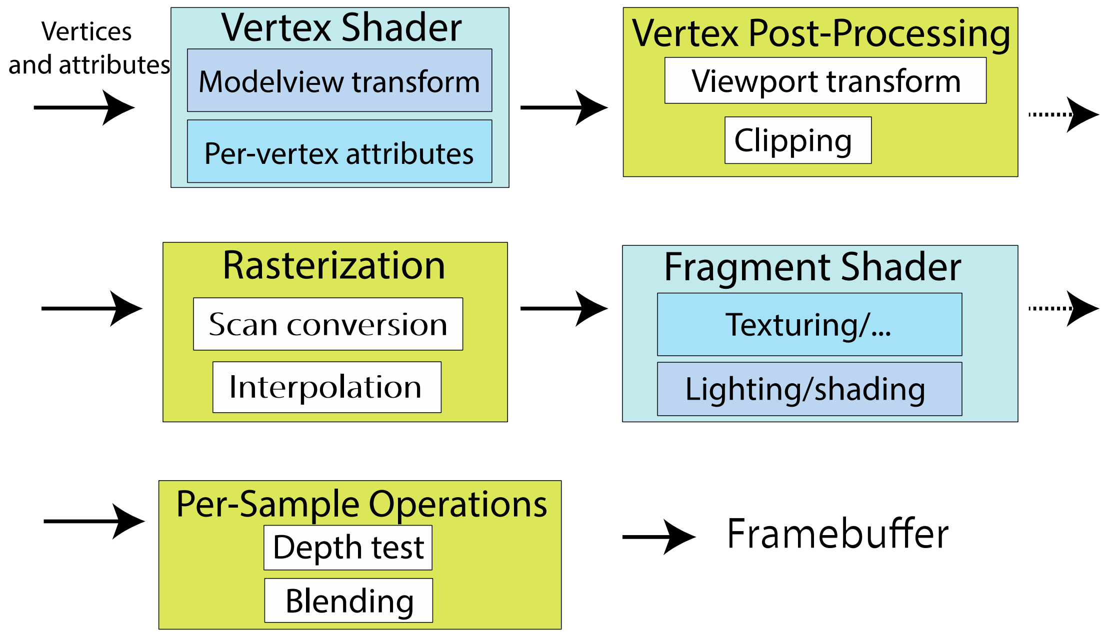
33 – BIG REVIEW

- This review is NOT everything you need to know
- This is just a list of questions you might want to answer in order to start preparation
- Now is a good time to start preparing!

RENDERING

- What is rendering?
- What is the input for the rendering process? Output?
- What are the stages of rendering?
  - Describe each one
- How do we make rendering real-time?
- How do we make rendering realistic?

THE RENDERING PIPELINE



HOMOGENEOUS COORDINATES

- Why do we use homogeneous coordinates?
- How to convert them from/to Euclidean coordinates?
  - Is such conversion 1-1?
- Where in the pipeline do we operate with HC/EC?
- How to tell a vector from a point in HC?

TRANSFORMATION MATRICES

- What's an affine transformation? Linear?
- Can all of them be represented as matrix operations?
- What's a structure of a transformation matrix?

