University of British Columbia
CPSC 314 Computer Graphics
Sep-Dec 2009

Tamara Munzner
(guest lecturer)

Viewing/Projections

Week 5, Wed Oct 6

News

- assignment 1 posted

Viewing
(Review?)

Using Transformations

- three ways
  - modelling transforms
    - place objects within scene (shared world)
    - affine transformations
  - viewing transforms
    - place camera
    - rigid body transformations: rotate, translate
  - projection transforms
    - change type of camera
    - projective transformation

Rendering Pipeline

Scene graph
Object geometry

↓

Modelling Transforms

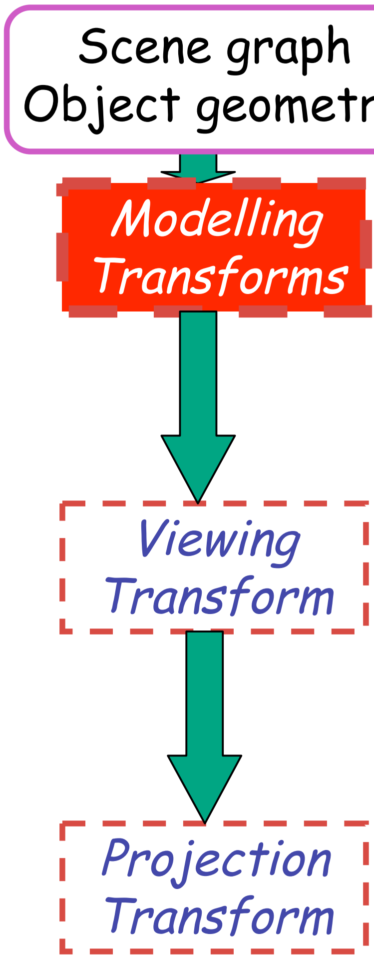
↓

Viewing Transform

↓

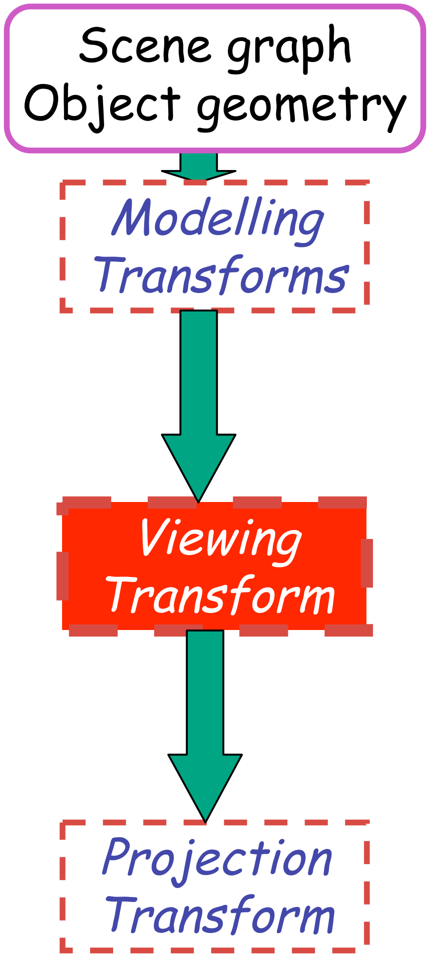
Projection Transform

Rendering Pipeline

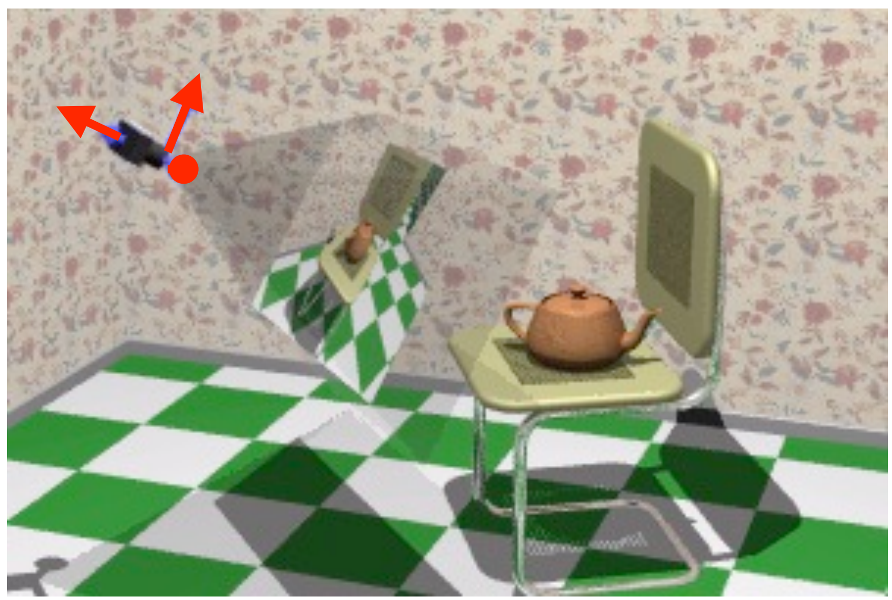


- result
  - all vertices of scene in shared 3D world coordinate system

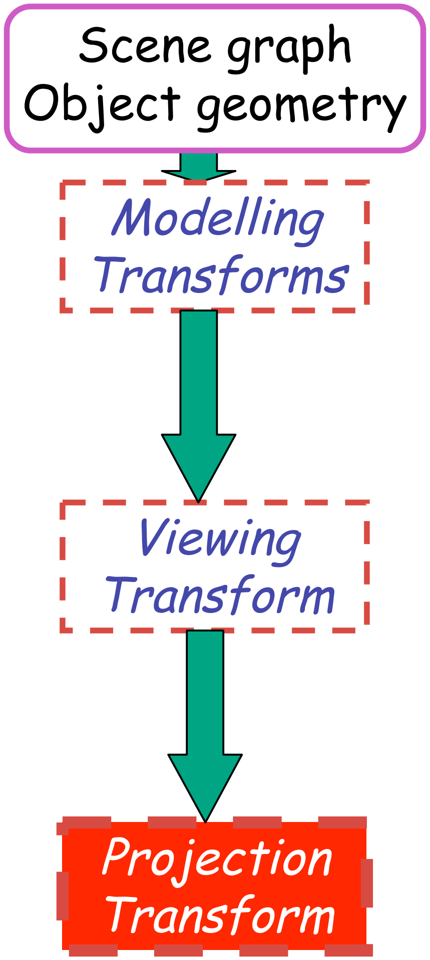
Rendering Pipeline



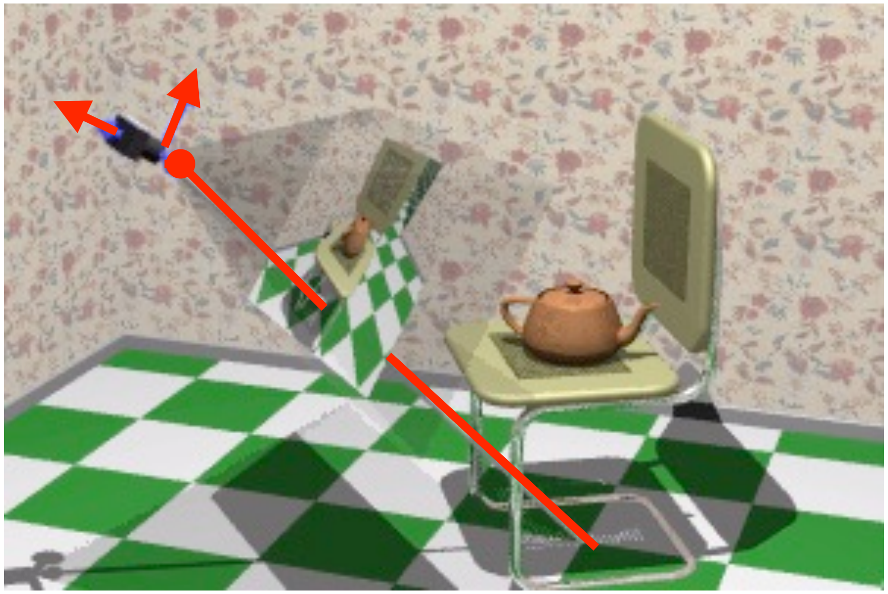
- result
  - scene vertices in 3D view (camera) coordinate system



