CPSC 314
05 – 3D TO 2D: A LONG JOURNEY

UGRAD.CS.UBC.CA/~CS314

TEXTBOOK: I.2

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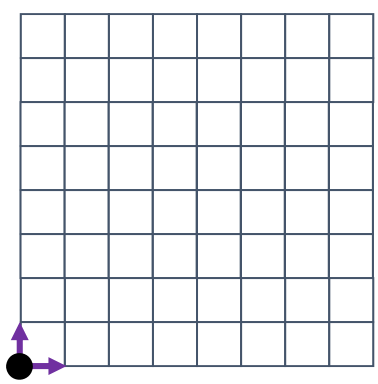
Sep 2016

COORDINATE SYSTEMS

- Coordinate system = Origin + Basis Vectors



COORDINATE SYSTEMS

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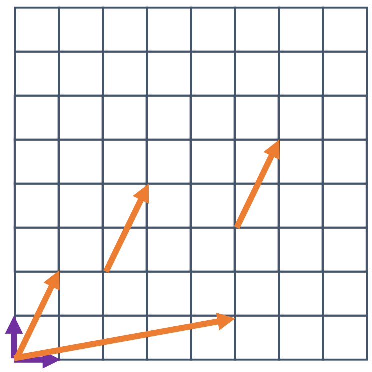
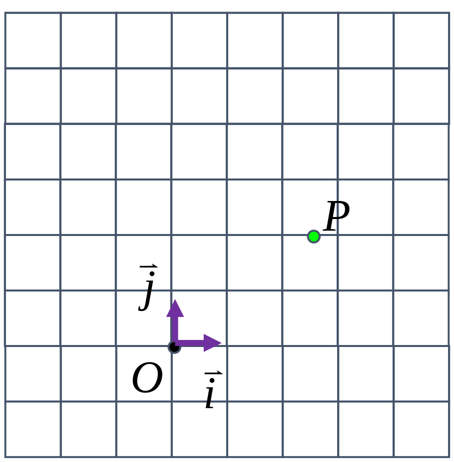


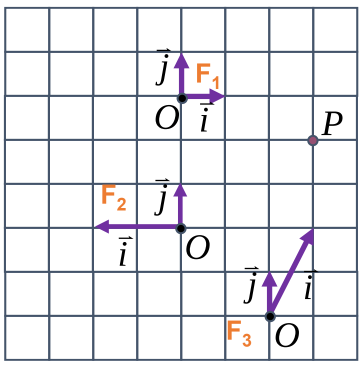
COORDINATE SYSTEMS



$P = O + x\vec{i} + y\vec{j}$

equivalent: $P = (x, y)$

COORDINATE SYSTEMS



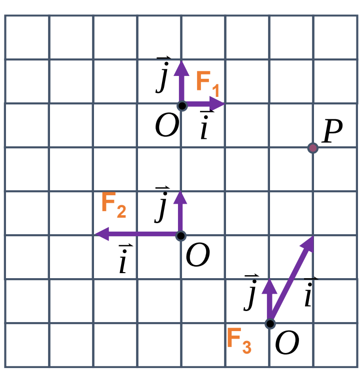
$$P = O + x\vec{i} + y\vec{j}$$

F_1

F_2

F_3

COORDINATE SYSTEMS



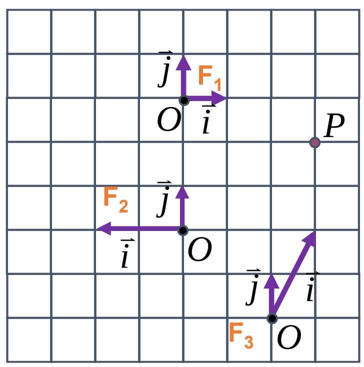
$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

