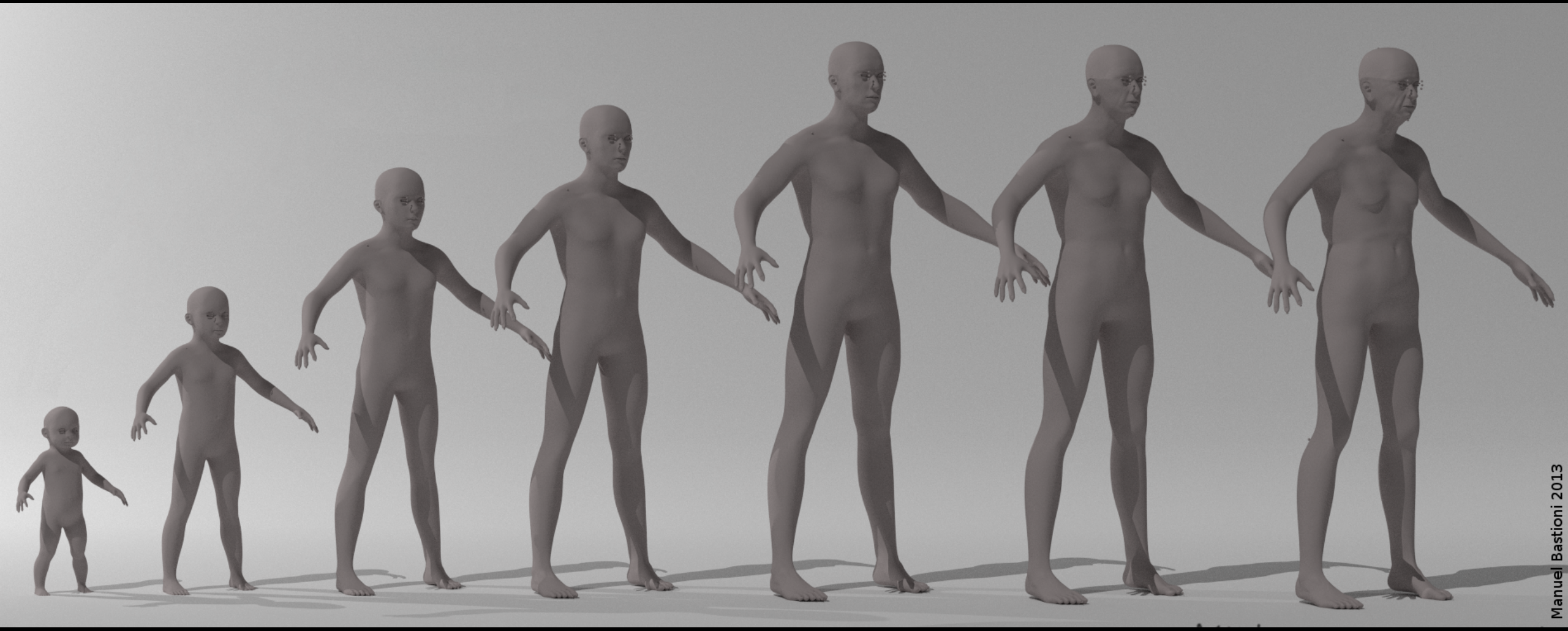
32 – GEOMETRIC MODELING

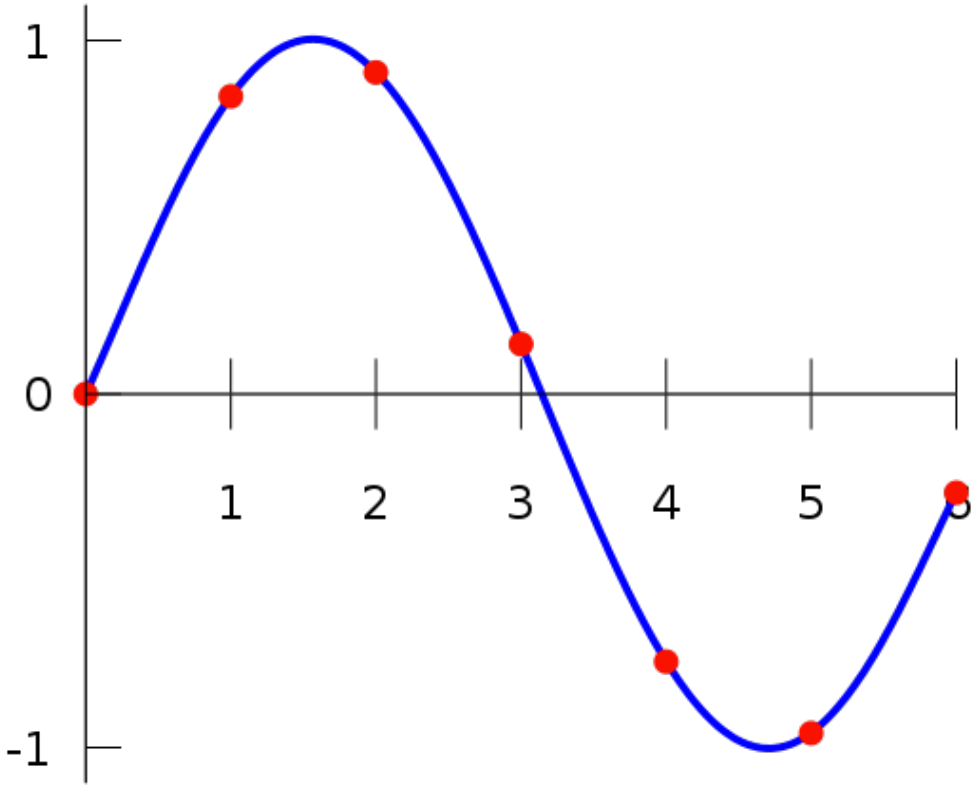


Textbook: 22

Mikhail Bessmeltsev

INTERPOLATION

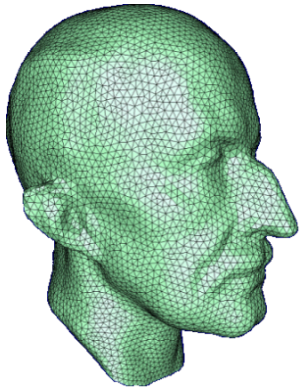
- While we were thinking how to interpolate a function,
- We invented ways to draw curves!