AUGMENTED MATRIX

Translation

Linear Transformation

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & w \end{bmatrix}$$

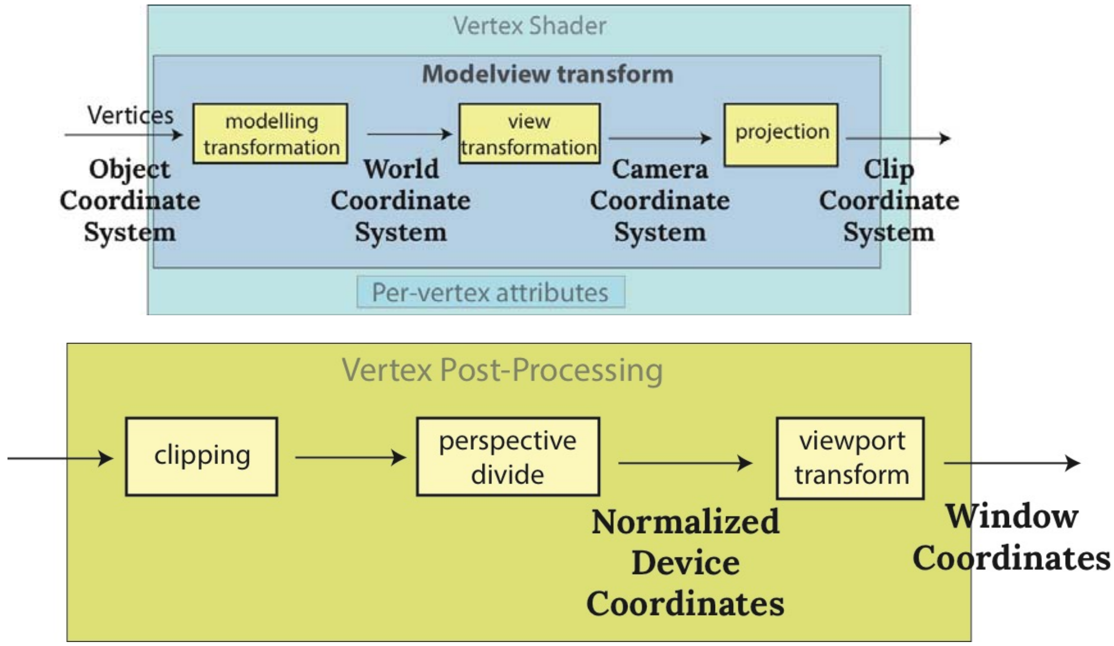
TRANSFORMING COORDINATE FRAME

Columns are new basis vectors (and new origin)!

$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x \cdot (1-\cos\theta) + p_y \cdot \sin\theta \\ \sin\theta & \cos\theta & p_y \cdot (1-\cos\theta) + p_x \cdot \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$

PIPELINE

- What are the transformations involved in the pipeline?
- What are the coordinate systems involved?
- Why do we do perspective divide?
- Why do we do clipping before perspective divide?
- Why do we need viewport transform?



MATH

- What are implicit, explicit, and parametric ways to define geometry?
  - What are their limitations?
- How to intersect two objects if they are
  - Both implicitly defined
  - Both explicitly defined
- How many parameters do we need to represent objects parametrically?

MATH

- How to calculate a normal to an implicit surface/curve?
- How to calculate a tangent plane?
- How to approximate surface area of some 2D shape?
- How to intersect a ray with a planar polygon in 2D? In 3D?

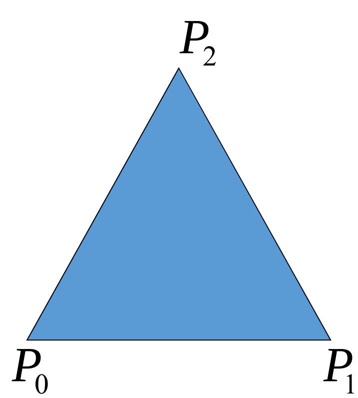
TRIANGLE

- Normal

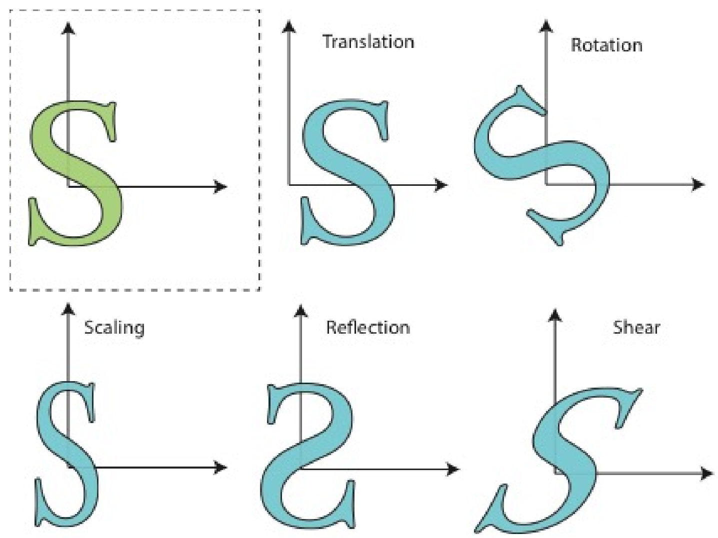
$$n = \frac{(P_1 - P_0) \times (P_2 - P_0)}{\left\| (P_1 - P_0) \times (P_2 - P_0) \right\|}$$