F_2

F_3

COORDINATE SYSTEMS



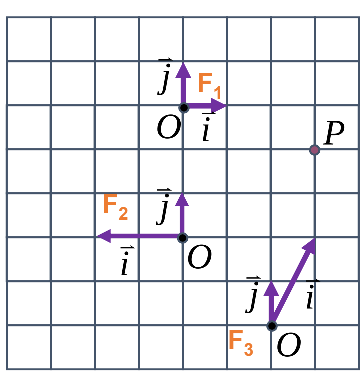
$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

F_2 **P(-1.5,2)**

F_3

COORDINATE SYSTEMS



$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

F_2 **P(-1.5,2)**

F_3 **P(1,2)**

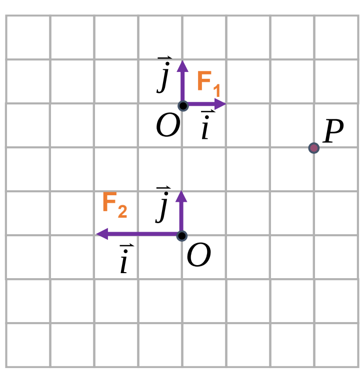
?

- For a given vector and a coordinate frame, are vector's coordinates unique? Why? Are they always defined?

TRANSFORMATIONS

- Transformations as a change of frame

$$P = O + x\vec{i} + y\vec{j}$$



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_1 + x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_2 + x_1 \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}_2 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2$$

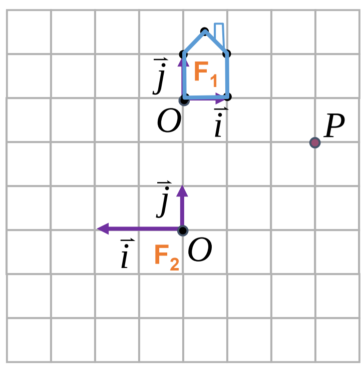
$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

check: $P_1(3,-1)$ becomes $P_2(-1.5,2)$

$$P_2 = MP_1$$

TRANSFORMATIONS

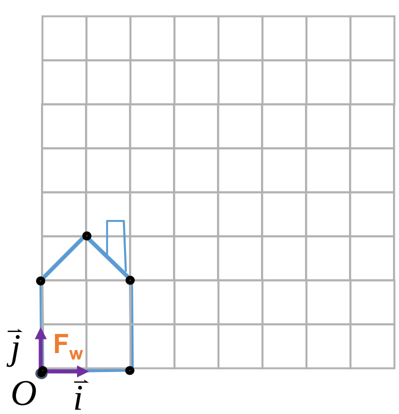
change of basis expressed using a matrix



$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

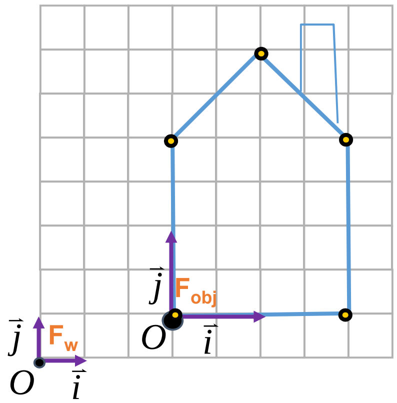
USAGE OF TRANSFORMATIONS



set up the modeling matrix M

for each vertex v
  v’ = Mv

USAGE OF TRANSFORMATIONS



$$P = O + x\vec{i} + y\vec{j}$$

$$\begin{bmatrix} x_{obj} \\ y_{obj} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{obj} + x_{obj}\begin{bmatrix} 1 \\ 0 \end{bmatrix}_{obj} + y_{obj}\begin{bmatrix} 0 \\ 1 \end{bmatrix}_{obj}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_w + x_{obj}\begin{bmatrix} 2 \\ 0 \end{bmatrix}_w + y_{obj}\begin{bmatrix} 0 \\ 2 \end{bmatrix}_w$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