3D GEOMETRY

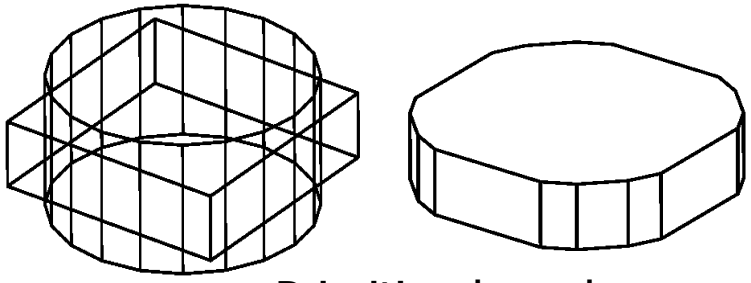
- Mathematical models of real world shapes
  - Most common: Boundary representations

Freeform – smooth surface

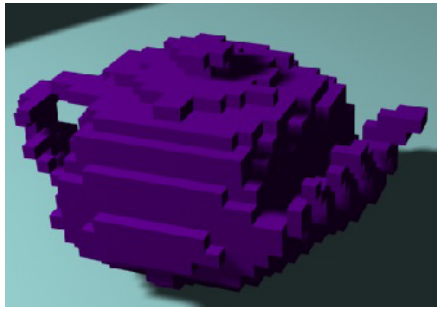


Mesh – polygonal surface

  - Alternative: Volumetric representations

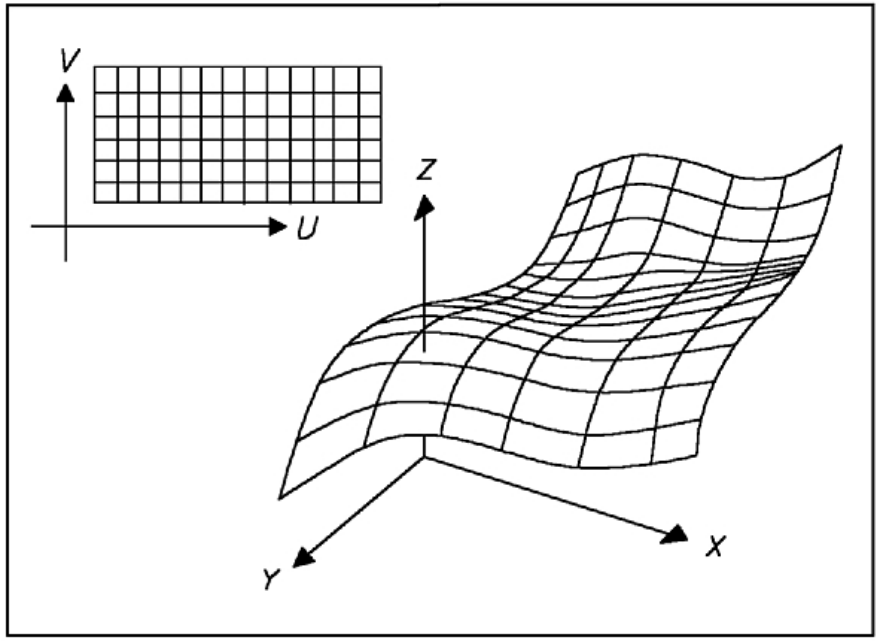


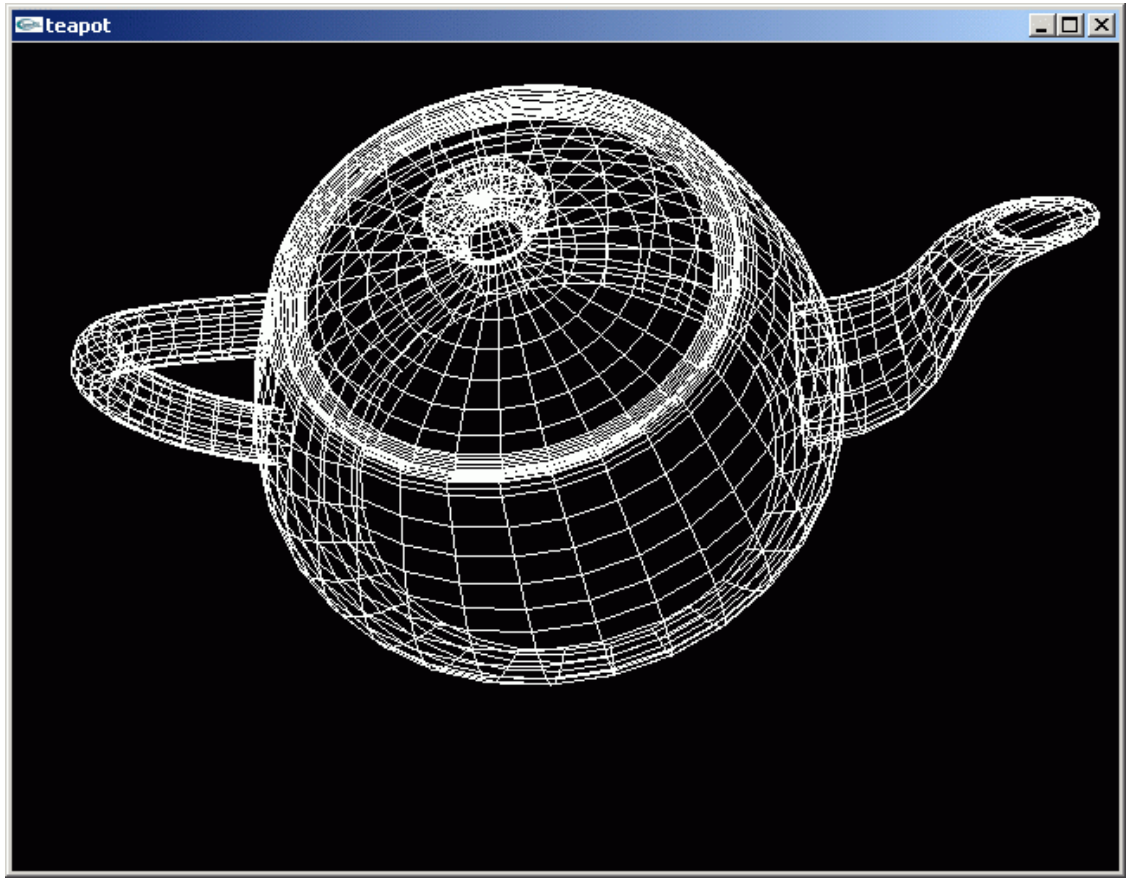
Primitive based



Voxel based

TENSOR SPLINE SURFACES





FROM CURVES TO SURFACES: TENSOR SPLINES

- Curve: linear combination of basis functions $B_i(u)$ with coefficients P_i

$$C(u)=\sum_{i=0}^{n} P_i B_i(u)$$

- Surfaces: treat surface as a curve of curves
- Assume P_i is not constant, but a function of second parameter v: $P_i(v)=\sum_{j=0}^{m} Q_{ij} B_j(v)$

