CPSC 314
Assignment 3

Due Wednesday Nov 9th, 2016, in class

> Answer the questions in the spaces provided on the question sheets. If you run out of space for an answer, use separate pages and staple them to your assignment.

Name: _____

Student Number: _____

Question 1		/ 5
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Question 4		/ 6
TOTAL		/ 49

1. (5 points) Scan Conversion
   Give the pseudocode for scan converting the region shown to the right.

   It is the portion of the circle on the right side of the line defined by fixed points $c = (1, 0)$ and $d = (2, 2)$. The circle is centered at (x_o, y_o) and has radius r. Use implicit equations to develop your solution.
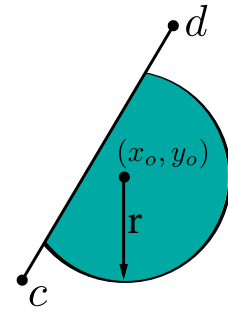
2 **Solution:**

For the non-vertical line cd, we can calculate its line equation as follows:

Let $y = kx + a$, then we have $k = \frac{y_d - y_c}{x_d - x_c} = 2$, $a = y_c - kx_c = -2$, so cd's line equation is $y = 2x - 2$.

Let $E_{cd}(x, y) = 2x - y - 2$, then for any point i with coordinate (x_i, y_i), if $E_{cd}(x_i, y_i) < 0$, i is on the left side of cd; if $E_{cd}(x_i, y_i) = 0$, i is on the boundary; if $E_{cd}(x_i, y_i) > 0$, i is on the right side of cd.

For circle, let $E_{circle}(x_i, y_i)$ be its equation, we have $E_{circle}(x_i, y_i) = (x_i - x_0)^2 + (y_i - y_0)^2 - r^2$. If $E_{circle}(x_i, y_i) < 0$, point i is inside this circle; if $E_{circle}(x_i, y_i) = 0$, point i is on the boundary; if $E_{circle}(x_i, y_i) > 0$, point i is outside.

We iterate over all points in the bounding box of the circle, which are points (x_i, y_i) satisfying $x_0 - r \leq x_i \leq x_0 + r$ and $y_0 - r \leq y_i \leq y_0 + ri$

So if the point i on the screen satisfied both $E_{circle}(x_i, y_i) < 0$ and $E_{cd}(x_i, y_i) > 0$, we will paint that pixel. Here's the pseudocode of the algorithm:


Algorithm 1 ScanConversion

1: **for** each pixel (x_i, y_i) in the bounding box **do**
2:     **if** $E_{ellipse}(x_i, y_i) < 0$ && $E_{cd}(x_i, y_i) > 0$
3:         **then** paint that pixel
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2. (9 points) Interpolation

   A triangle has device coordinates $P_1 = (1,4)$, $P_2 = (9,2)$, and $P_3 = (7,9)$. You wish to interpolate a value v for point $P = (7,5)$, given the value of v at the vertices:

   $v_1 = 2, v_2 = 6, v_3 = 5$.

   (a) (1 point) Sketch the triangle and the point P (this drawing will help you further down).

   Solution:
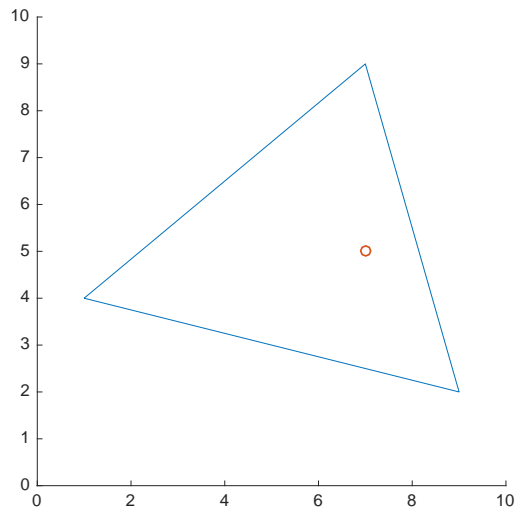
   (b) (4 points) Derive a plane equation for v as a function of x and y, i.e. $Ax + By + Cv + D = 0$. To do this, determine the constraints based on the given triangle points and their respective values v_i. You can use Matlab or an online linear solver to solve a set of linear equations for your plane parameters. Write down your derived equation. Compute the value v for the point P using the plane equation.