Rendering Pipeline



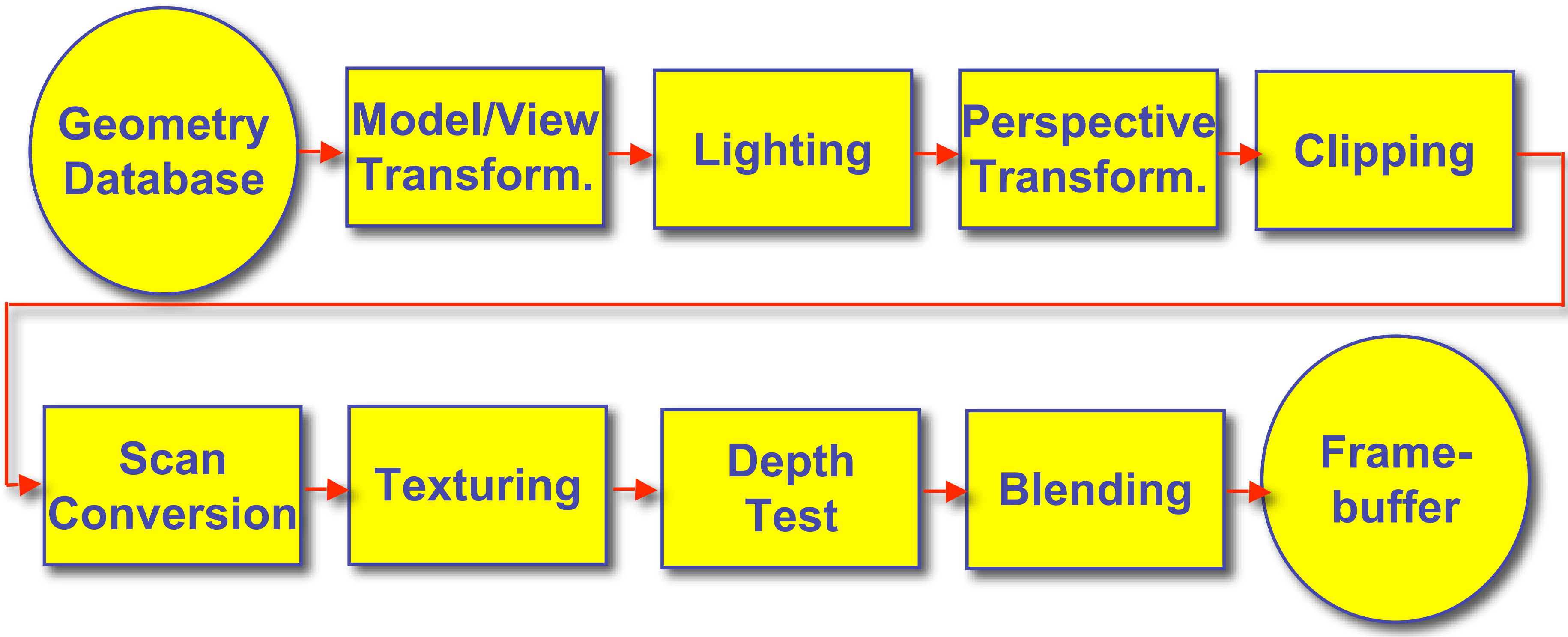
- result
  - 2D screen coordinates of clipped vertices



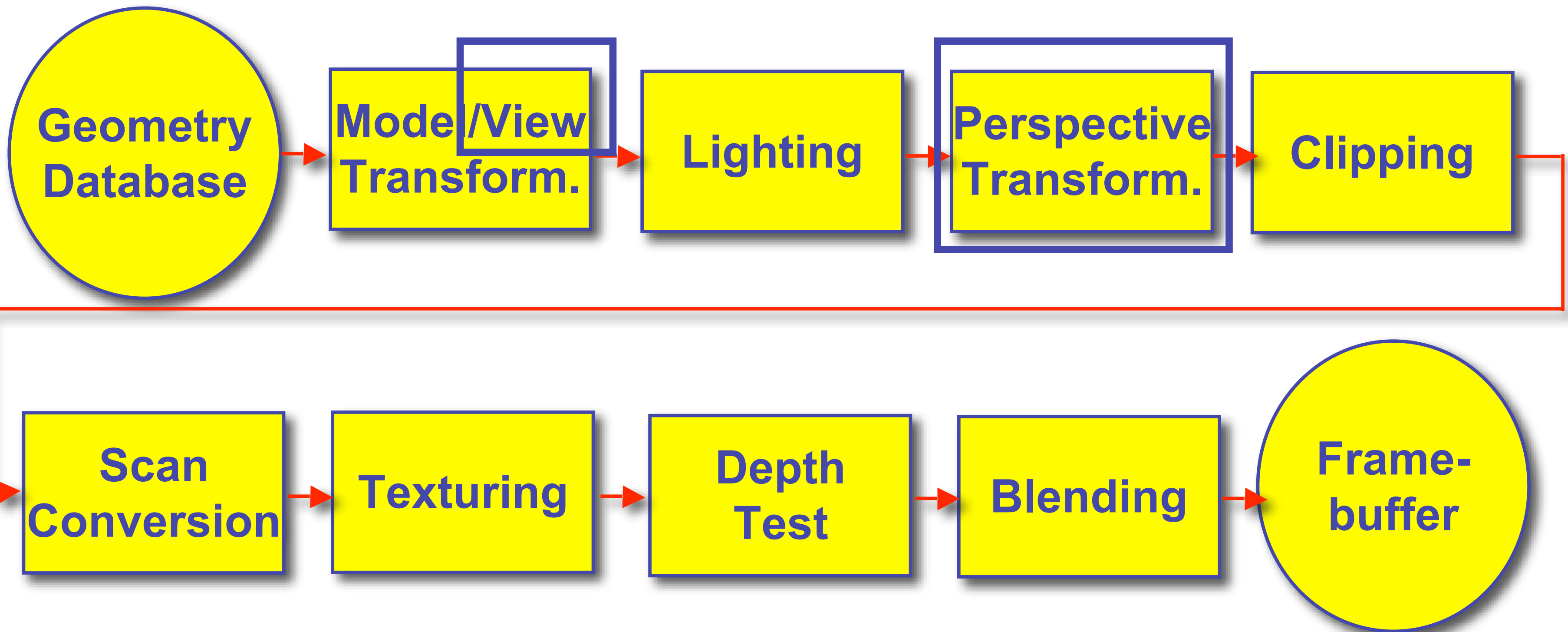
Viewing and Projection

- need to get from 3D world to 2D image
- projection: geometric abstraction
  - what eyes or cameras do
- two pieces
  - viewing transform:
    - where is the camera, what is it pointing at?
  - perspective transform: 3D to 2D
    - flatten to image

Rendering Pipeline



Rendering Pipeline



OpenGL Transformation Storage

- modeling and viewing stored together
  - possible because no intervening operations
- perspective stored in separate matrix

- specify which matrix is target of operations
  - common practice: return to default modelview mode after doing projection operations

```
glMatrixMode(GL_MODELVIEW);
glMatrixMode(GL_PROJECTION);
```

Coordinate Systems

- result of a transformation
- names
  - convenience
    - mouse: leg, head, tail
  - standard conventions in graphics pipeline
    - object/modelling
    - world
    - camera/viewing/eye
    - screen/window
    - raster/device

Projective Rendering Pipeline

object
OCS
O2W
modeling transformation
world
WCS
W2V
viewing transformation
viewing
VCS
V2C
projection transformation
clipping
CCS
C2N
perspective divide
normalized device
NDCS
N2D
viewport transformation
device
DCS

OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

Projections I

Pinhole Camera

- ingredients
  - box, film, hole punch
- result
  - picture

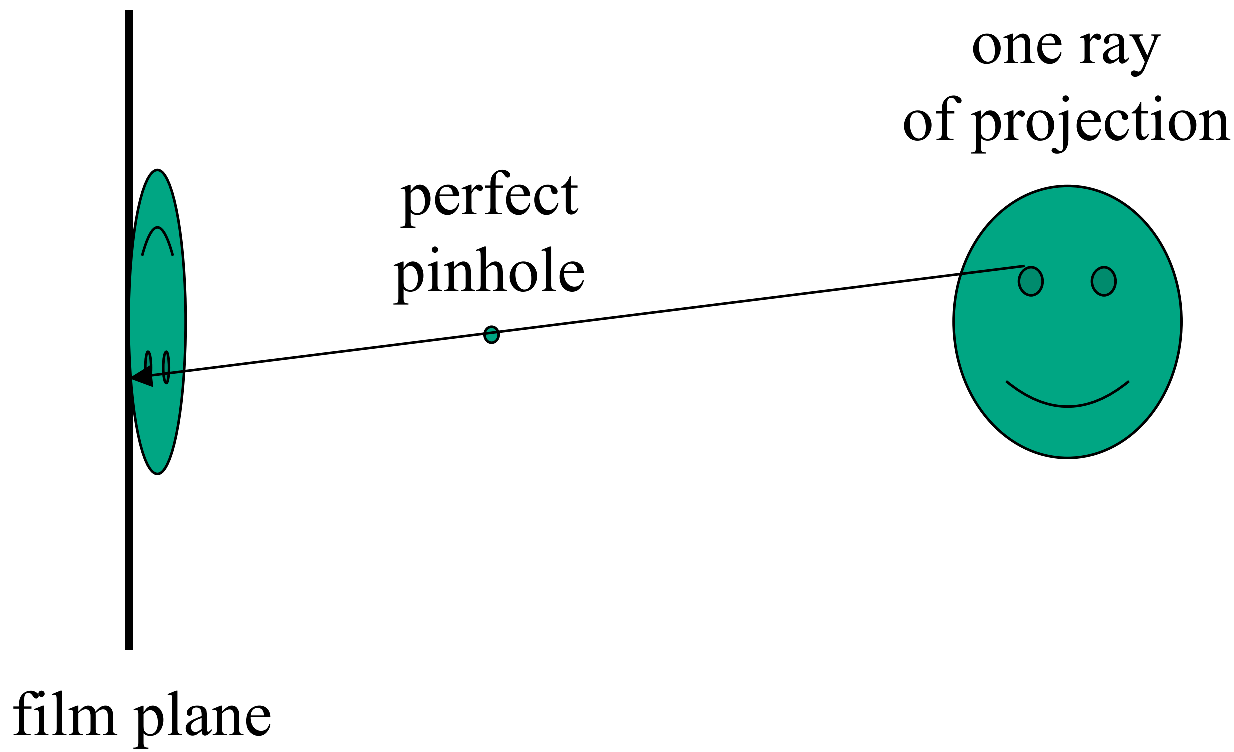
www.kodak.com

www.pinhole.org

www.debevec.org/Pinhole

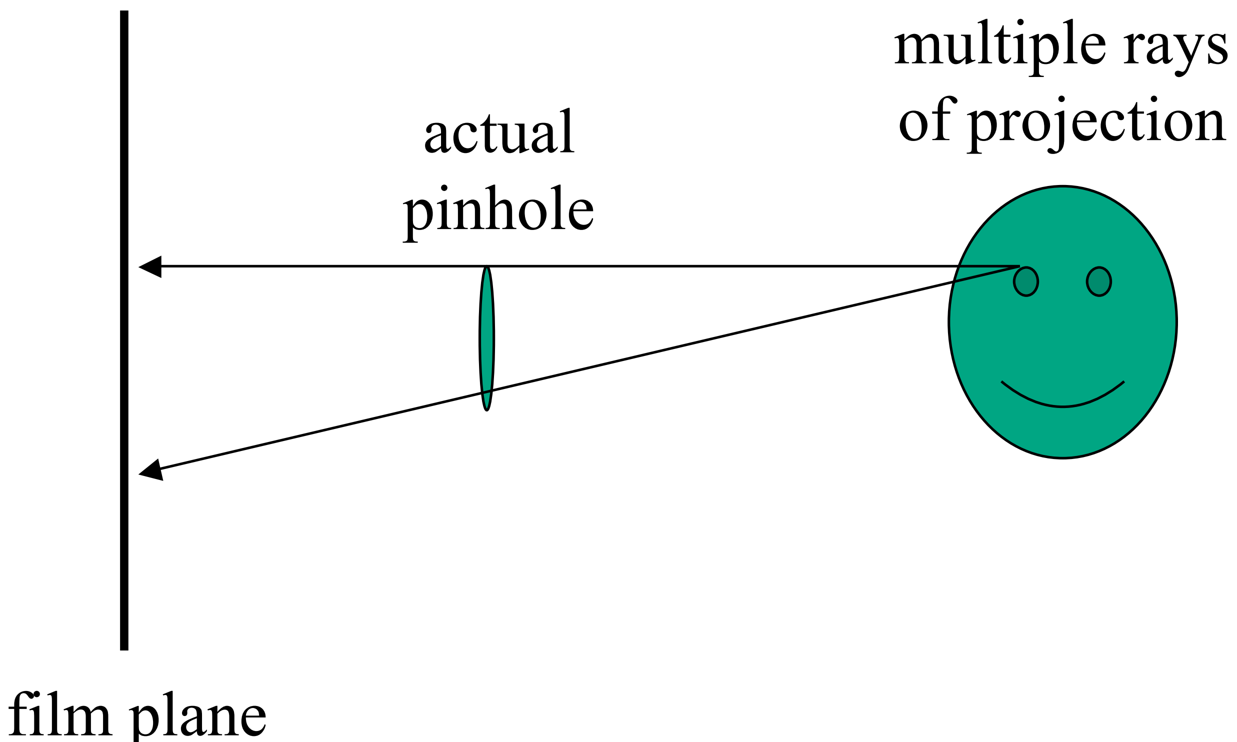
Pinhole Camera

- theoretical perfect pinhole
  - light shining through tiny hole into dark space yields upside-down picture



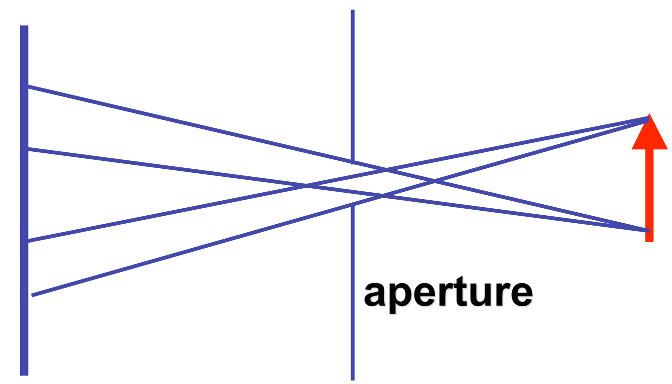
Pinhole Camera

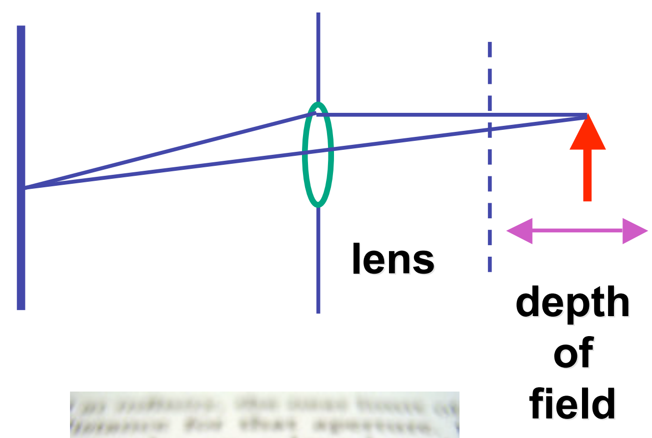
- non-zero sized hole
  - blur: rays hit multiple points on film plane

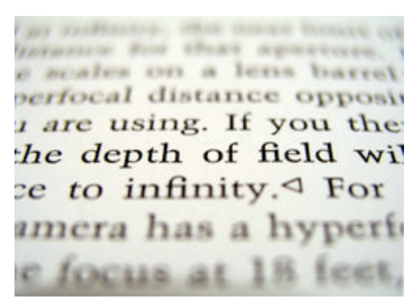


Real Cameras

- pinhole camera has small aperture (lens opening)
  - minimize blur
- problem: hard to get enough light to expose the film
- solution: lens
  - permits larger apertures
  - permits changing distance to film plane without actually moving it
    - cost: limited depth of field where image is in focus



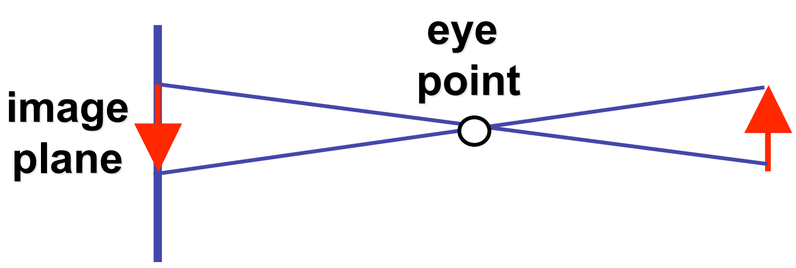




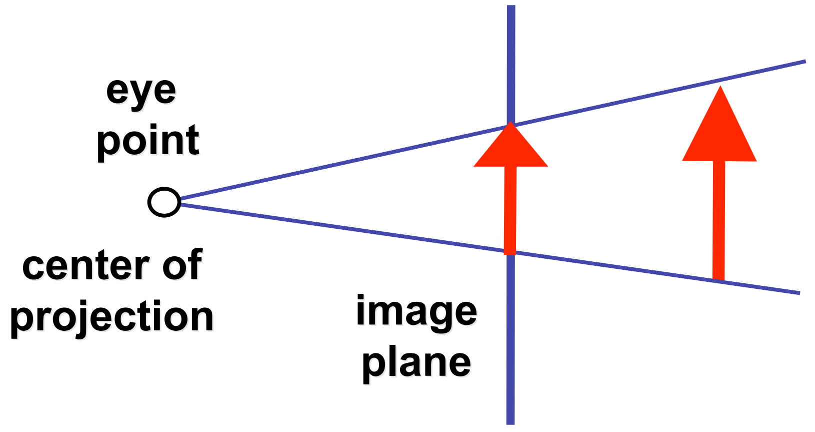
http://en.wikipedia.org/wiki/Image:DOF-ShallowDepthofField.jpg

