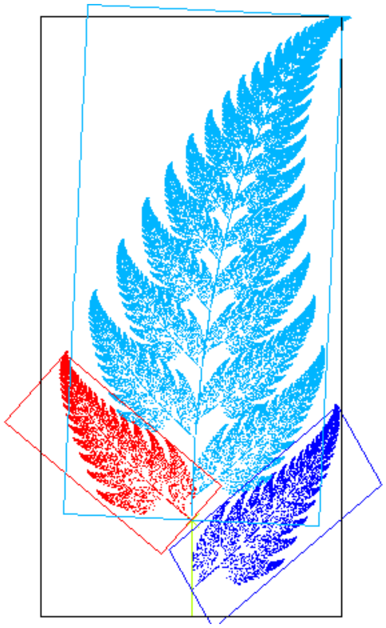


CPSC 314
07 – AFFINE TRANSFORMATIONS

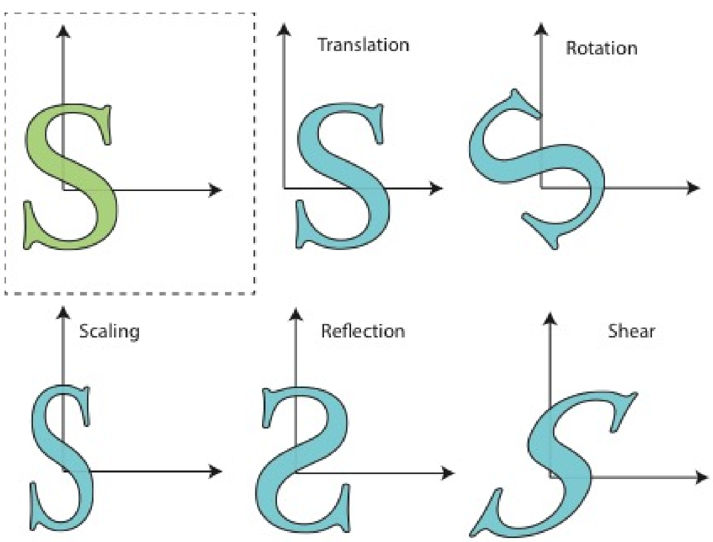
TEXTBOOK: I.3

UGRAD.CS.UBC.CA/~CS314

Alla Sheffer

2016

TRANSFORMATIONS WE WANT



TRANSLATION

- There's a minor glitch.

- Translation:

$$L: u \rightarrow u + b$$
$$L(0) = b \neq 0$$

Translation is not a linear transformation and can't be represented as a matrix operation in this space.

TRANSLATION

- There's a minor glitch.

- Translation:

$$L: u \rightarrow u + b$$
$$L(0) = b \neq 0$$

We need more power.

GENERAL TRANSFORMATIONS

We need to represent all the
linear transformations + translation.

Ideas?

GENERAL TRANSFORMATIONS

We need to represent all the
linear transformations + translation.

Ideas?

$$T(\boldsymbol{v}) = M\boldsymbol{v} + \boldsymbol{b}$$

COMPOSITING SNEAK PEEK

- How do we first rotate, then scale?

- How do we first rotate, then translate?

UNIFYING

- We want something more universal:

$$T(v) = Mv$$

Then we could easily combine different transformations or invert

Let's extend the vector to have one artificial coordinate: (x,y,1)

Then what should the matrix be?

AUGMENTED MATRIX

$$M_{2x2}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x'\\y'\end{bmatrix}$$

$$\begin{bmatrix}M_{2x2} & & 0\\ & & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{bmatrix}x'\\y'\\1\end{bmatrix}$$

Haven't changed much, have we?

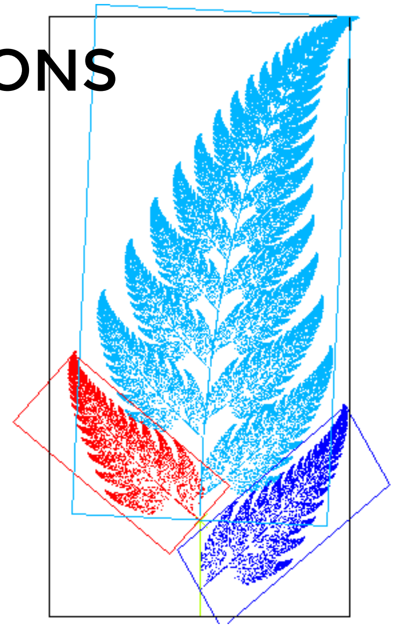
AUGMENTED MATRIX

$$\begin{bmatrix}M_{2x2} & & b_x\\ & & b_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{bmatrix}x'+b_x\\y'+b_y\\1\end{bmatrix}=\begin{bmatrix}x'\\y'\\1\end{bmatrix}+\boldsymbol{b}$$