USING TRANSFORMATIONS

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

2D

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

3D

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{obj}$$

PIPELINE: MORE DETAILS

Vertices and attributes →

Vertex Shader
- Modelview transform
- Per-vertex attributes

→

Vertex Post-Processing
- Viewport transform
- Clipping

→

Rasterization
- Scan conversion
- Interpolation

→

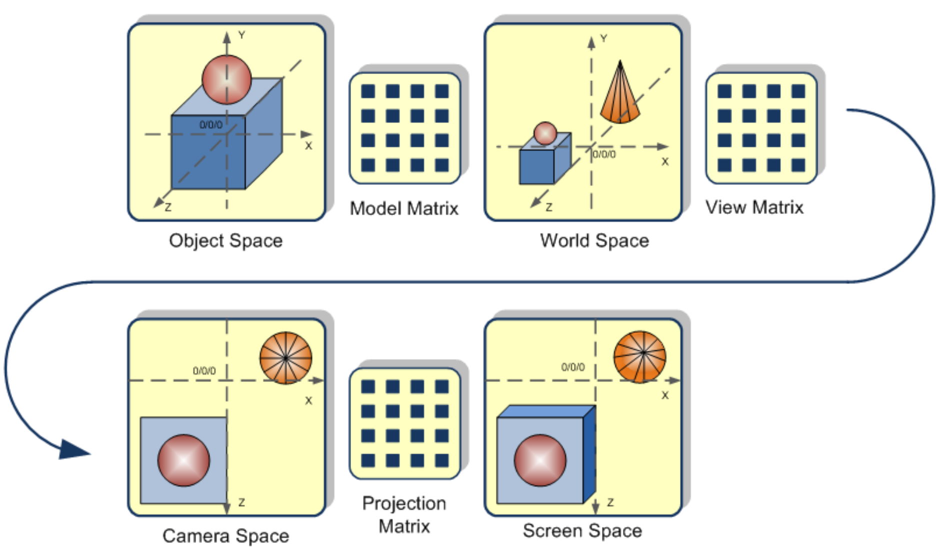
Fragment Shader
- Texturing/...
- Lighting/shading