- Area

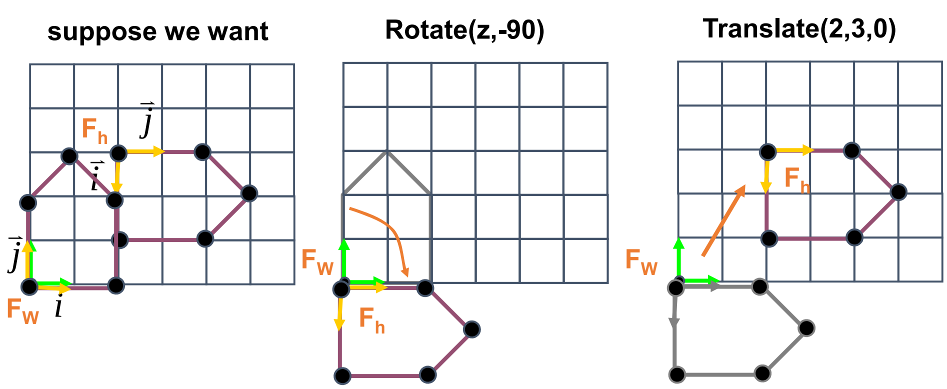
$$A = \frac{1}{2} \left\| \overrightarrow{P_0 P_1} \times \overrightarrow{P_0 P_2} \right\|$$



AFFINE TRANSFORMATIONS



COMPOSING TRANSFORMATIONS



$$P_A = Rot(z,-90)\,P_h \qquad P_W = Trans(2,3,0)\,P_A$$

F_h

$$P_W = Trans(2,3,0)Rot(z,-90)\,P_h$$

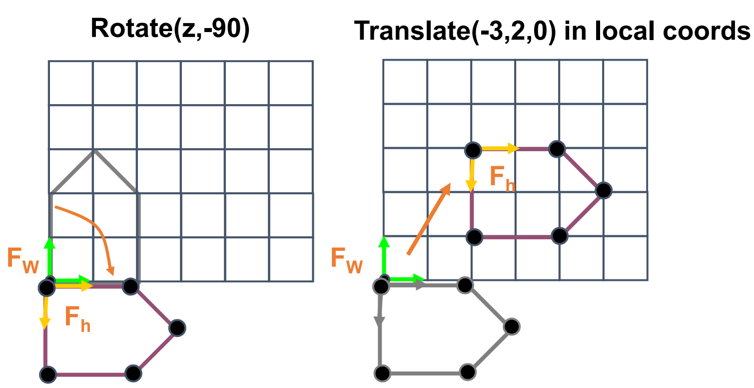
COMPOSING TRANSFORMATIONS

$$P_W = Trans(2,3,0)Rot(z,-90)\,P_h$$

- R-to-L: interpret operations wrt fixed coords
  - moving object
- L-to-R: interpret operations wrt local coords
  - changing coordinate system