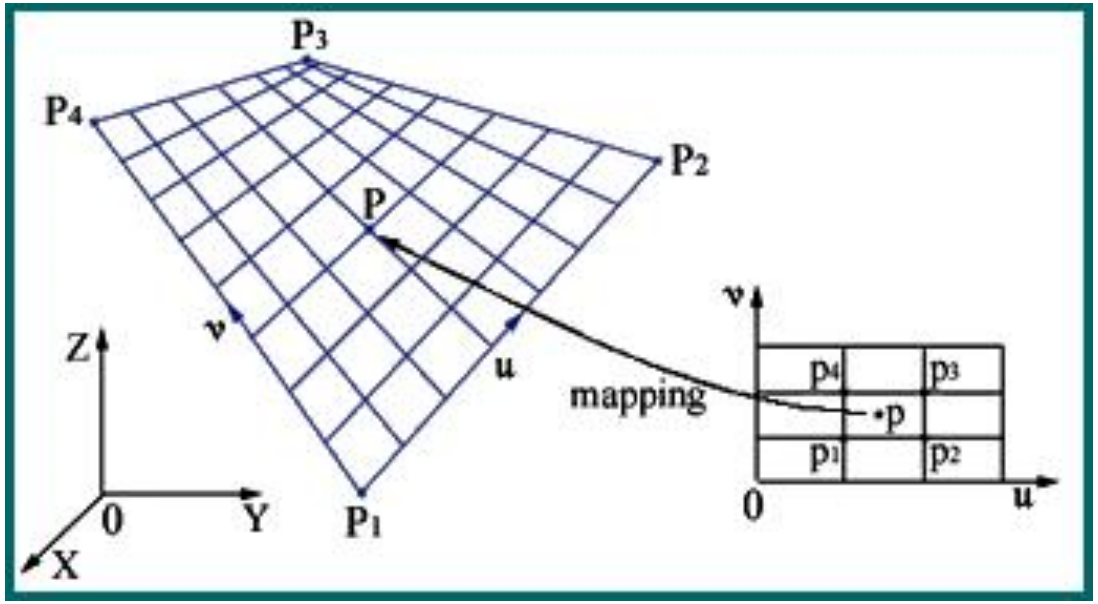
$$C(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m} Q_{ij} B_j(v) B_i(u)$$

BILINEAR PATCHES

- Bilinear interpolation of four 3D points

$$P_{00}, P_{01}, P_{10}, P_{11}$$

  - similar to how we defined a line segment



BILINEAR PATCHES

Given $P_{00}, P_{01}, P_{10}, P_{11}$ associated parametric bilinear surface is:

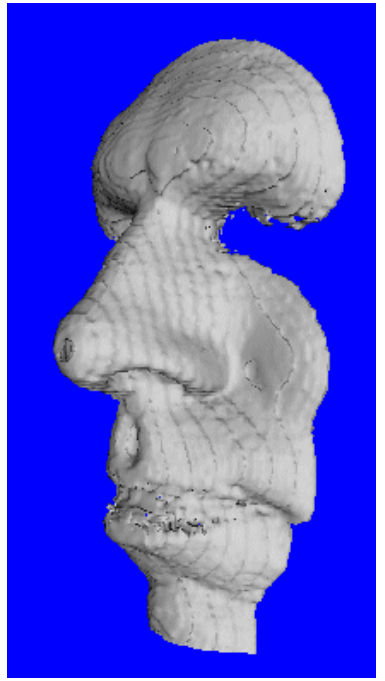
$$u, v \in [0,1]$$

$$P(u,v) = (1-u)(1-v)P_{00} + (1-u)vP_{01} + u(1-v)P_{10} + uvP_{11}$$

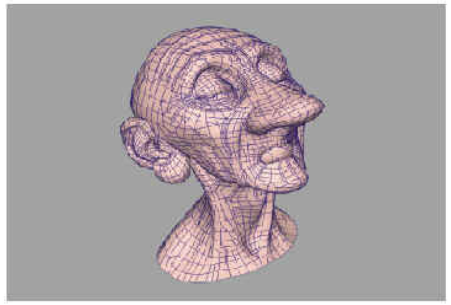
- Questions:
  - What does an isoparametric curve of a bilinear patch look like ?
  - When is a bilinear patch planar?