Graphics Cameras

- real pinhole camera: image inverted

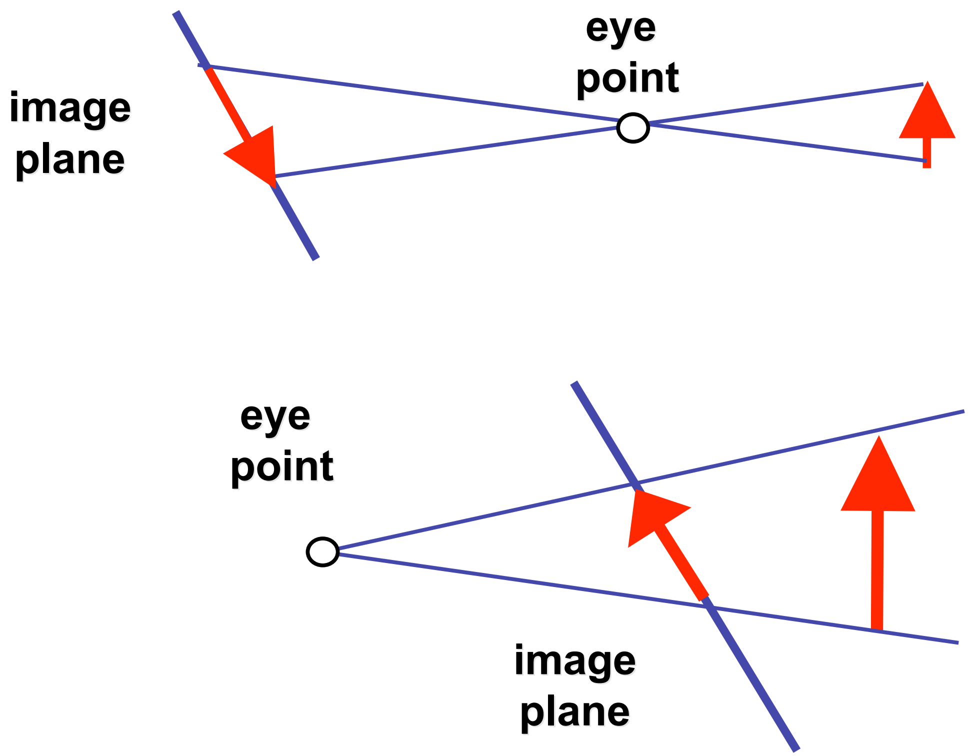


- computer graphics camera: convenient equivalent



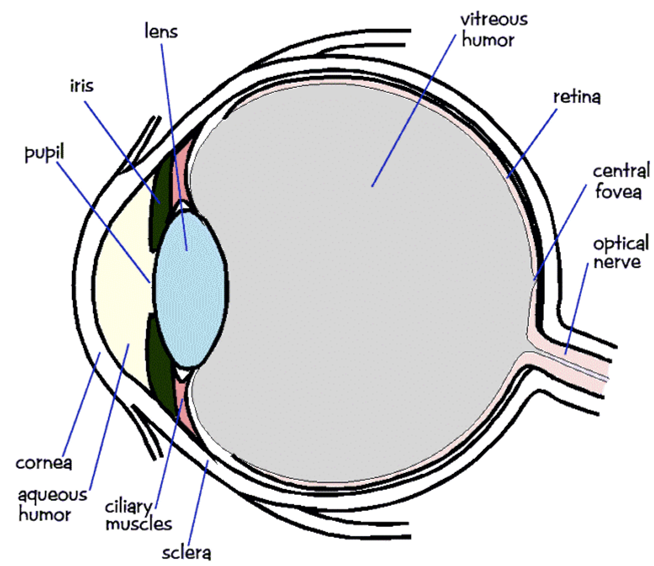
General Projection

- image plane need not be perpendicular to view plane



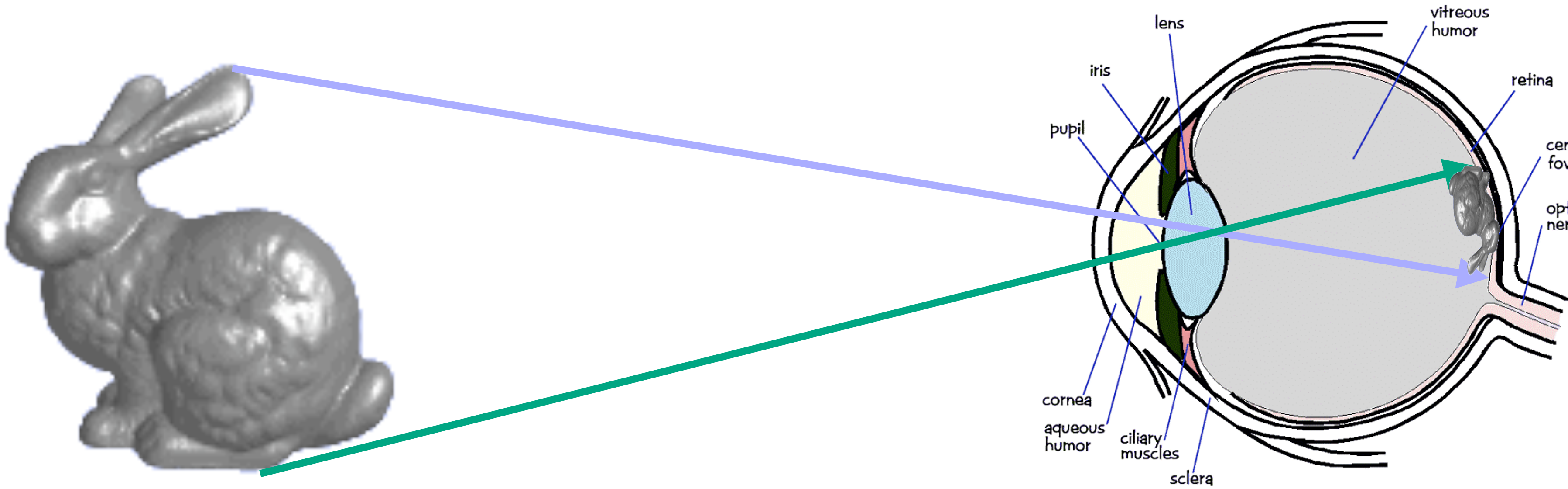
Perspective Projection

- our camera must model perspective



Perspective Projection

- our camera must model perspective



Projective Transformations

- planar geometric projections
  - planar: onto a plane
  - geometric: using straight lines
  - projections: 3D -> 2D
- aka projective mappings

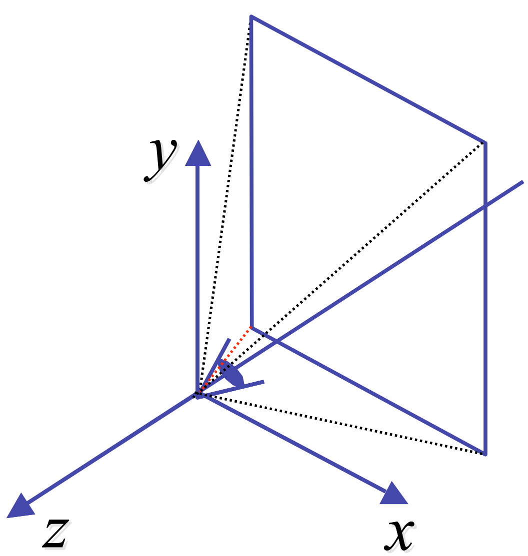
Projective Transformations

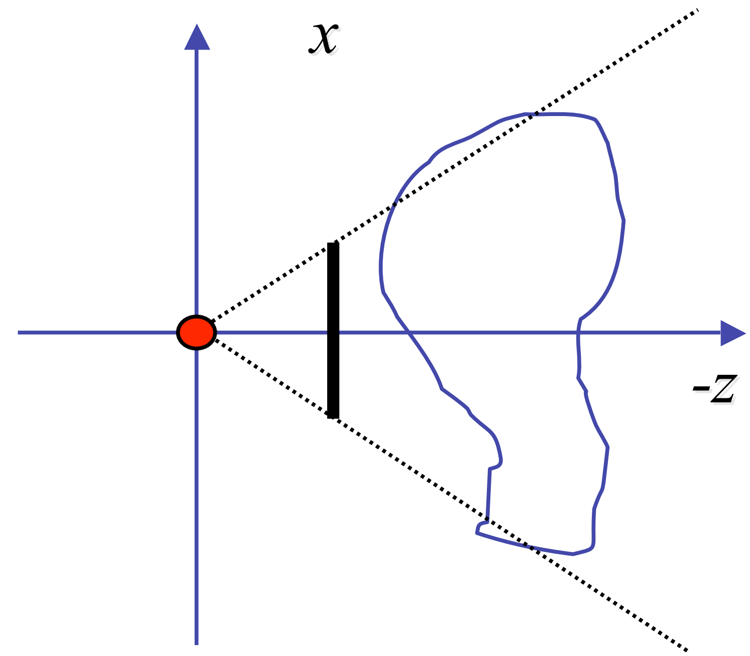
- properties
  - lines mapped to lines and triangles to triangles
  - parallel lines do NOT remain parallel
    - e.g. rails vanishing at infinity

  - affine combinations are NOT preserved
    - e.g. center of a line does not map to center of projected line (perspective foreshortening)

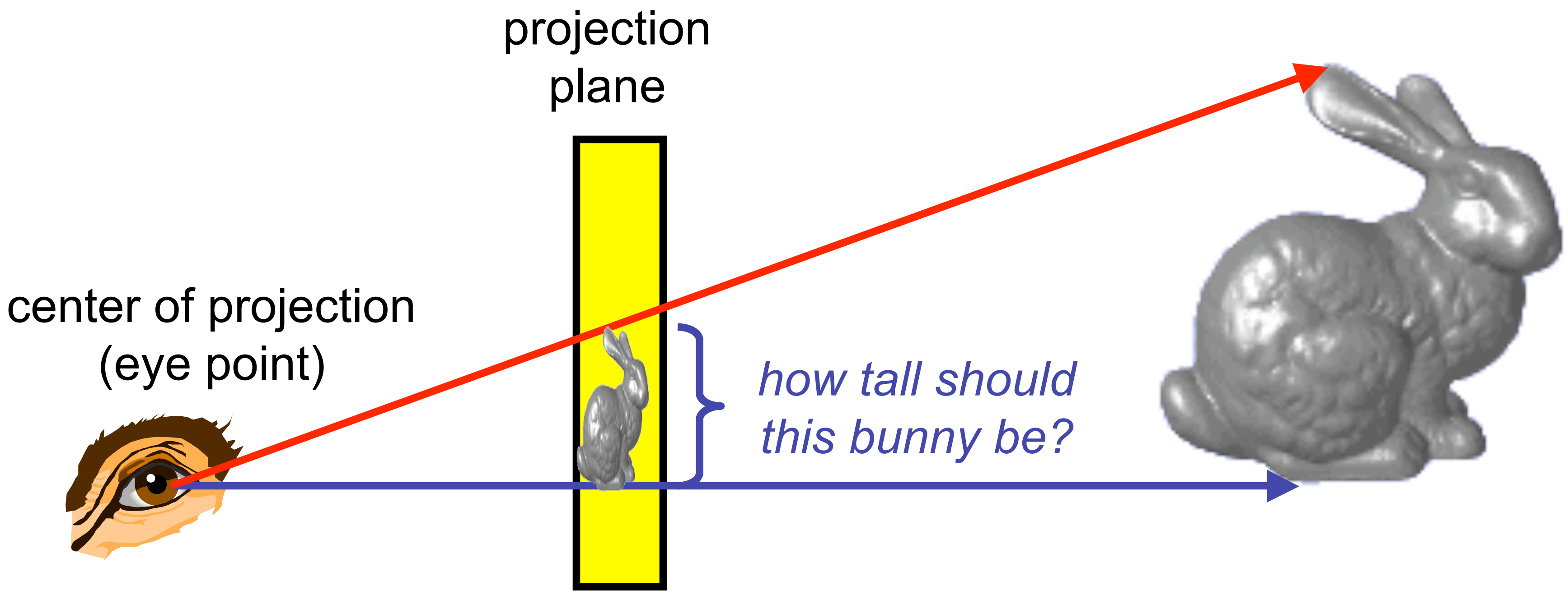
Perspective Projection

- project all geometry
  - through common center of projection (eye point)
  - onto an image plane