→

Per-Sample Operations
- Depth test
- Blending

→ Framebuffer

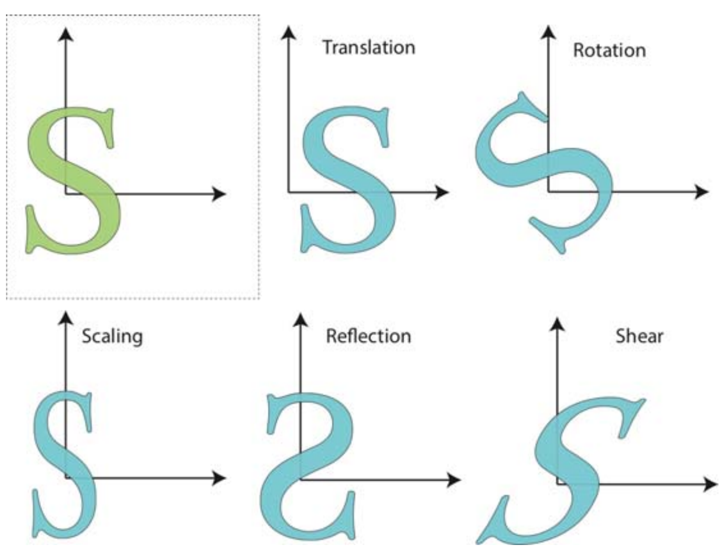
COORDINATE SYSTEMS



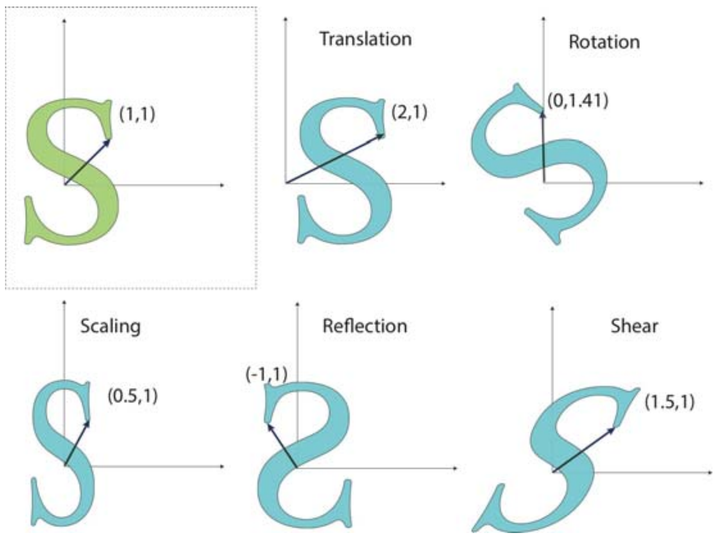
HOW TO TRANSFORM COORDINATES

- Between coordinate frames
- Or animate objects

LINEAR TRANSFORMATIONS



LINEAR TRANSFORMATIONS: Radius vectors



MATRICES

- A more compact representation
- To transform a vector, multiply it by the transformation matrix:

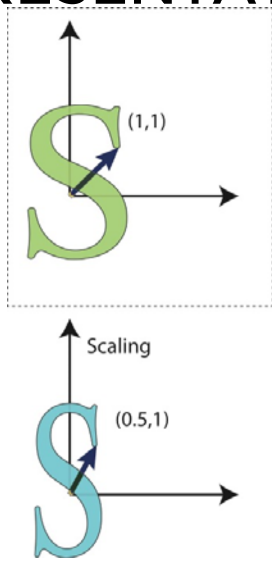
$$v' = M \cdot v$$

- Here we represent vectors as columns:

$$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

MATRIX REPRESENTATIONS

- Scale:



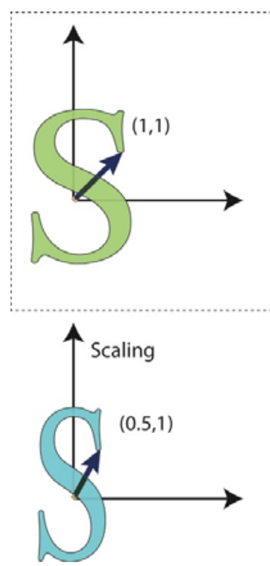
MATRIX REPRESENTATIONS

- Scale:

$$M = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

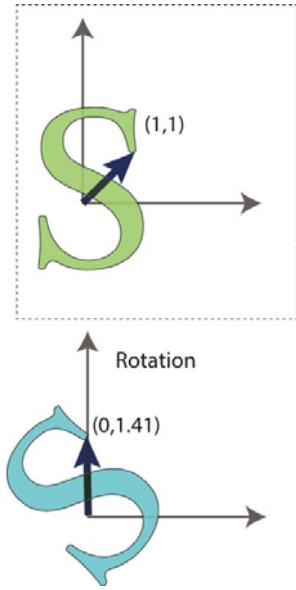
Example:

$$\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \alpha \\ 2\beta \end{pmatrix}$$



MATRIX REPRESENTATIONS

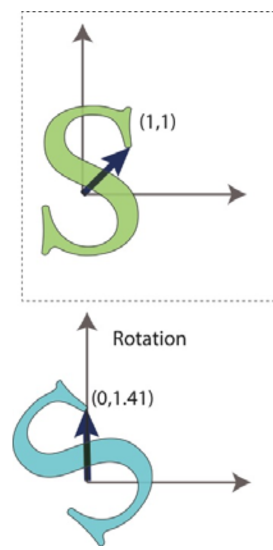
- Rotation



MATRIX REPRESENTATIONS

- Rotation

$$R(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$



Example:

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) - \sin(\alpha) \\ \cos(\alpha) + \sin(\alpha) \end{pmatrix}$$

