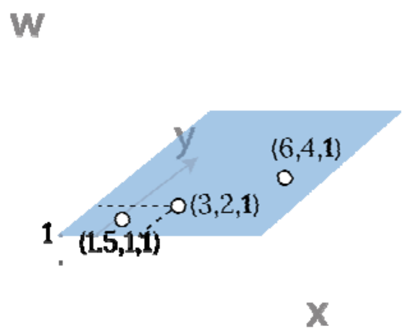


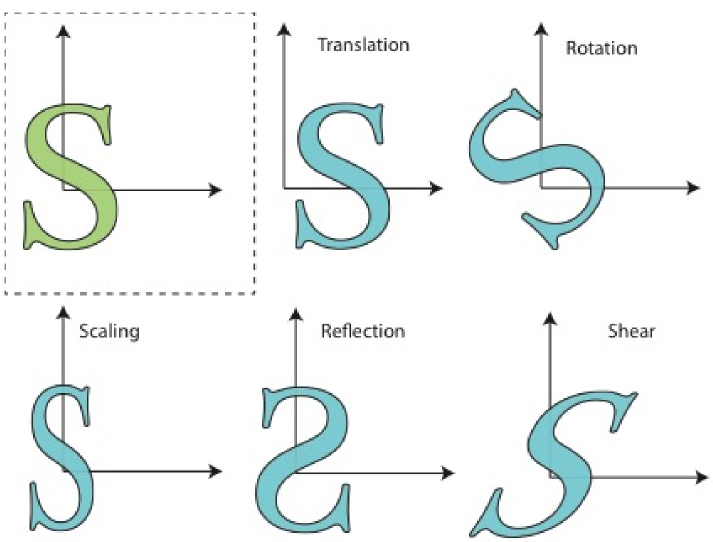
CPSC 314
08 –HOMOGENEOUS COORDINATES & TRANSFORMATIONS CONT.

UGRAD.CS.UBC.CA/~CS314

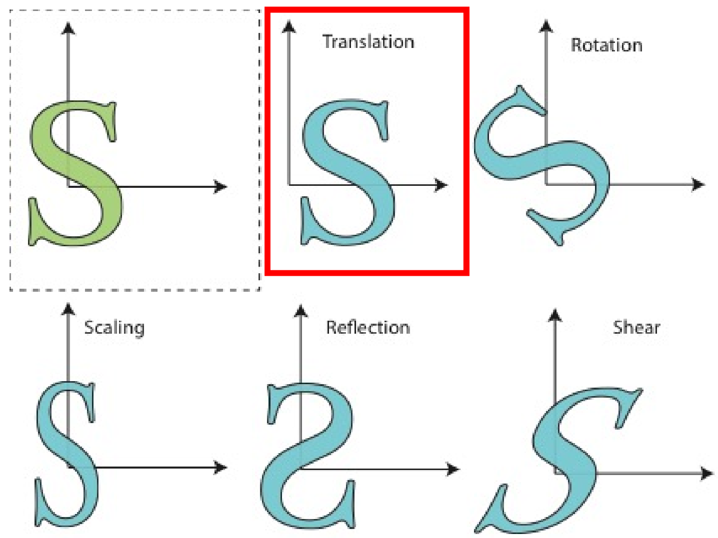
Alla Sheffer

2016

AFFINE TRANSFORMATIONS



AFFINE TRANSFORMATIONS



IDEA

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

AUGMENTED MATRIX

$$M\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Translation

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' + b_x \\ y' + b_y \\ z' + b_z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} + \boldsymbol{b}$$

AUGMENTED MATRIX

Linear Transformation

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

AUGMENTED MATRIX

Translation

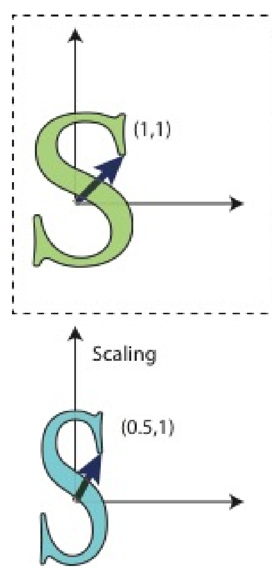
Linear Transformation

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

AFFINE TRANSFORMATIONS

- Scale:

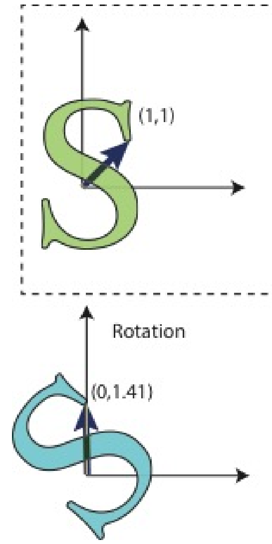
$$M = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



AFFINE TRANSFORMATIONS

- Rotation

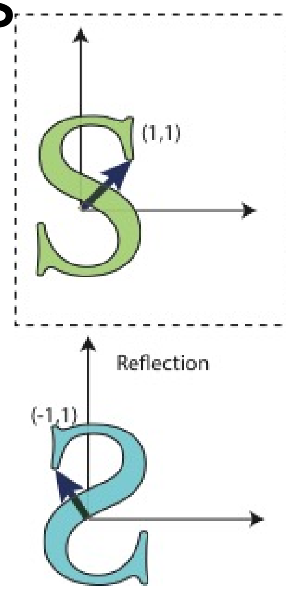
$$M = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



AFFINE TRANSFORMATIONS

- Reflection (y)

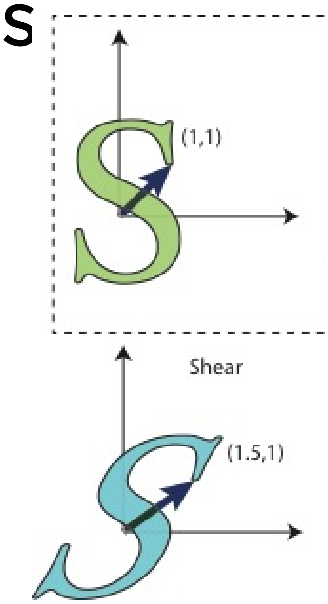
$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



AFFINE TRANSFORMATIONS

- Shear (along x)

$$M = \begin{bmatrix} 1 & \lambda & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



AFFINE TRANSFORMATIONS

- Translation

$$M = \begin{bmatrix} 1 & 0 & 0 & c_x \\ 0 & 1 & 0 & c_y \\ 0 & 0 & 1 & c_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

WHAT'S THAT 1?

- Now our points are represented by $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ - homogeneous coordinates
- Those are vectors from ANOTHER SPACE.

HOMOGENEOUS COORDINATES

Homogeneous coordinates $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \leftarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Euclidean coordinates

HOMOGENEOUS COORDINATES

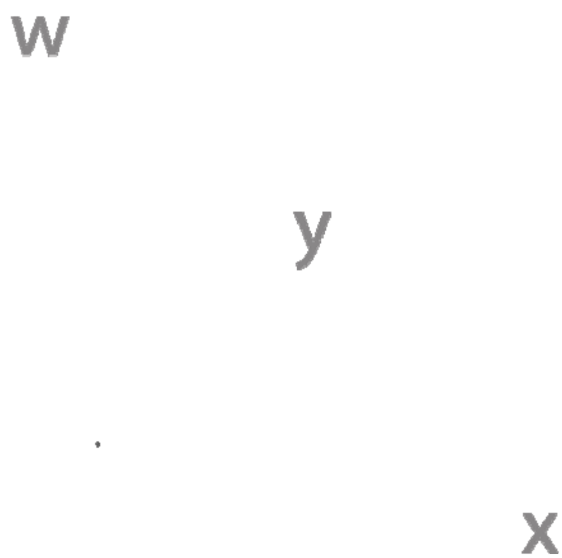
Homogeneous coordinates $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \rightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \end{bmatrix}$ Euclidean coordinates