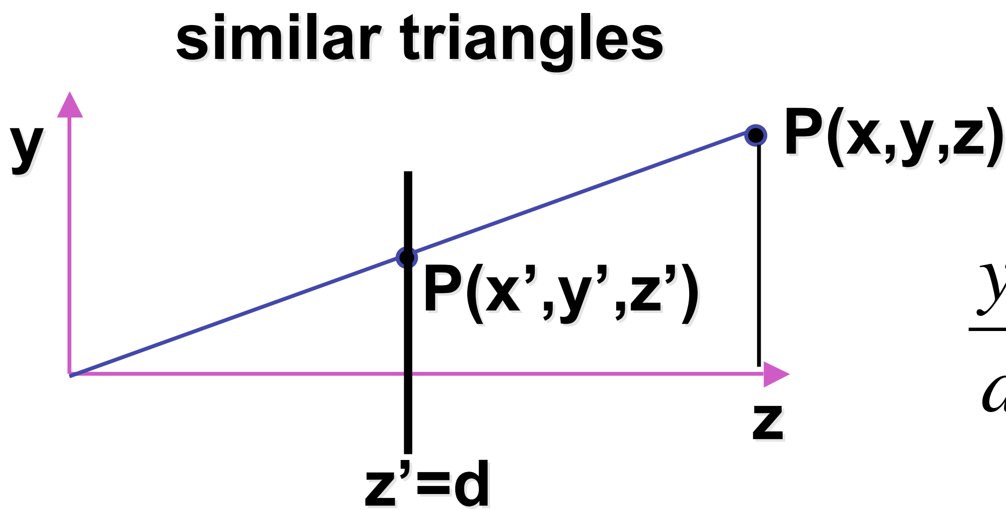




Perspective Projection



Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

$$\frac{x'}{d} = \frac{x}{z} \rightarrow x' = \frac{x \cdot d}{z}$$

but $z' = d$

- nonuniform foreshortening
  - not affine

Perspective Projection

- desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{x \cdot d}{z} = \frac{x}{z/d}, \quad y' = \frac{y \cdot d}{z} = \frac{y}{z/d}, \quad z' = d$$

- what could a matrix look like to do this?

Simple Perspective Projection Matrix

$$\begin{bmatrix} \dfrac{x}{z/d} \\ \dfrac{y}{z/d} \\ d \end{bmatrix}$$

Simple Perspective Projection Matrix

$$\begin{bmatrix} \dfrac{x}{z/d} \\ \dfrac{y}{z/d} \\ d \end{bmatrix}$$

is homogenized version of

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

where w = z/d

Simple Perspective Projection Matrix

$$\begin{bmatrix} \dfrac{x}{z/d} \\ \dfrac{y}{z/d} \\ d \end{bmatrix}$$ is homogenized version of $$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

where w = z/d

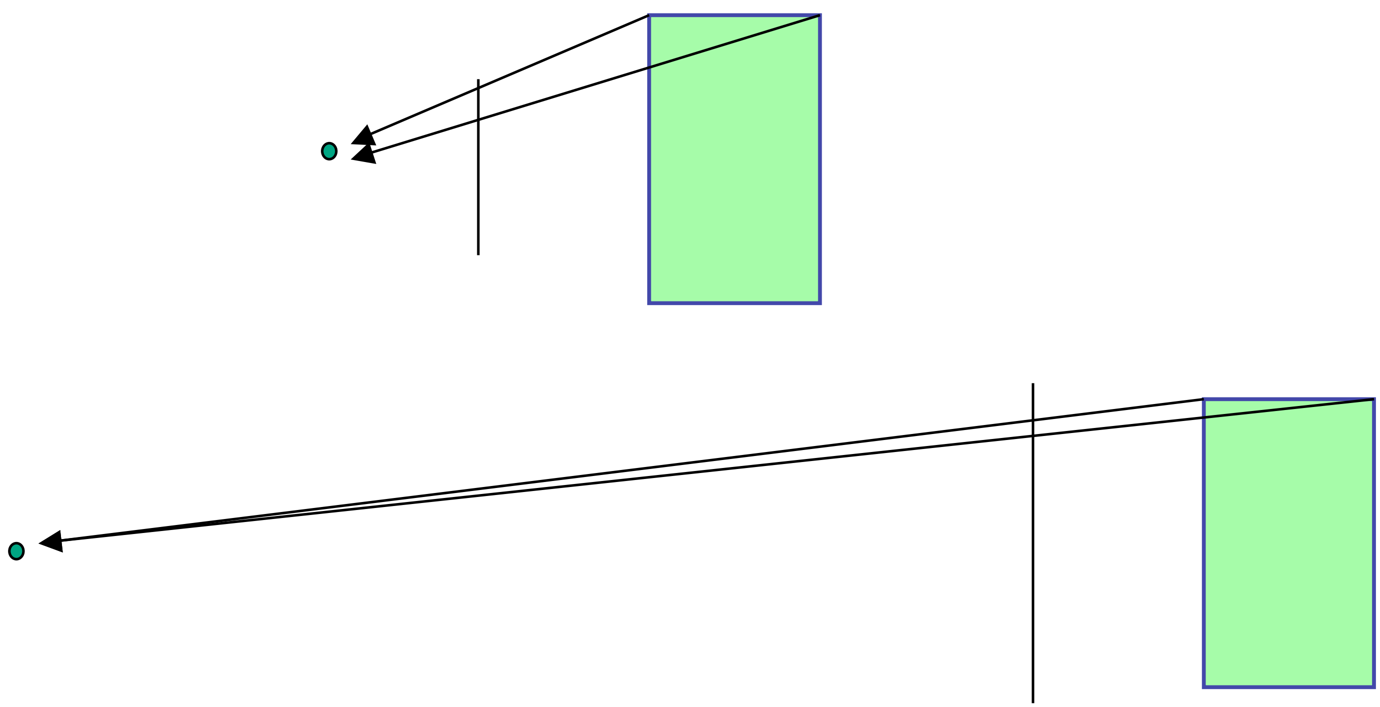
$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Projection

- expressible with 4x4 homogeneous matrix
  - use previously untouched bottom row
- perspective projection is irreversible
  - many 3D points can be mapped to same (x, y, d) on the projection plane
  - no way to retrieve the unique z values

Moving COP to Infinity

- as COP moves away, lines approach parallel
- when COP at infinity, orthographic view



Orthographic Camera Projection

- camera's back plane parallel to lens
- infinite focal length
- no perspective convergence

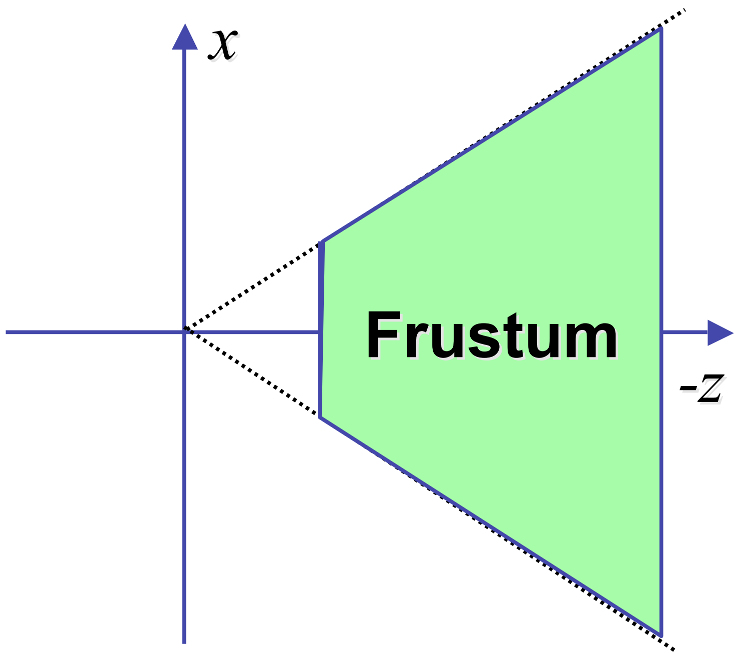
$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