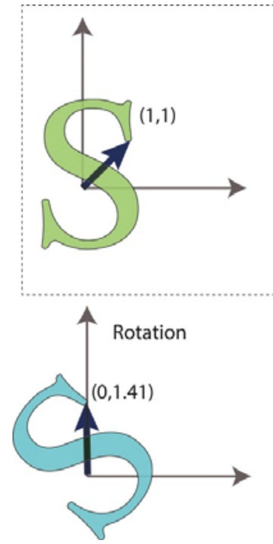
MATRIX REPRESENTATIONS

- Rotation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



ROTATION PROPERTIES

- $R(\alpha)$ is orthogonal matrix

$$R(\alpha)^{-1} = R(\alpha)^T$$

ROTATION PROPERTIES

- $R(\alpha)$ is orthogonal matrix

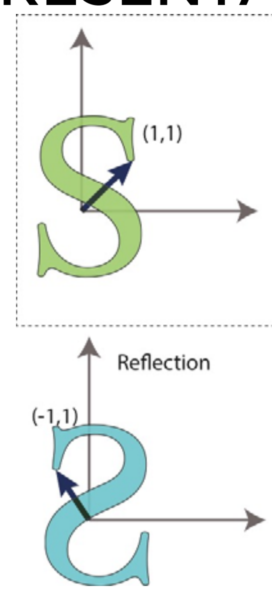
$$R(\alpha)^{-1} = R(\alpha)^T$$

- So to rotate back (i.e. to rotate by $-\alpha$) is

$$R(-\alpha) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) \\ \sin(-\alpha) & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} = R(\alpha)^T$$

MATRIX REPRESENTATIONS

- Reflection



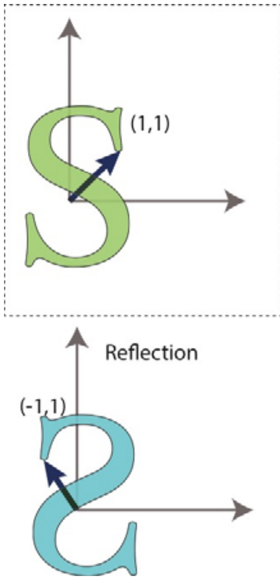
MATRIX REPRESENTATIONS

- Reflection

$$M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$



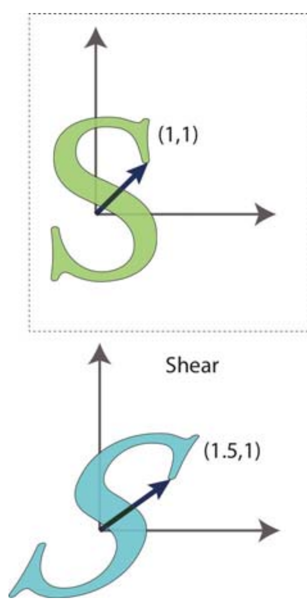
MATRIX REPRESENTATIONS

- Shear

$$M = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} \text{ or } M = \begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \lambda y \\ y \end{pmatrix}$$



LINEAR TRANSFORMATION

A transform L is ***linear***, iff for any two vectors u, v

$$L(u) + L(v) = L(u + v)$$ **additivity**

and for a scalar α

$$\alpha \cdot L(u) = L(\alpha u)$$ **homogenity of degree 1**

LINEAR TRANSFORMATION

A transform L is ***linear***, iff for any two vectors $u, v,$ and scalar α

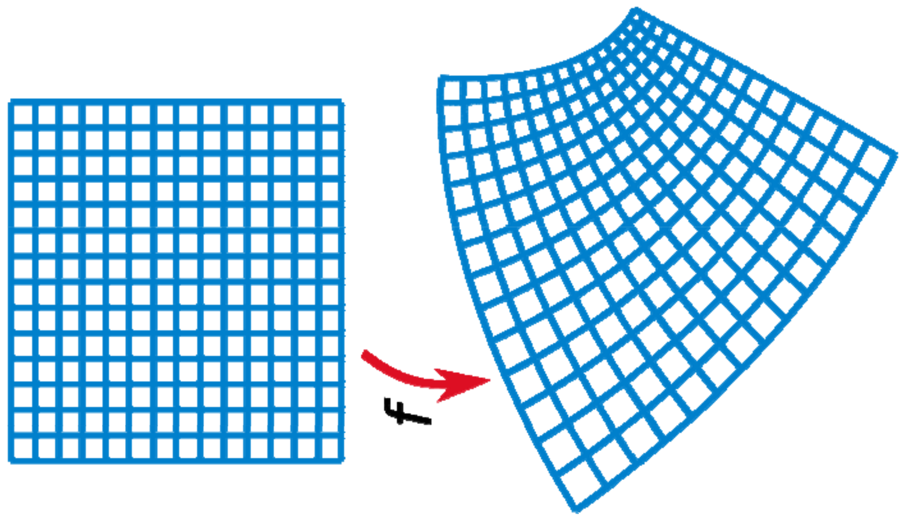
$$L(u) + L(v) = L(u + v)$$ **additivity**