HOMOGENEOUS COORDINATES

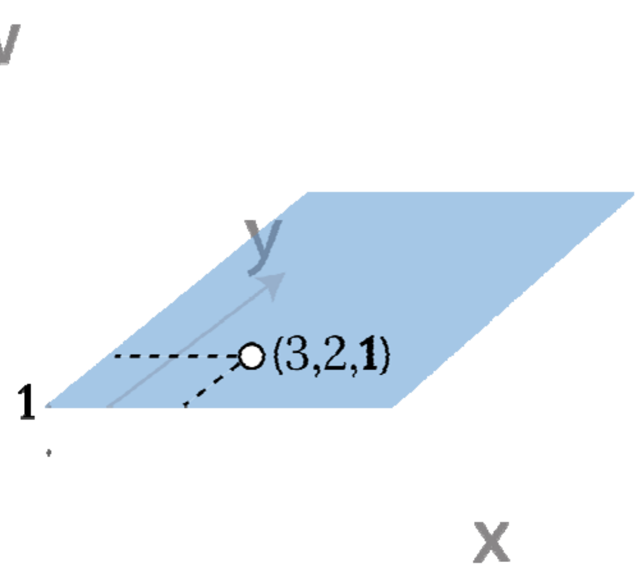
Homogeneous coordinates $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \rightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \end{bmatrix}$ Euclidean coordinates

Not one-to-one!