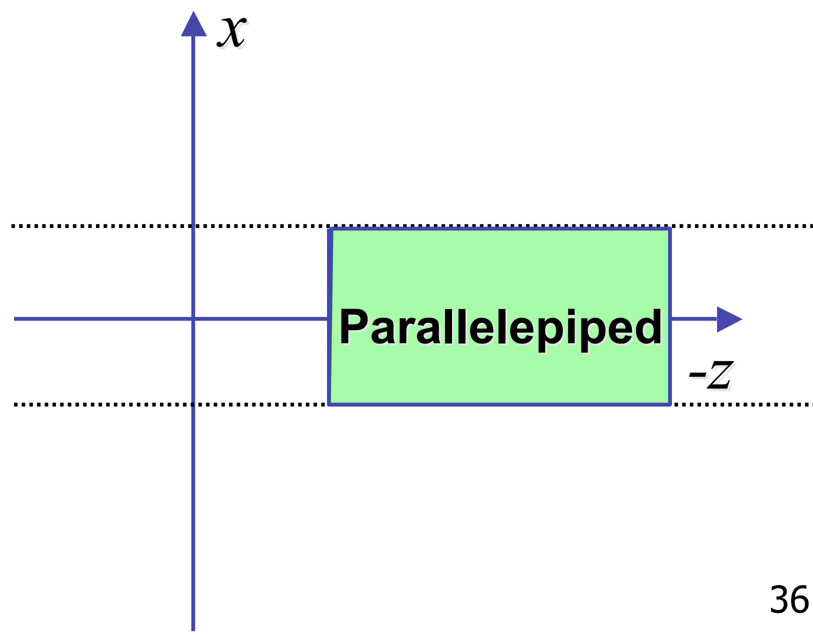
- just throw away z values

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective to Orthographic

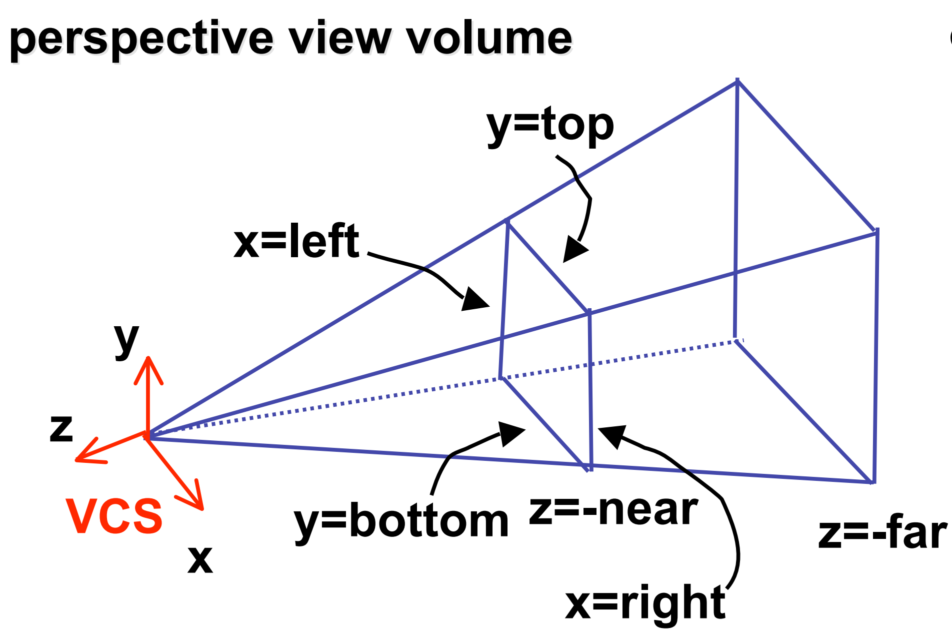
- transformation of space
  - center of projection moves to infinity
  - view volume transformed
    - from frustum (truncated pyramid) to parallelepiped (box)

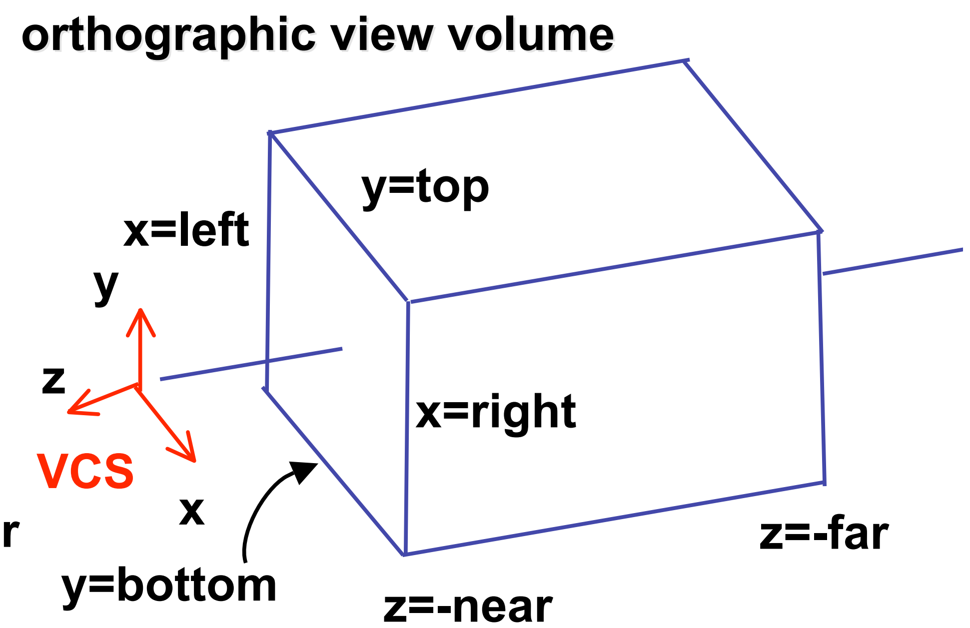




View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test





Demo: Perspective and Ortho Volumes

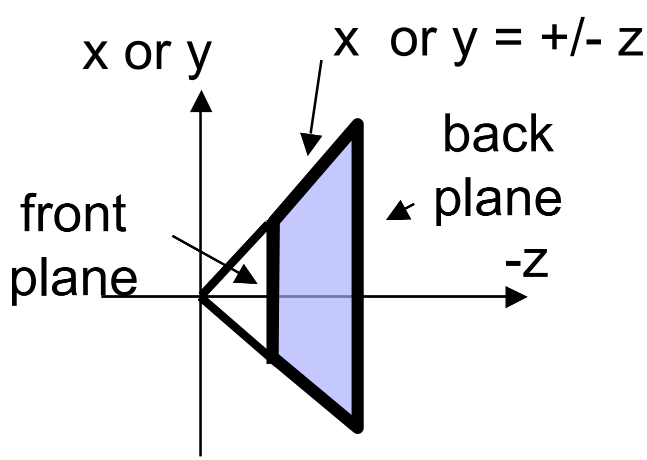
- Nate Robins tutorial (projection)
  - http://www.xmission.com/~nate/tutors.html

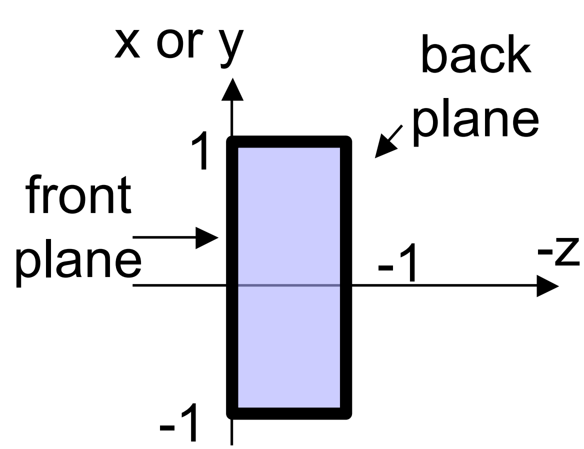
Canonical View Volumes

- standardized viewing volume representation

perspective

orthographic
orthogonal
parallel





Why Canonical View Volumes?

- permits standardization
  - clipping
    - easier to determine if an arbitrary point is enclosed in volume with canonical view volume vs. clipping to six arbitrary planes
  - rendering
    - projection and rasterization algorithms can be reused

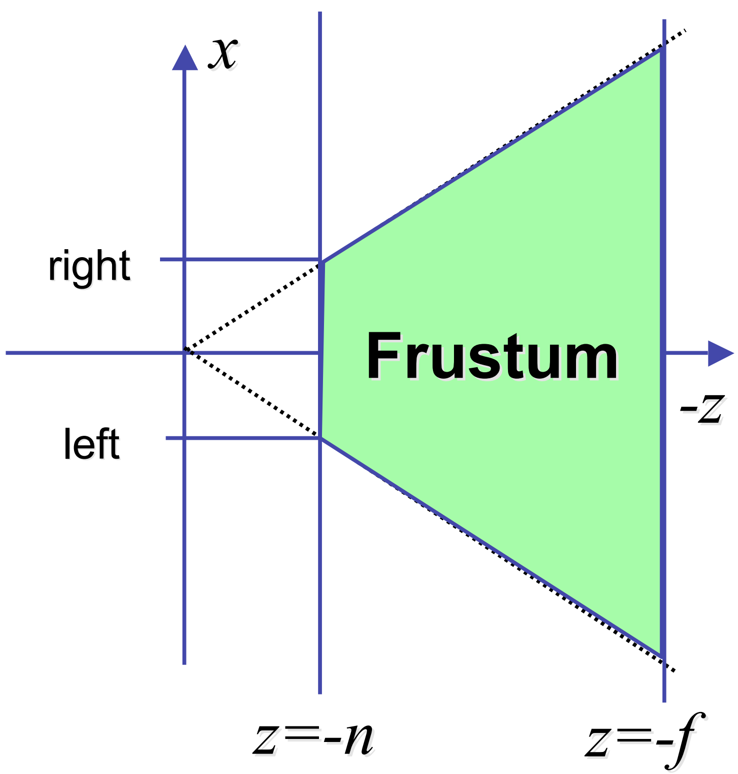
Normalized Device Coordinates

- convention
  - viewing frustum mapped to specific parallelepiped
    - Normalized Device Coordinates (NDC)
    - same as clipping coords
  - only objects inside the parallelepiped get rendered
  - which parallelepiped?
    - depends on rendering system