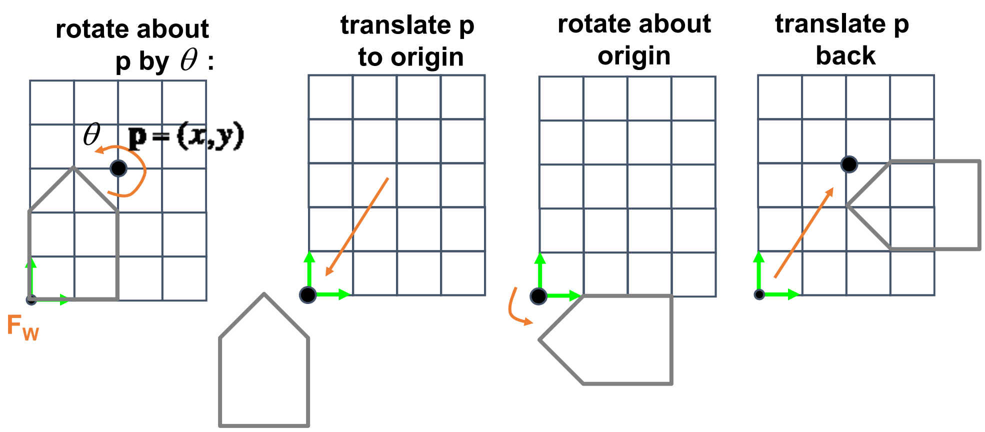
$$M_{MV} = Trans(2,3,0)\cdot M_{MV}$$
$$M_{MV} = Rot(z,-90)M_{MV}$$

COMPOSING TRANSFORMATIONS



$$P_W = Rot(z,-90)Trans(-3,2,0)\,P_h$$

ROTATION ABOUT A POINT: MOVING OBJECT



$$\mathbf{T}(x,y,z)\mathbf{R}(z,\theta)\mathbf{T}(-x,-y,-z)$$

SIMPLE COMPOSITIONS

$$Tr(x_1, y_1, z_1) \cdot Tr(x_2, y_2, z_2) = Tr(x_1 + x_2, y_1 + y_2, z_1 + z_2)$$
$$Tr(x_2, y_2, z_2) \cdot Tr(x_1, y_1, z_1) = Tr(x_2, y_2, z_2) \cdot Tr(x_1, y_1, z_1)$$

$$Scale(a, b, c) \cdot Scale(d, e, f) = Scale(ad, be, cf)$$
$$Scale(a, b, c) \cdot Scale(d, e, f) = Scale(d, e, f) \cdot Scale(a, b, c)$$

$$Rot(\alpha, 0,0,1) \cdot Rot(\beta, 0,0,1) = Rot(\alpha + \beta, 0,0,1)$$
$$Rot(\alpha, 0,0,1) \cdot Rot(\beta, 0,0,1) = Rot(\beta, 0,0,1) \cdot Rot(\alpha, 0,0,1)$$

MORE COMPLICATED COMPOSITIONS

$$Tr(x, y, z) \cdot Scale(a, b, c) \neq Scale(a, b, c) \cdot Tr(x, y, z)$$
$$Tr(x, y, z) \cdot Scale(a, b, c) = Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right)$$

$$Tr(x, y, z) \cdot Rot(\alpha, 0,0,1) \neq Rot(\alpha, 0,0,1) \cdot Tr(x, y, z)$$
$$Rot(\alpha, 0,0,1) \cdot Rot(\beta, 0,1,0) \neq Rot(\beta, 0,1,0) \cdot Rot(\alpha, 0,0,1)$$
$$Scale(a, a, a) \cdot Rot(\beta, 0,0,1) = Rot(\beta, 0,0,1) \cdot Scale(a, a, a)$$

$$Scale(a, b, c) \cdot Rot(\beta, 0,0,1) \neq Rot(\beta, 0,0,1) \cdot Scale(a, b, c)$$

INVERSE TRANSFORMS

$$Tr(x, y, z)^{-1} = Tr(-x, -y, -z)$$

$$Rot(\alpha, 0,0,1)^{-1} = Rot(-\alpha, 0,0,1) = Rot(\alpha, 0,0,1)^T \text{ (orthogonal!)}$$

$$Scale(a, b, c)^{-1} = Scale\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$$