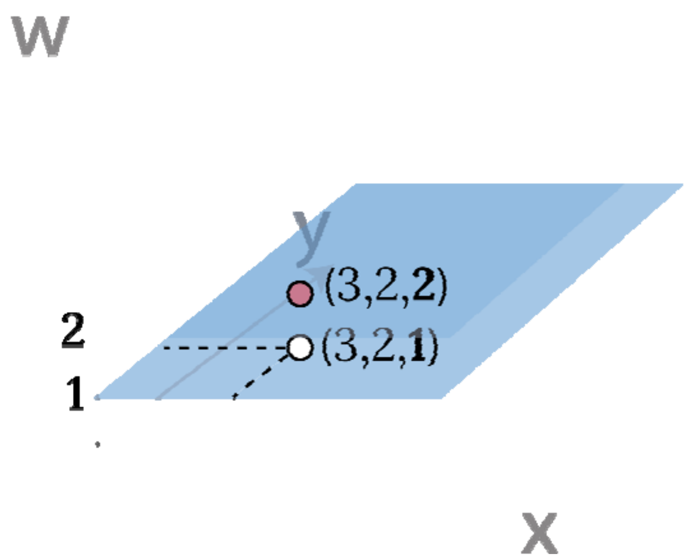
HOMOGENEOUS COORDINATES



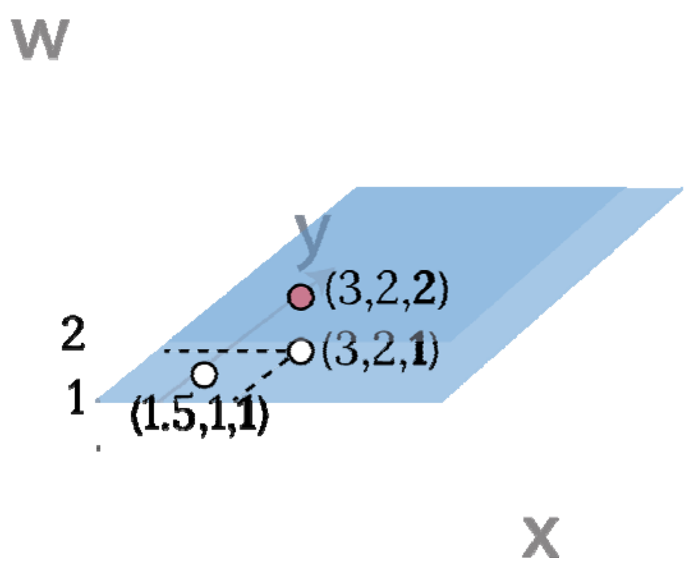
HOMOGENEOUS COORDINATES



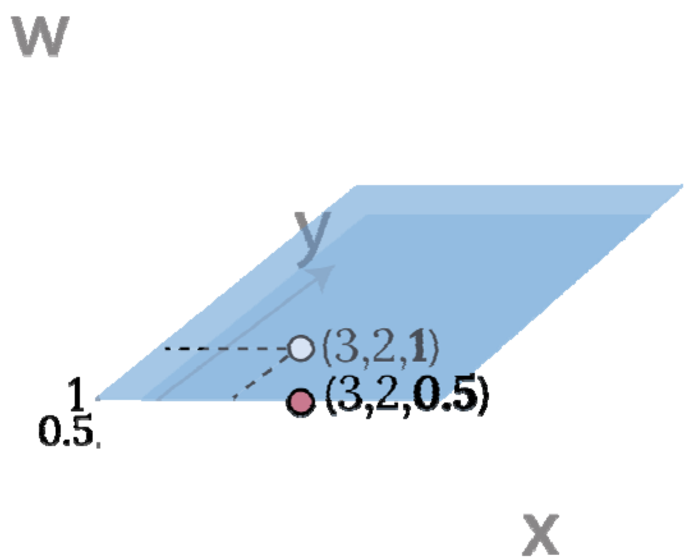
HOMOGENEOUS COORDINATES



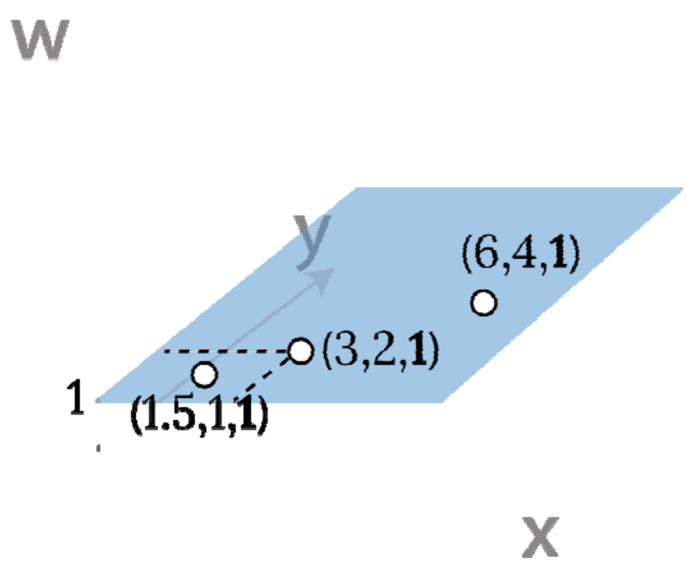
HOMOGENEOUS COORDINATES



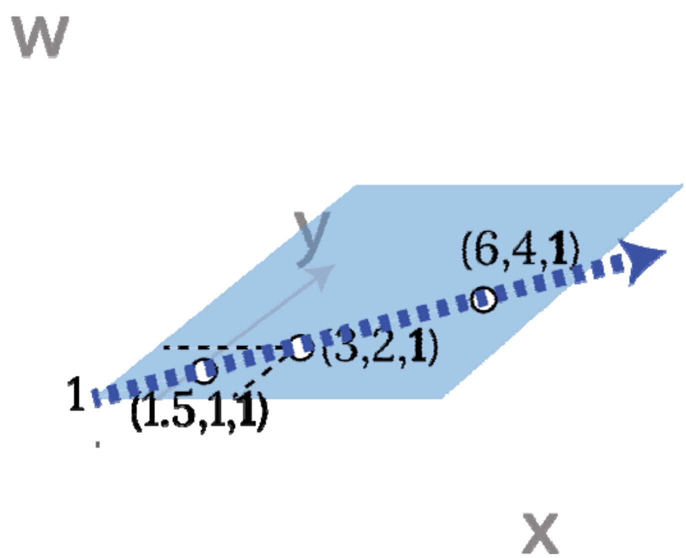
HOMOGENEOUS COORDINATES



HOMOGENEOUS COORDINATES



HOMOGENEOUS COORDINATES



POINTS AND VECTORS

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} \text{ are vectors!}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}, w \neq 0 \text{ are points}$$

