Chapter 13
Geometric Modeling
Meshes & Subdivision

Meshes
- Simplest boundary representation—polygonal mesh
- Properties
  - Triangular/Quad
  - Manifold
- Simplicity of representation & manipulation
- Base representation for scanned data
- Input to hardware rendering algorithms (Z-buffer, polygon fill, etc.)
- Manipulation algorithms well defined (computational geometry)

Processing
- Construction
  - From scans
  - From free-form/volumetric data
- Compression—typical meshes are very large due to
  - Origin (scan)
  - Required LOD
- Manipulation
  - Note: No (u,v) parameterization
- Smoothing
  - Simulate via lighting methods
  - Refine - subdivision

Subdivision Curves and Surfaces
- Subdivision—given polyline/polygon/polyhedron recursively modify its vertices to achieve smooth curve

Corner Cutting
The 4-point scheme

The 4-point scheme

The 4-point scheme

The 4-point scheme

The 4-point scheme

The 4-point scheme
The 4-point scheme

Subdivision curves

- Non interpolatory subdivision schemes
  - Corner Cutting

- Interpolatory subdivision schemes
  - The 4-point scheme
Subdivision curves

Basic concepts of Subdivision
- Subdivision curve generated by repeatedly applying subdivision operator to given polygon
- Each iteration:
  - Increase number of vertices (approximately) * 2
- Initial polygon - control polygon
- Central questions:
  - Convergence: Given a subdivision operator and a control polygon, does the subdivision process converge?
  - Smoothness: Does subdivision converge to smooth curve?

Subdivision schemes for surfaces
- Each iteration:
  - Subdivision refines control net (mesh)
  - Increase number of vertices (approximately) * 4
  - Mesh vertices converge to limit surface
  - Every subdivision method has:
    - Method to generate net topology
    - Rules to calculate location of new vertices

Triangular subdivision
- Works only for triangular meshes (control nets)
- Every face replaced by 4 new triangular faces
- Two kinds of new vertices:
  - Green vertices are associated with old edges
  - Blue vertices are associated with old vertices

Loop’s scheme
- New vertex = weighted average of old vertices
- List of weights - subdivision mask or stencil
  - Rule for new blue vertices (n - vertex valence)
  - Rule for new green vertices

The original control net
After 1st iteration

After 2nd iteration

After 3rd iteration

The limit surface

Limit surfaces of Loop’s subdivision is \( C^2 \) almost everywhere

Butterfly scheme

Interpolatory scheme

New blue vertices inherit location of old vertices

New green vertices calculated by following stencil:

The original control net
Limit surfaces of Butterfly subdivision are $C^1$, but do not have second derivative.

Properties:
- Require regular connectivity (valence 6) to work well
- Easy to implement (efficiency...)
- Local support
- Allow LOD
- Continuous
Drawbacks

- Not always intuitive
- Can have artifacts
- Hard to control