ROTATION AFTER NON-UNIFORM SCALE

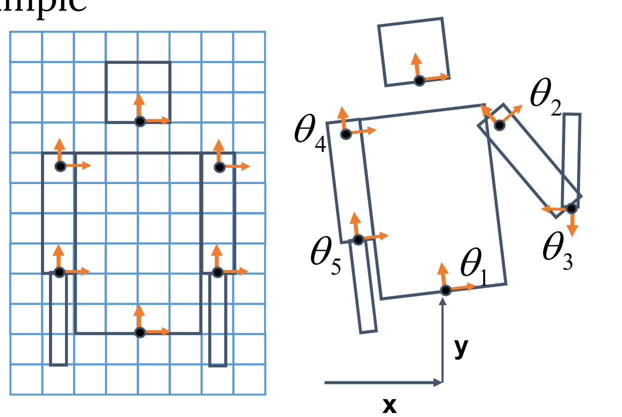
- Not what you'd expect!
- $M = Scale(a,b,c) \cdot Rot(\beta,0,0,1) =$

$$\begin{pmatrix} \mathrm{a} \cdot \cos(\beta) & -a \cdot \sin(\beta) & 0 & 0 \\ \mathrm{b} \cdot \sin(\beta) & b \cdot \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Basis vector not orthogonal

TRANSFORMATION HIERARCHIES

- Example



$M_1 = Tr_{(x,y)} \cdot Rot\theta_1$

$M_2 = M_1 \cdot Tr_{(2.5,5.5)} \cdot Rot\ \theta_2$

$M_3 = M_2 \cdot Tr_{(0,-3.5)} \cdot Rot\ \theta_3$

PROJECTIONS

- What is the purpose of projections?
- What's the difference between ortho- and perspective projections?
- Who chooses which projection to use?
- Can we get a nearly orthographic projection while using a perspective projection matrix?
- What happens to z in perspective projection?
- What happens to the view volumes?

CLIPPING

- What happens to points during clipping? Triangles?
- What are the equations of the frustum planes?
- How can we test if a triangle should be clipped?

RASTERIZATION

- What's rasterization?
- How do we rasterize a polygon?
- Why do we interpolate?
- What are the values we typically interpolate?
- How?
- How is it done in ray/path tracing?

LIGHTING & SHADING

- What's a Gouraud shading?
- What are Lambert/Phong materals?
- If the scene is lit with only ambient light, what will we see?
  - Only diffuse/specular?
- How can we control size of the specular highlight?
- How do we shade in ray tracing? in path tracing?
- In path tracing, how can we simulate more complex materials?

TEXTURING

- How can we tile a wall with bricks?
  - If a texture contains a single brick, what should be texture coordinates for wall's corners?
- Why do we use mipmaps?
- How much storage do we need for them?
- How do we generate mipmaps?
- Where do we get texture coordinates?
- How do we interpolate them?

BUMP AND NORMAL MAPPING

- Why?
- Which mapping would you use to add scales to a fish?
- Bullets on the walls?
- Fur on an animal?
- How do we apply bump mapping?

ENVIRONMENT MAPS

- Why do we need them?
- What are the types?
- How do we generate them?
- How do we apply them?
- When do we re-generate them?

SHADOW MAPS

- Why do we need them?
- How does it fit into pipeline?
- What's the algorithm?

ILLUMINATION MODELS/ALGORITHMS

Local illumination - Fast
Ignore real physics, approximate the look
Interaction of each object with light
- Compute on surface (light to viewer)

Global illumination – Slow
Physically based
Interactions between objects

BASIC RAY-TRACING ALGORITHM

```
RayTrace(r,scene)
obj = FirstIntersection(r,scene)

if (no obj) return BackgroundColor;
else {
     if (Reflect(obj))
        reflect_color = RayTrace(ReflectRay(r,obj));
     else
        reflect_color = Black;

    if (Transparent(obj))
        refract_color = RayTrace(RefractRay(r,obj));
     else
        refract_color = Black;

    return Shade(reflect_color, refract_color, obj);
}
```

WHEN TO STOP?

