Vector Graphics and Topology

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Vector Graphics Complexes

SIGGRAPH slides: http://www.dalboris.com/research/vgc
Demo of VPaint: http://vpaint.org
What is topology?
Why is it relevant to vector graphics?

Topology 101: neighbourhood, open set, closed set, interior, boundary, compactness, ... (cf. Wikipedia 😊)

Topology 102: homeomorphism (things that “look” similar)
Surface = compact space everywhere locally homeomorphic to $\mathbb{R}^2$
What is topology?
Why is it relevant to vector graphics?

Topology is the formal framework to define the concept of surfaces. Vector Graphics is all about manipulating surfaces (defined by a 2D boundary).
So understanding the topology of surfaces is of primary importance for vector graphics (and many other fields of computer graphics).

Different surfaces => Different properties

Let’s have some fun with paper and scissors!

Take a rectangle, choose two points on its boundary, and cut the surface along a curve connecting those two points.
You get two surfaces!

Do the same with a cylinder...
Möbius strips

Well... that was unfair: a cylinder has two “edges”, while a rectangle/disk has only one. Obviously these surfaces are different.

What about a Möbius strip? (class activity. Summary: build one, observe that it has only one edge, like a disk. Cut it by joining two points on the boundary: you get either one or two surfaces)

With Vector Graphics, you don’t really know if you have a disk or a Möbius strip.... It’s more a matter of interpretation. (demo)

Polygonal scheme
The classification of surfaces

Every surface is homeomorphic to one of the following:

- Sphere with holes
- One or several torus glued together
- One or several projective planes glued together

$k = \text{number of holes / boundaries}$

$g = \text{genus}$

What happens when you cut them?
What happens when you cut them?

Cut it differently?
Cut it differently?

Cutting other things?

Experiment by yourself 😊
Or have a look at the VGC technical report!