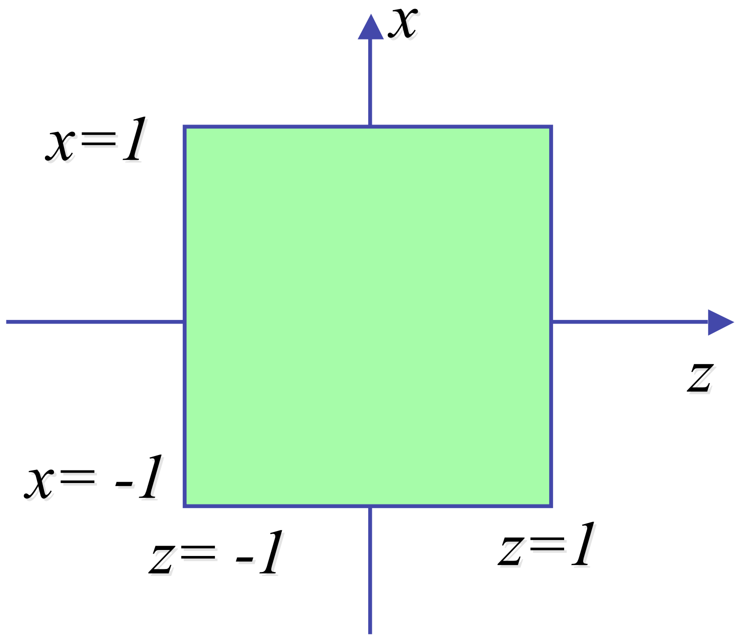
Normalized Device Coordinates

left/right x =+/- 1, top/bottom y =+/- 1, near/far z =+/- 1

Camera coordinates

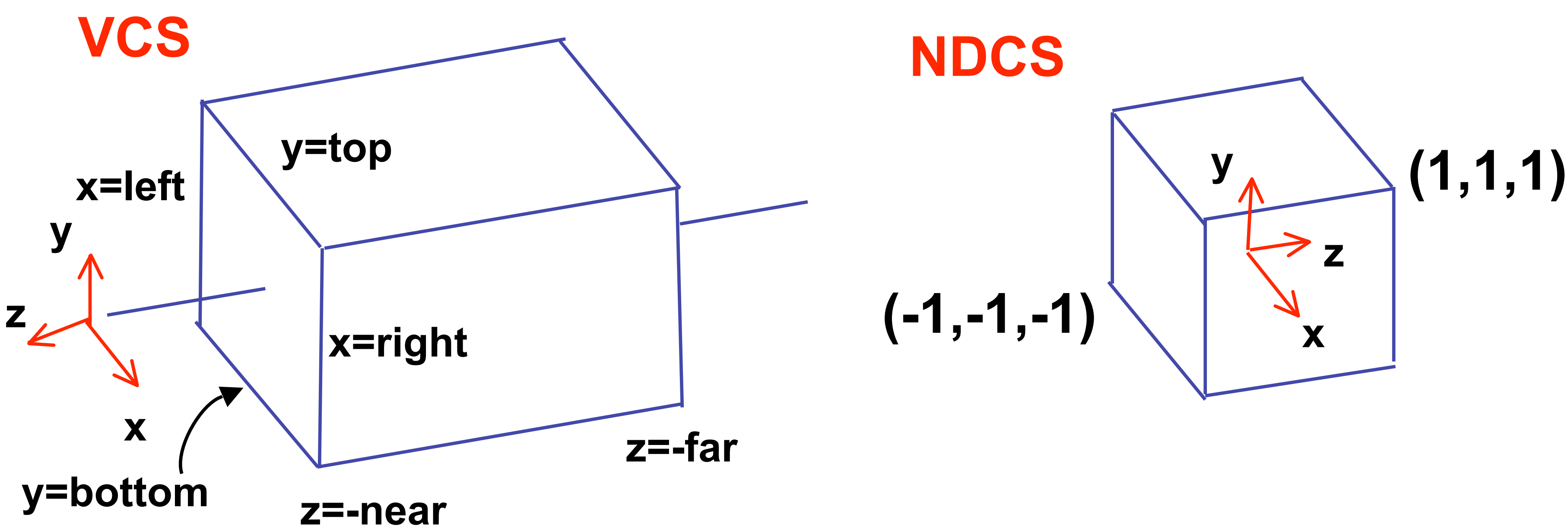


NDC



Understanding Z

- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)

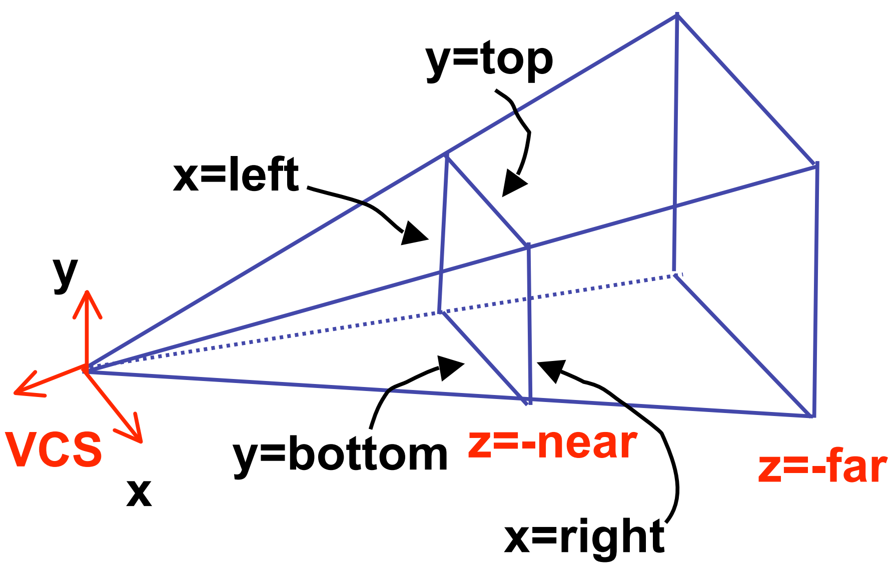


Understanding Z

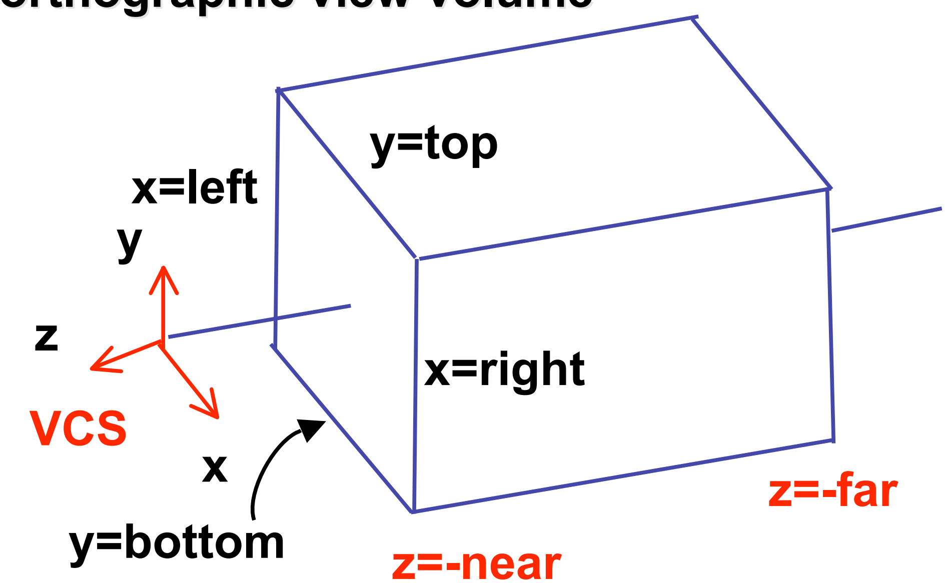
near, far always positive in OpenGL calls

```
glOrtho(left,right,bot,top,near,far);
glFrustum(left,right,bot,top,near,far);
glPerspective(fovy,aspect,near,far);
```

perspective view volume



orthographic view volume

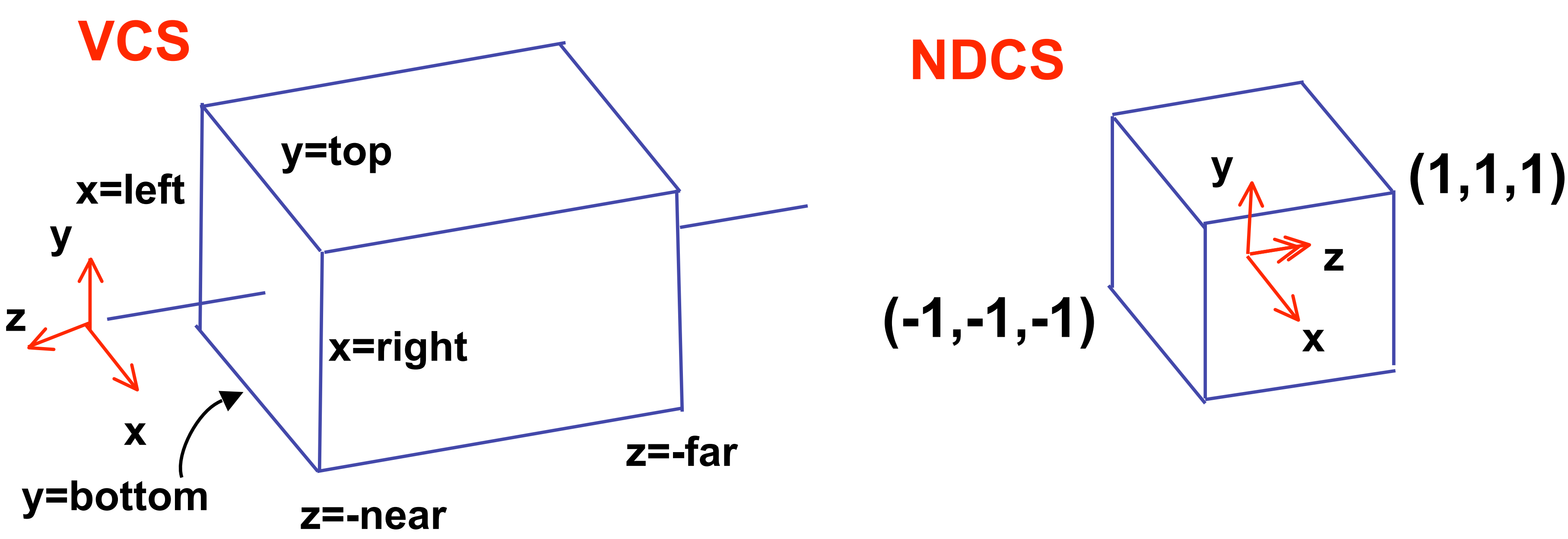


Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)
    - avoid/reduce numerical precision artifacts for distant objects