   Solution:

   In order to express v by x and y, we let $C = -1$, then we have $v = Ax + By + D$. Substituting the x, y and v in the equation by P_1, P_2 and P_3's values, we get a linear system:

   $$\begin{bmatrix} 1 & 4 & 1 \\ 9 & 2 & 1 \\ 7 & 9 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ D \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}.$$

   Solving this linear system gives us $A = 0.5, B = 0, D = 1.5$. So we have:

   $$v = 0.5x + 1.5$$

   Then we substitute the x and y by P's coordinate, and it gives us $v_P = 5$

(c) (4 points) Compute the barycentric coordinates for point P. Compute v for point P using the Barycentric coordinates.

Solution:

According to the definition of barycentric coordinate, we can express P's coordinate using the coordinates of P_1, P_2, and P_3 with:

$$P = \frac{A_{PP_2P_3}}{A_{P_1P_2P_3}}P_1 + \frac{A_{PP_1P_3}}{A_{P_1P_2P_3}}P_2 + \frac{A_{PP_1P_2}}{A_{P_1P_2P_3}}P_3$$

where $A_{XYZ} = \frac{1}{2}||\vec{XY} \times \vec{XZ}||$. So we have:

$$A_{P_1P_2P_3} = 0.5||(8,-2,0) \times (6,5,0)|| = 26$$
$$A_{PP_1P_2} = 0.5||(-6,-1,0) \times (2,-3,0)|| = 10$$
$$A_{PP_2P_3} = 0.5||(2,-3,0) \times (0,4,0)|| = 4$$
$$A_{PP_1P_3} = 0.5||(-6,-1,0) \times (0,4,0)|| = 12$$
$$P = \frac{4}{26}P_1 + \frac{12}{26}P_2 + \frac{10}{26}P_3$$

which means the barycentric coordinate of P is $(\frac{2}{13}, \frac{6}{13}, \frac{5}{13})$, and we can now calculate:

$$v_P = \frac{2}{13}v_1 + \frac{6}{13}v_2 + \frac{5}{13}v_3 = 5$$

the same results with (b).

3. (10 points) Clipping

   Suppose that a perspective view-volume is defined by

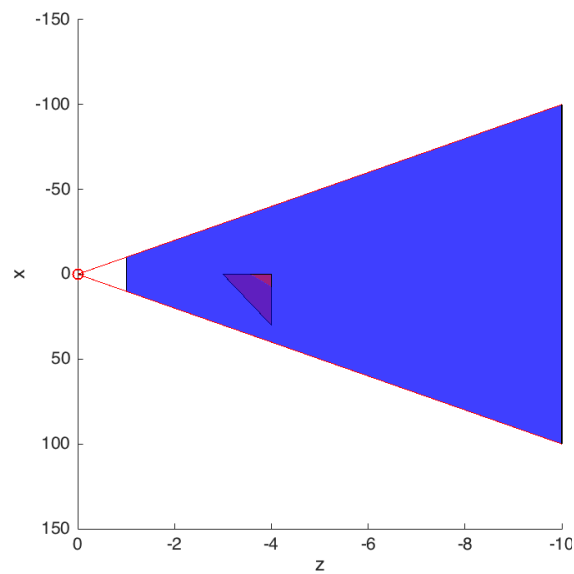   `near = -1, far = -10, bottom = -5, top = 5, left = -10, right = 10`.