Translation

AFFINE TRANSFORMATIONS

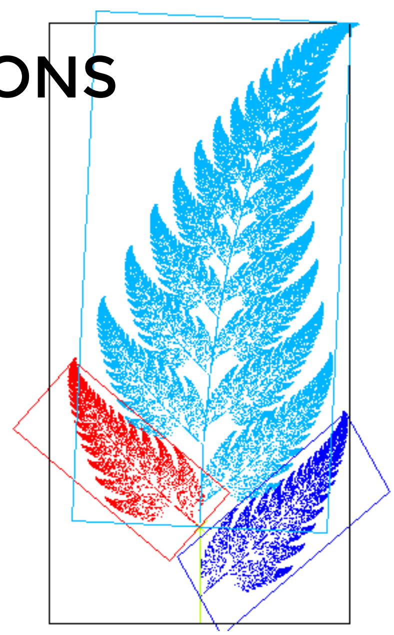
Linear (rotation, scaling, shear, reflections)
+ TRANSLATION



AFFINE TRANSFORMATIONS

Linear (rotation, scaling, shear, reflections)
+ TRANSLATION

How to convert a linear transformation matrix into affine matrix?



AFFINE TRANSFORMATIONS

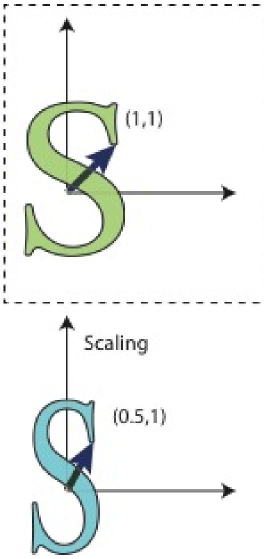
- Scale:

$$M = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

$$M = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ 2\beta \\ 3\gamma \\ 1 \end{pmatrix}$$



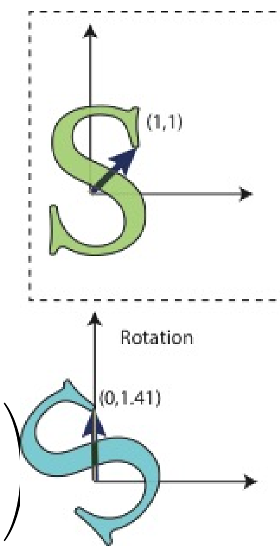
AFFINE TRANSFORMATIONS

- Rotation

$$M = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) - \sin(\alpha) \\ \cos(\alpha) + \sin(\alpha) \\ 0 \\ 1 \end{pmatrix}$$



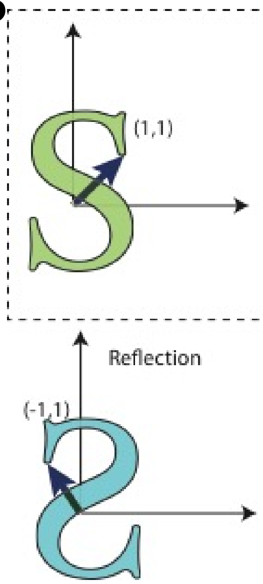
AFFINE TRANSFORMATIONS

- Reflection (y)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} -x \\ y \\ z \\ 1 \end{pmatrix}$$



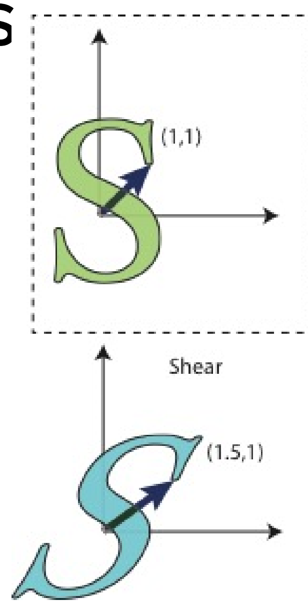
AFFINE TRANSFORMATIONS

- Shear (along x)

$$M = \begin{bmatrix} 1 & \lambda & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & \lambda & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + \lambda y \\ y \\ z \\ 1 \end{pmatrix}$$



AFFINE TRANSFORMATIONS

- Translation

$$M = \begin{bmatrix} 1 & 0 & 0 & c_x \\ 0 & 1 & 0 & c_y \\ 0 & 0 & 1 & c_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 0 & 0 & c_x \\ 0 & 1 & 0 & c_y \\ 0 & 0 & 1 & c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + c_x \\ y + c_y \\ z + c_z \\ 1 \end{pmatrix}$$

AFFINE TRANSFORMATIONS: 2D

$$M = \begin{bmatrix} 1 & 0 & \color{red}{0} & c_x \\ 0 & 1 & \color{red}{0} & c_y \\ \color{red}{0} & \color{red}{0} & \color{red}{1} & \color{red}{c_z} \\ 0 & 0 & \color{red}{0} & 1 \end{bmatrix}$$

AFFINE TRANSFORMATIONS: 2D

$$M = \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

WHAT'S THAT 1?

- Now our points are represented by $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ - homogeneous coordinates
- Those are vectors from ANOTHER SPACE.