QUESTION

- What do these affine transformations do?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

WHICH PAIR IS EQUIVALENT?

A. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

D. A & B

E. A & C

TRANSFORMING POINTS & VECTORS

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' + b_x \\ y' + b_y \\ z' + b_z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix}$$

TRANSFORMING POINTS & VECTORS

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' + b_x \\ y' + b_y \\ z' + b_z \\ 1 \end{bmatrix}$$

Point: **Linear transformation** + **translation**

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix}$$

Vector: **Linear transformation**

TRANSFORMATION COMPOSITION

TRANSFORMATION COMPOSITION

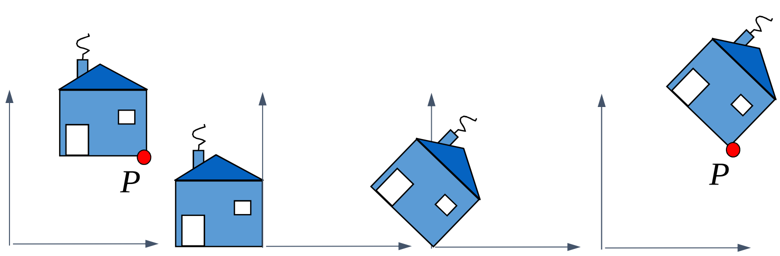
- What operation rotates XY by θ around $P = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$?

TRANSFORMATION COMPOSITION

- What operation rotates XY by θ around $P = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$?
- Answer:
  - Translate P to origin

TRANSFORMATION COMPOSITION

- What operation rotates XY by θ around $P = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$?
- Answer:
  - Translate P to origin
  - Rotate around origin by θ
  - Translate back



TRANSFORMATION COMPOSITION

$$T^{(p_x,p_y)} R^{\theta} T^{(-p_x,-p_y)}(V)$$

$$= \begin{bmatrix} 1 & 0 & p_x \\ 0 & 1 & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -p_x \\ 0 & 1 & -p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$

GENERAL IDEA

- In general:
  - Transform geometry into coordinate system where operation becomes simpler
  - Perform operation
  - Transform geometry back to original coordinate system

- Note: composition of affine transformations is an affine transformation

TWO INTERPRETATIONS OF COMPOSITE

1) read from inside-out as transformation of object

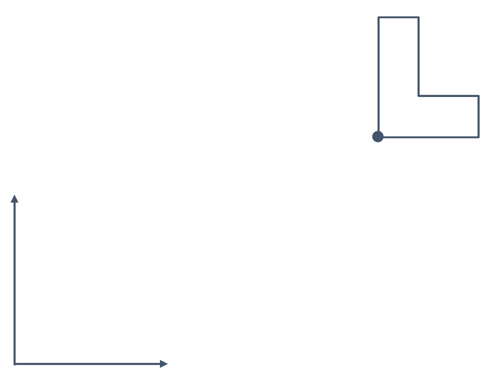
- Translate object by −t
- Rotate object by Φ
- Translate object by t

2) read from outside-in as transformation of the coordinate frame

- Translate frame by t
- Rotate frame by −Φ (i.e. rotate object by Φ)
- Translate frame by −t