   Consider the triangle defined by the view coordinate frame coordinates

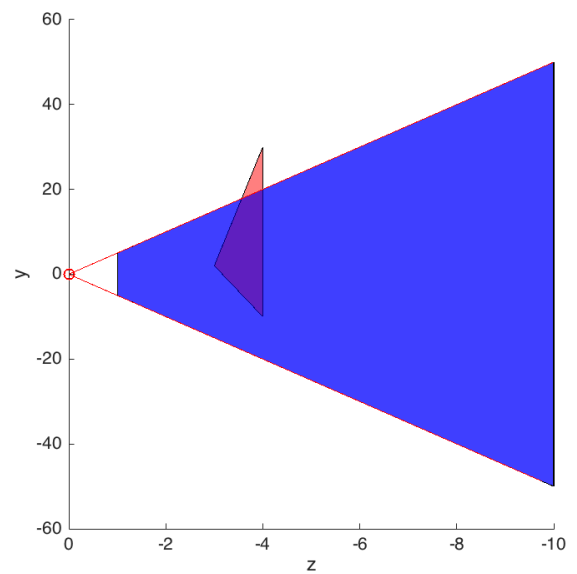 $$P_1 = (0, 2, -3), \qquad P_2 = (30, -10, -4), \qquad P_3 = (0, 30, -4).$$

   (a) (3 points) Sketch a side-view and top-view of the view-volume and the triangle.
       Solution:

Topview:

Sideview:

(b) (3 points) Determine if view-frustum culling should be applied to the triangle, i.e., if any vertices are "outside" with respect to any one of the six view frustum planes. Use the implicit plane equation

$$Ax + By + Cz + D = 0$$

Hint: remember what we discussed in class about plane equation and its sign.

Hint 2: If you find you have to do a lot of tedious calculations, think more. **Solution:**

So we need to calculate the plane equations for the 6 surfaces of the frustum to test their intersection with the triangle.

For the top plane, we already know three points on it. They are $(0, 0, 0)$, $(10, 5, -1)$, and $(-10, 5, -1)$. Because the top plane is not parallel to the y axis, we can let $B = -1$ and the plane equation becomes $Ax + Cz + D = y$. Then we can construct the following linear system by plug the coordinates of the three points into the function:

$$\begin{bmatrix} 0 & 0 & 1 \\ 10 & -1 & 1 \\ -10 & -1 & 1 \end{bmatrix} \begin{bmatrix} A \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}.$$

Then we got the top plane's equation $y = -5z$.

Similarly, we can calculate out the bottom plane's equation $y = 5z$, the left plane's equation $x - 10z = 0$, the right plane's equation $x + 10z = 0$, the near plane's equation $z = -1$, and the far plane's equation $z = -10$.

Then we can form a constraint for points inside the frustum:

$$\begin{cases} 5z \leq y \leq -5z \\ 10z \leq x \leq -10z \\ -10 \leq z \leq -1 \end{cases}$$

So there's only $P_3 = (0, 30, -4)$ not in the frustum, because it fails the test $5z \leq y \leq -5z$, which means P_3 is just above the frustum. This also tells us the polygon inside the frustum is a quadrangle, which is consisted of P_1, P_2 and the two intersection points on the top plane.

(c) (1 point) Based on your work for the question above, determine the view-frustum planes that the triangle intersects. **Solution:**

See solution in 3(b).

(d) (3 points) **Solution:**

To calculate the two intersection points, we need P_3P_1's and P_3P_2's line equations. They are $y = -28z - 82, x = 0$ for P_3P_1 and $4x + 3y = 90, z = -4$ for P_3P_2.[1] Combining them with the top plane's equation $y = -5z$ gives us the two intersection points $P_4 = (0, \frac{410}{23}, -\frac{82}{23})$ and $P_5 = (\frac{15}{2}, 20, -4)$. So the clipped polygon is a quadrangle consisted of $P_1P_2P_4P_5$.

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[1]See http://tutorial.math.lamar.edu/Classes/CalcIII/EqnsOfLines.aspx for 3D line equation.

4. (6 points) Local Illumination

Sketch the illumination for the following scene when computed using the Phong illumination model. The scene is viewed from above using an orthographic projection and is lit by the single light source L. Draw 4 curves (overlay these in the graph below), one for each of ambient, diffuse, specular, and total illumination. The Phong illumination model is given by:

$$I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (R \cdot V)^n$$

with the following values:

$$I_a = I_d = I_s = 1.0, k_a = 0.2, k_d = 0.8, k_s = 0.8, n = 10$$

Hint: it may be useful to draw vertical lines marking critical points the surface. We do not expect numerically accurate plots, but do expose the characteristics of the curves.