GEOMETRY CREATION (MESHES)

- Reconstruction:
  - Capture real life shapes & convert to mesh
  - Inputs:
    - Points (laser scanner)
    - 3D images
- Modeling (user driven)

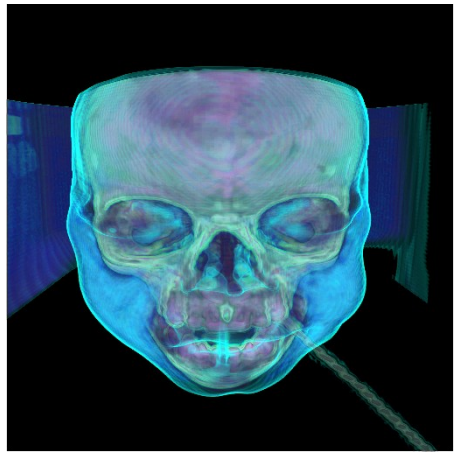


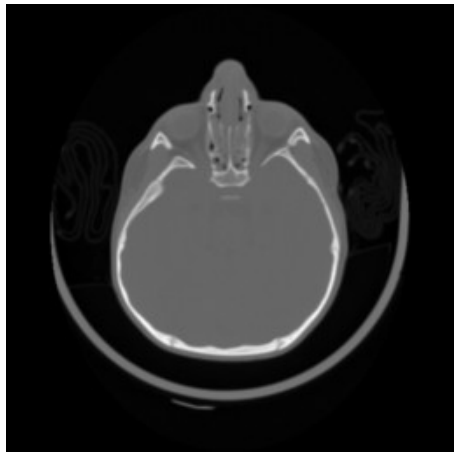
- Will see two examples
  - Marching Cubes – reconstruction from images
  - Subdivision – generating smooth meshes from coarse user-given “cages”



RECONSTRUCTION FROM VOLUME DATA

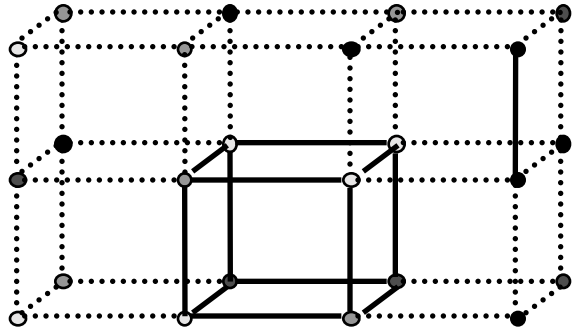
- Volume data: view as voxel grid with values at vertices
  - Defines implicit function in 3D
    - interpolate grid values
- Shape defined by isosurface
  - isosurface = set of points with constant isovalue α
  - separates values above α from values below
- Reconstruction – Extract triangulation approximating isosurface





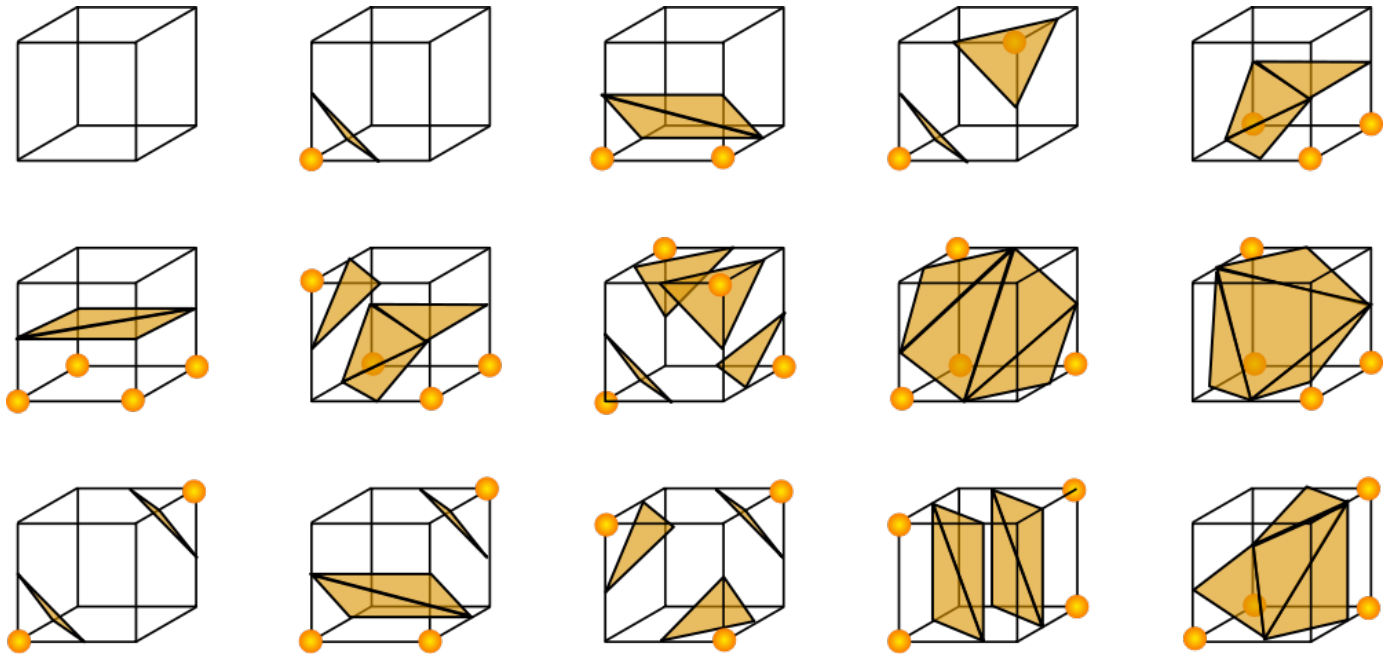
VOXELS

- Voxel – cube with values at eight corners
  - Each value is above or below isovalue α
- 2^8=256 possible configurations (per voxel)
  - reduced to 15 (symmetry and rotations)
- Each voxel is either:
  - Entirely inside isosurface
  - Entirely outside isosurface
  - Intersected by isosurface
- MC main observation: Can extract triangulation independently per voxel



BASIC MC ALGORITHM

- For each voxel produce set of triangles
  - Based on above/below corner configuration
  - Empty for non-intersecting voxels
  - Approximate surface inside voxel



CONFIGURATIONS

- For each configuration add 1-4 triangles to isosurface

- Isosurface vertices computed by:
  - Interpolation along edges (according to grid values)
    - better shading, smoother surfaces
  - Default – mid-edges