$$\alpha \cdot L(u) = L(\alpha u)$$ **homogenity of degree 1**

Corollary: Lines stay lines ("Linear transformations preserve collinearity")

- Not true for an **arbitrary** transformation.

NON-LINEAR TRANSFORM



LINEAR TRANSFORMATION

homogenity of degree 1

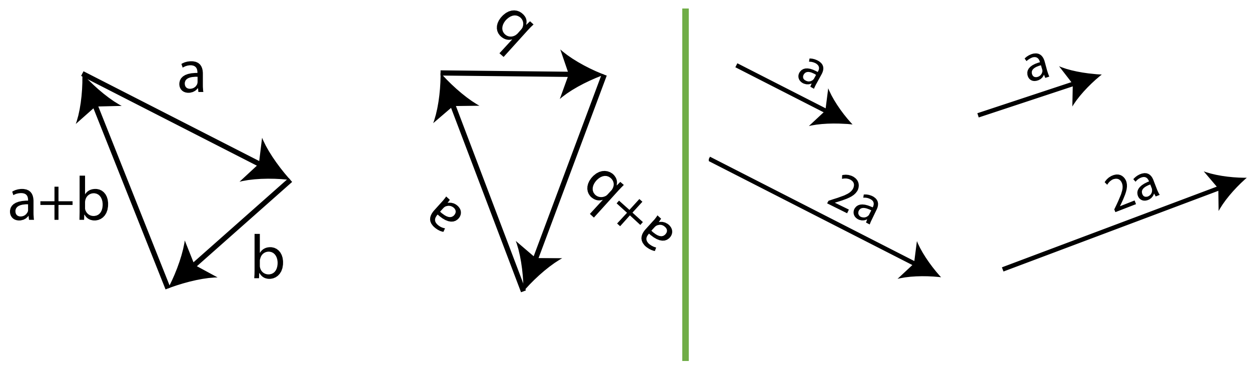
$$\alpha \cdot L(u) = L(\alpha u)$$

- In particular,

$$L(0) = 0$$

JUST TO MAKE SURE

- Rotations are linear
  - They are representable by a rotation matrix
  - But also we can check it directly:



MATRIX MULTIPLICATION

- Matrix multiplication satisfies those requirements:

additivity

$L(u) + L(v) = L(u + v)$ $\qquad$ $M(u + v) = Mu + Mv$

homogenity of degree 1

$\alpha \cdot L(u) = L(\alpha u)$ $\qquad$ $\alpha \cdot Mu = M \cdot (\alpha u)$

- So all the transformations expressed as matrices are LINEAR

What does this 2D transformation do?

A. Rotates by 90 deg
B. Scales by a factor of 2
C. Rotates by -90 deg
D. Nothing

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What does this 2D transformation do?

A. Rotates by 90 deg
B. Reflects the object
C. Rotates by -90 deg
D. Scales the object

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

TRANSLATION

- There's a minor glitch.
- Translation:

$$L: u \to u + b$$
$$L(0) = b \neq 0$$

Translation is not a linear transformation and can't be represented as a matrix operation in this space.

TRANSLATION

- There's a minor glitch.

- Translation:

$$L: u \to u + b$$

$$L(\alpha u) = \alpha u + b \neq \alpha(u + b) = \alpha L(u)$$

We need more power.