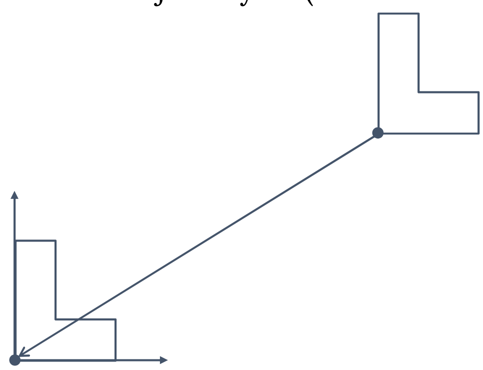
COMPOSITING OF AFFINE TRANSFORMATIONS

■Example scene:



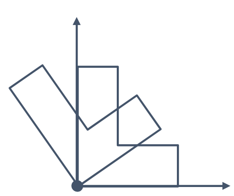
COMPOSITING OF AFFINE TRANSFORMATIONS

- First Interpretation:
  - Step 1: translate object by −t (move to origin)



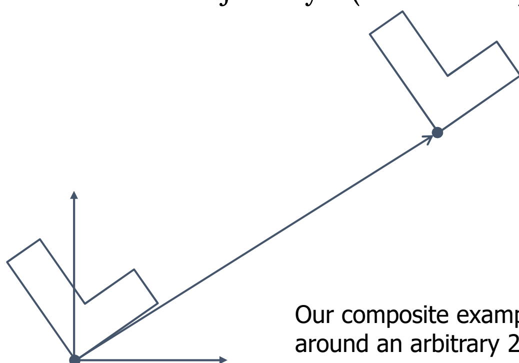
COMPOSITING OF AFFINE TRANSFORMATIONS

- First Interpretation:
  - Step 2: rotate object by Φ



COMPOSITING OF AFFINE TRANSFORMATIONS

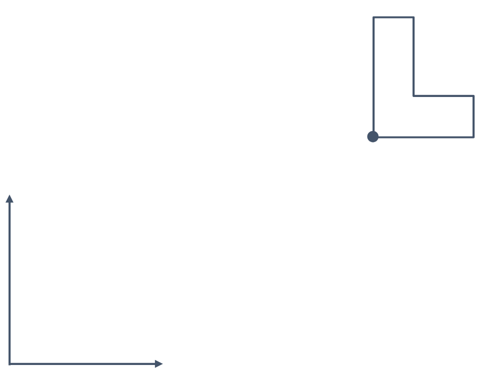
- First Interpretation:
  - Step 3: translate object by t (move back)



Our composite example is a rotation around an arbitrary 2D point with position t!

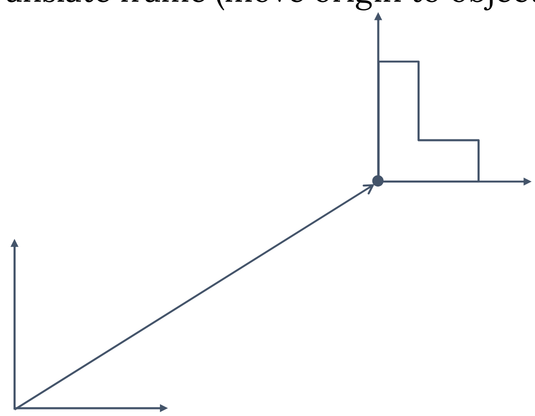
COMPOSITING OF AFFINE TRANSFORMATIONS

- Example scene, second interpretation:



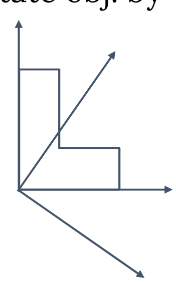
COMPOSITING OF AFFINE TRANSFORMATIONS

- Second interpretation:
  - Step 1: translate frame (move origin to object)



COMPOSITING OF AFFINE TRANSFORMATIONS

- Second interpretation:
  - Step 2: rotate frame by -Φ (i.e rotate obj. by Φ)



COMPOSITING OF AFFINE TRANSFORMATIONS

- Second interpretation:
  - Step 3: translate frame back (vector -t in new frame!)