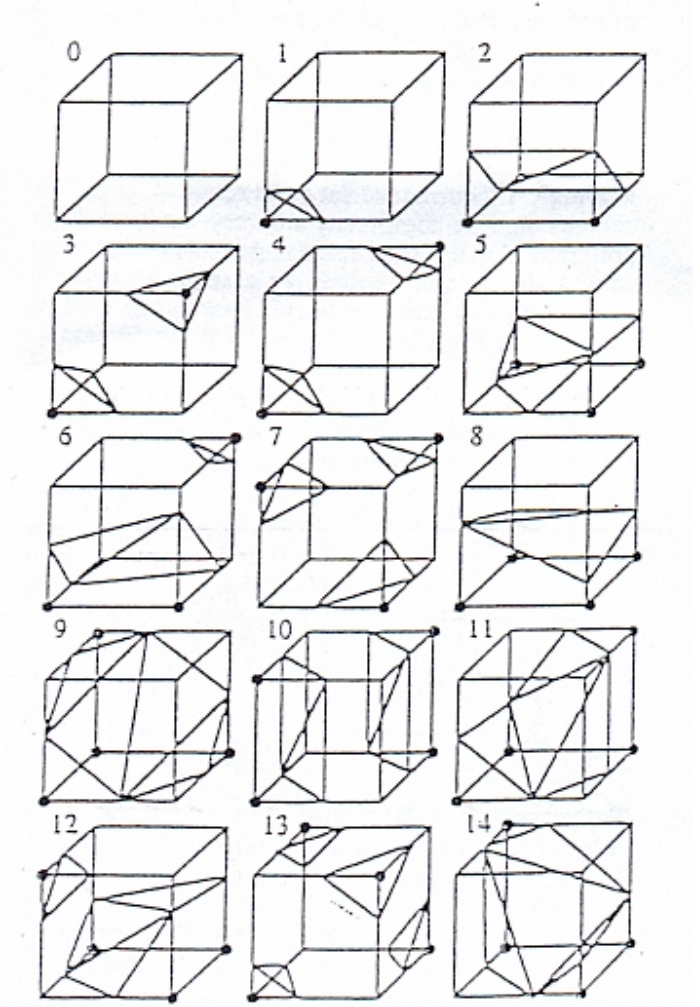
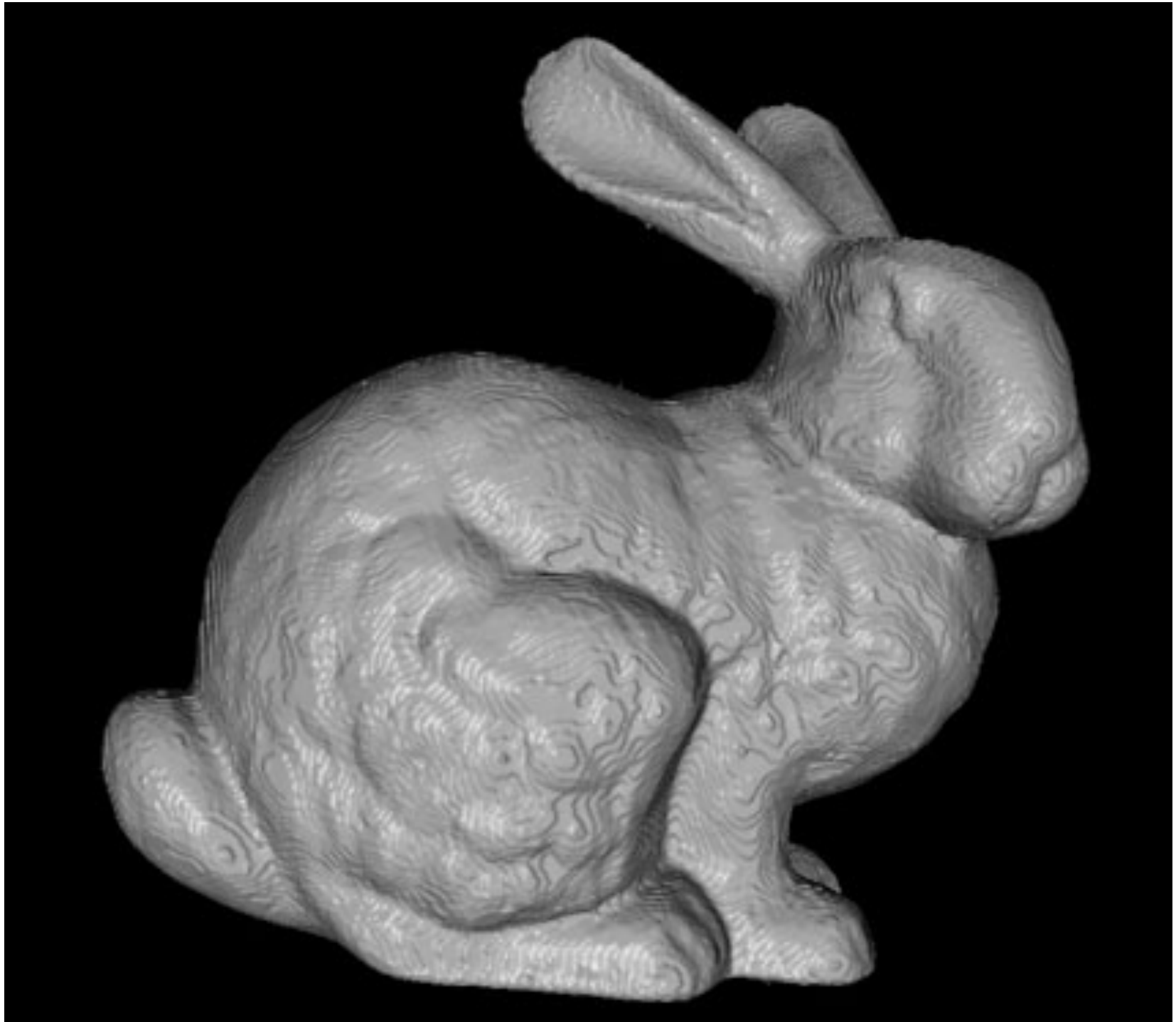
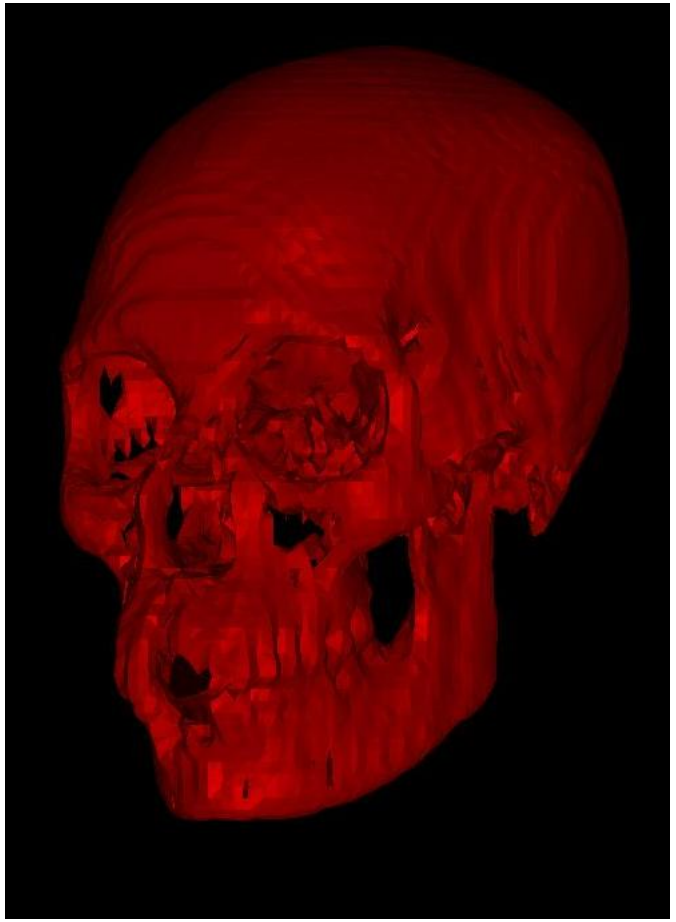