- Algorithm above does not terminate

- Termination Criteria
  - No intersection
  - Contribution of secondary ray attenuated below threshold – each reflection/refraction attenuates ray
  - Maximal depth is reached

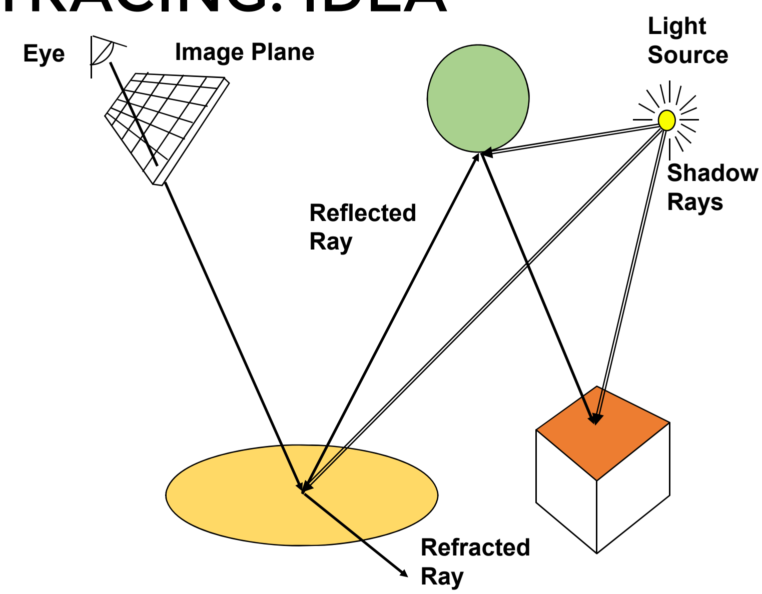
SIMULATING SHADOWS

- Trace ray from each ray-object intersection point to light sources
  - If the ray intersects an object in between ⇒ point is shadowed from the light source

```
shadow = RayTrace(LightRay(obj,r,light));

return Shade(shadow,reflect_color,refract_color,obj);
```

RAY TRACING: IDEA



RAY-OBJECT INTERSECTIONS

- Core of ray-tracing $\Rightarrow$ must be extremely efficient
- Usually involves solving a set of equations
  - Using implicit formulas for primitives

Example: Ray-Sphere intersection

ray: $x(t) = p_x + v_x t,\ \ y(t) = p_y + v_y t,\ \ z(t) = p_z + v_z t$

(unit) sphere: $x^2 + y^2 + z^2 = 1$

quadratic equation in t :

$$\begin{aligned} 0 &= (p_x + v_x t)^2 + (p_y + v_y t)^2 + (p_z + v_z t)^2 - 1 \\ &= t^2(v_x^2 + v_y^2 + v_z^2) + 2t(p_x v_x + p_y v_y + p_z v_z) \\ &\quad + (p_x^2 + p_y^2 + p_z^2) - 1 \end{aligned}$$

v

p

RAY-TRACING: DIRECT ILLUMINATION

- Local surface information (normal…)
  - For implicit surfaces $F(x,y,z)$=0:
    normal $\mathbf{n}(x,y,z)$ is gradient of F:

$$n(x,y,z) = \nabla F(x,y,z) = \begin{pmatrix} \partial F(x,y,z)/\partial x \\ \partial F(x,y,z)/\partial y \\ \partial F(x,y,z)/\partial z \end{pmatrix}$$

  - Example:

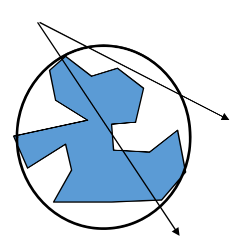
$$F(x,y,z) = x^2 + y^2 + z^2 - r^2$$

$$\mathbf{n}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

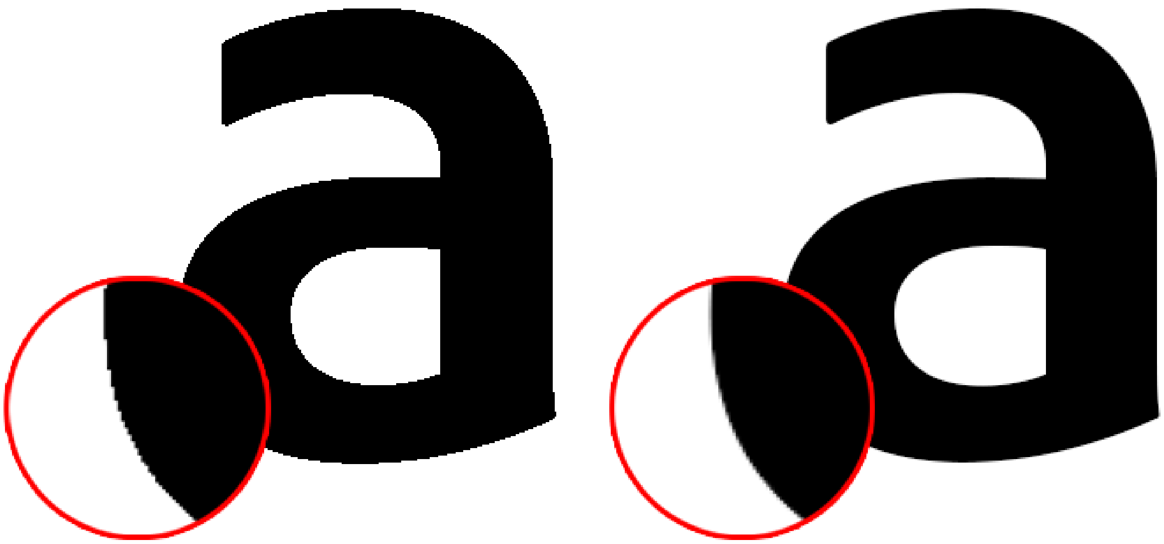
Needs to be normalized!

OPTIMIZED RAY-TRACING

- Basic algorithm is simple but VERY expensive
- Optimize…
  - Reduce number of rays traced
  - Reduce number of ray-object intersection calculations
- Parallelize
  - Cluster
  - GPU
- Methods
  - Bounding Boxes
  - Spatial Subdivision
    - Visibility, Intersection/Collision
  - Tree Pruning

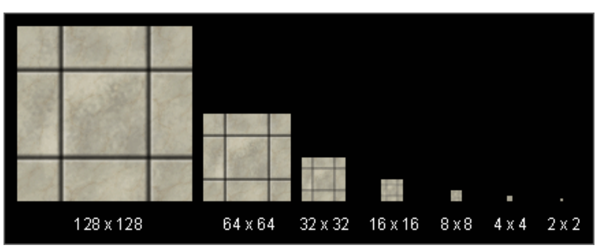


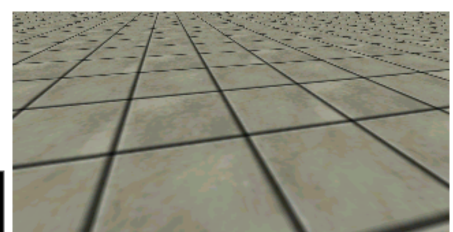
ALIASING & ANTI-ALIASING



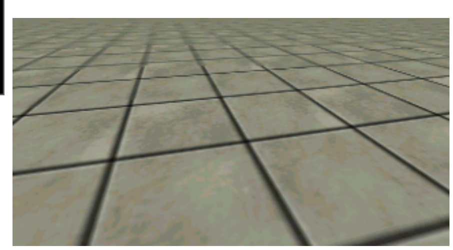
MIPMAPPING

use "image pyramid" to precompute averaged versions of the texture





Without MIP-mapping



With MIP-mapping

THANK YOU AND GOOD BYE!

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