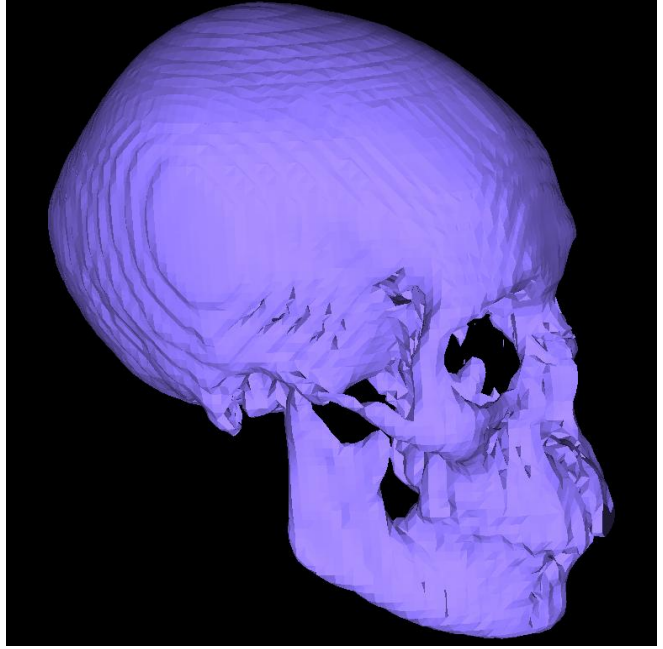
Figure 2. Configurations.

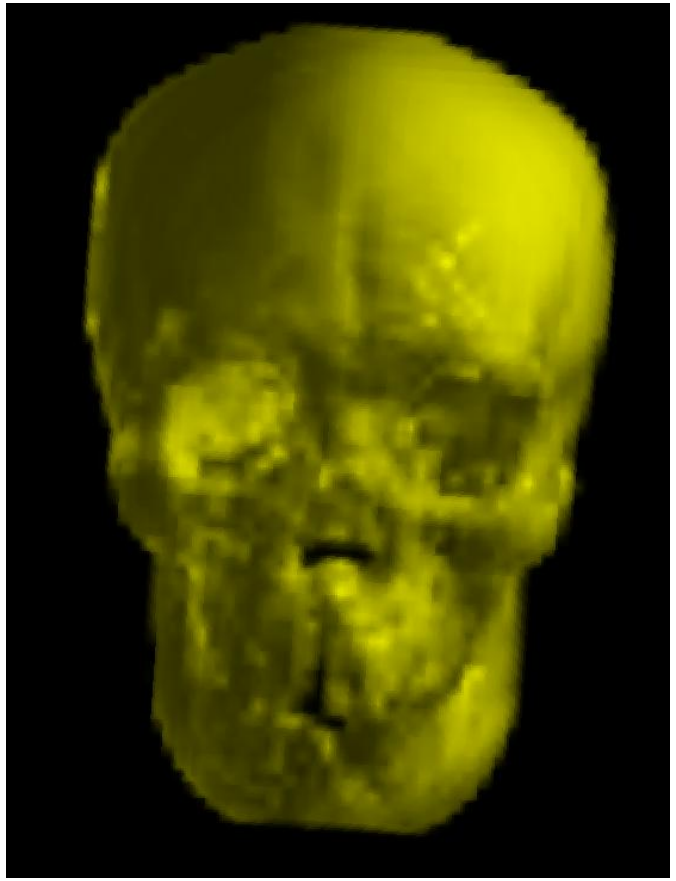
EXAMPLE



EXAMPLE



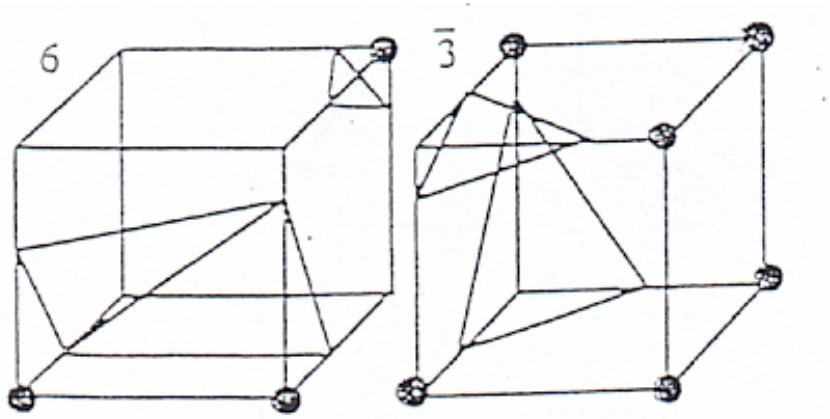




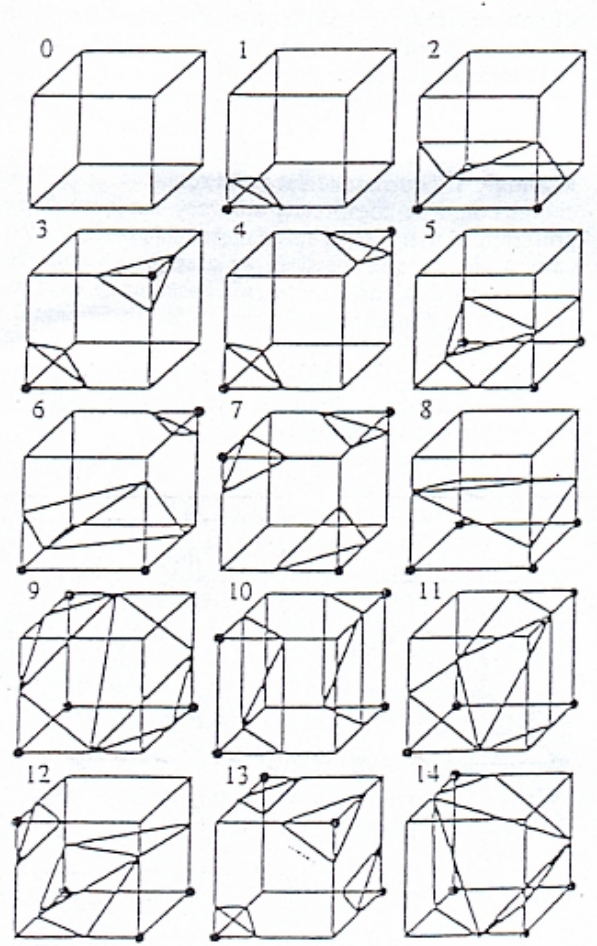
CONSISTENCY PROBLEM

- Can produce non-manifold results
  - Isovalue surfaces with “holes”

- Example:

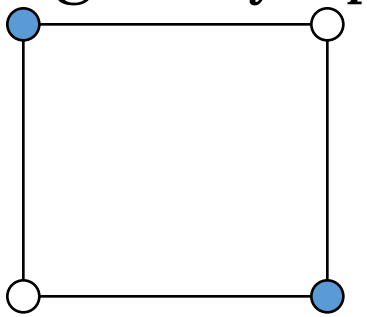


  - Voxel with configuration 6 sharing face with complement of configuration 3

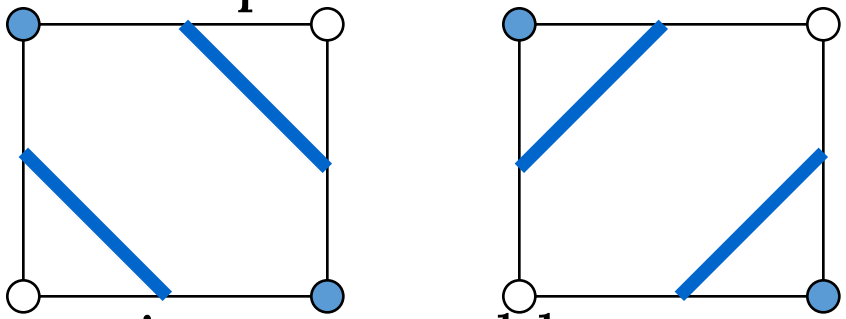


AMBIGUOUS FACES

- Face containing two diagonally opposite marked grid points and two unmarked ones



- Two locally valid interpretations



- Source of MC consistency problem

CONSISTENCY

- Problem:
  - Connection of isosurface points on shared face done one way on one face & another way on the other
- Need consistency → use different triangulations
- If choices are consistent get topologically correct surface

SOLUTION

- For each problematic configuration have more than one triangulation

- Distinguish different cases by choosing pairwise connections of four vertices on common face
  - Example:

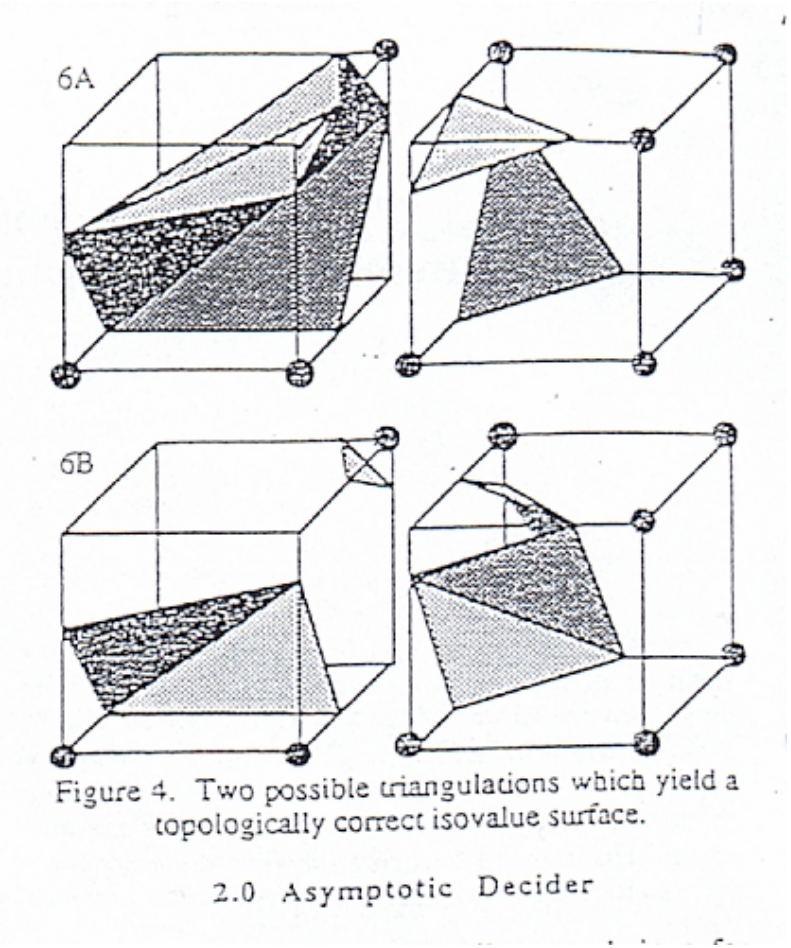
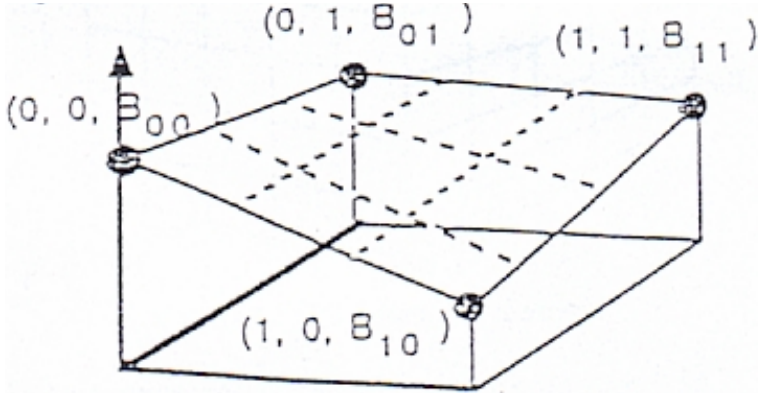


Figure 4. Two possible triangulations which yield a topologically correct isovalue surface.

2.0 Asymptotic Decider

ASYMPTOTIC DECIDER

- Interpolate value at quad center
- Use bilinear interpolation



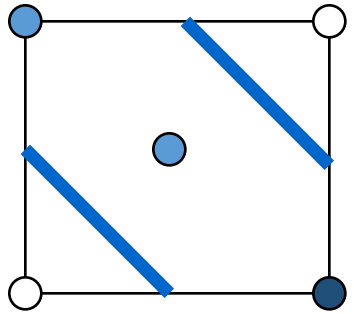
$$B(s,t) = \begin{pmatrix} 1-s & s \end{pmatrix} \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \begin{pmatrix} 1-t \\ t \end{pmatrix}$$

$$\{(s,t): 0 \le s \le 1, \quad 0 \le t \le 1\}$$

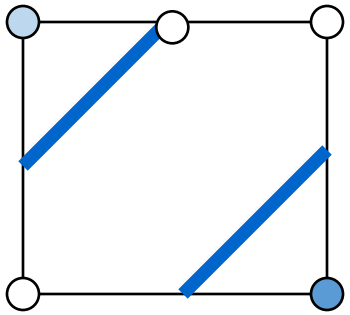
- B_{ij} - isovalues at face corners

AMBIGUOUS FACES

If center value closer to “blue” choose



Else



VARIOUS CASES

- Some configurations have no ambiguous faces → no modifications
- Other configurations need modifications according to number of ambiguous faces
  - Apply decider to each face to decide on triangulation template

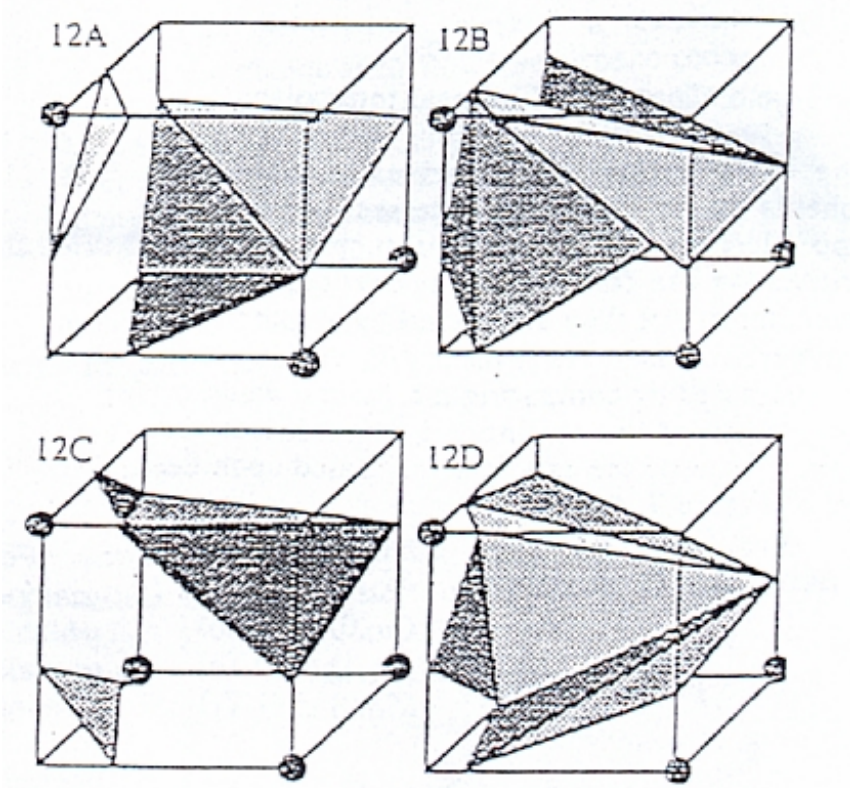


Figure 11.