Orthographic Derivation

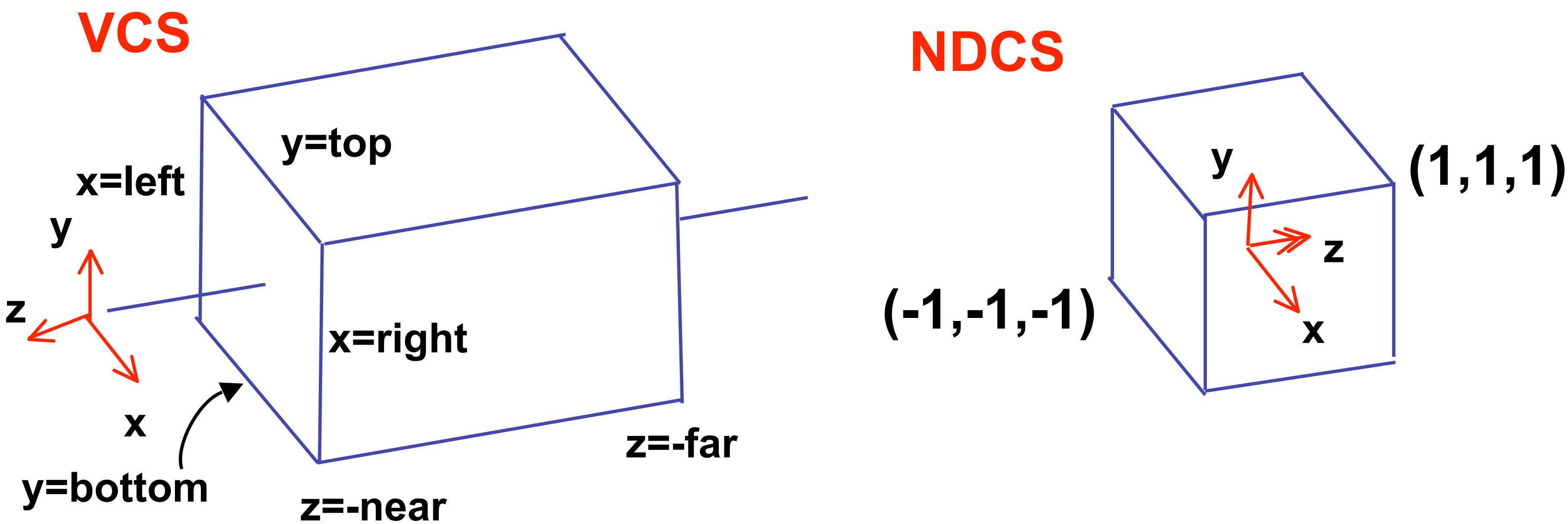
- scale, translate, reflect for new coord sys



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad \begin{array}{l} y = top \rightarrow y' = 1 \\ y = bot \rightarrow y' = -1 \end{array}$$



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b$$

$$y = top \rightarrow y' = 1 \qquad 1 = a \cdot top + b$$

$$y = bot \rightarrow y' = -1 \qquad -1 = a \cdot bot + b$$

$$b = 1 - a \cdot top, b = -1 - a \cdot bot$$

$$1 - a \cdot top = -1 - a \cdot bot$$

$$1 - (-1) = -a \cdot bot - (-a \cdot top)$$

$$2 = a(-bot + top)$$

$$a = \frac{2}{top - bot}$$

$$1 = \frac{2}{top - bot} top + b$$

$$b = 1 - \frac{2 \cdot top}{top - bot}$$

$$b = \frac{(top - bot) - 2 \cdot top}{top - bot}$$

$$b = \frac{-top - bot}{top - bot}$$

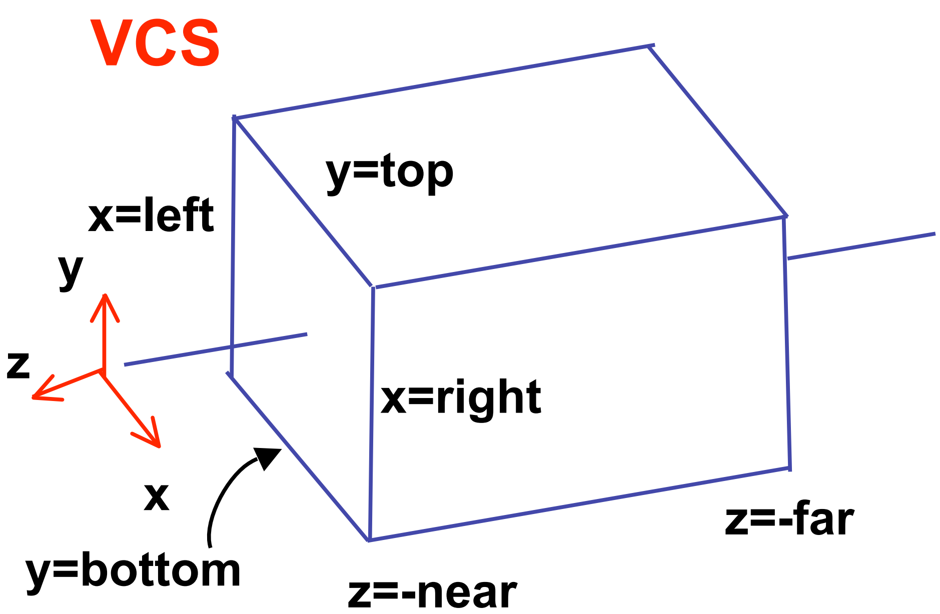
Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b$$

$$y = top \rightarrow y' = 1$$

$$y = bot \rightarrow y' = -1$$



$$a = \frac{2}{top - bot}$$

$$b = -\frac{top + bot}{top - bot}$$

same idea for right/left, far/near

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- $\boxed{\text{scale,}}$ translate, reflect for new coord sys

$$P' = \begin{bmatrix} \boxed{\dfrac{2}{right-left}} & 0 & 0 & -\dfrac{right+left}{right-left} \\ 0 & \boxed{\dfrac{2}{top-bot}} & 0 & -\dfrac{top+bot}{top-bot} \\ 0 & 0 & \boxed{\dfrac{-2}{far-near}} & -\dfrac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, $\boxed{\text{reflect}}$ for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{\boxed{-}2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```

Demo

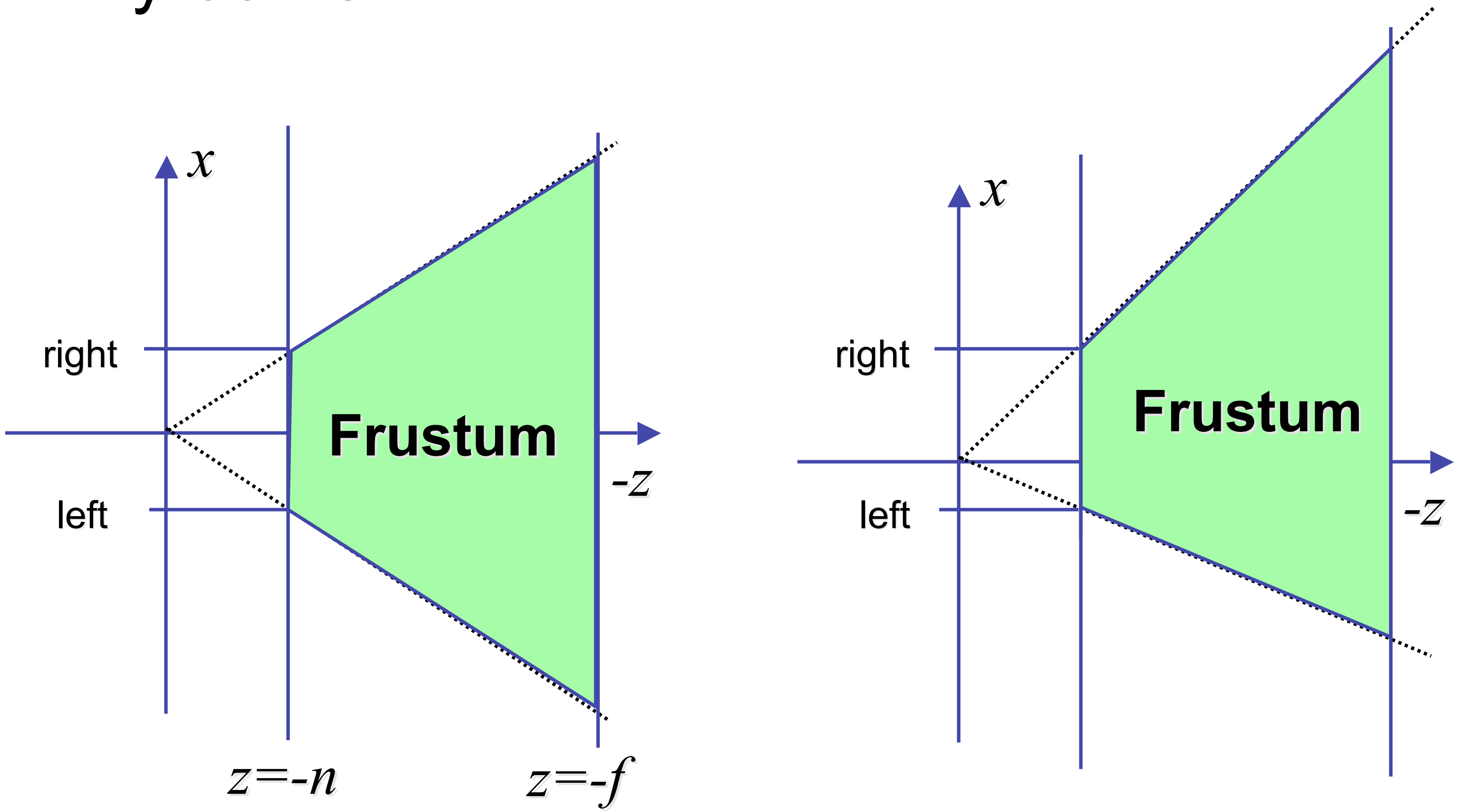
- Brown applets: viewing techniques
  - parallel/orthographic camera transformations

- http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/viewing_techniques.html

Projections II

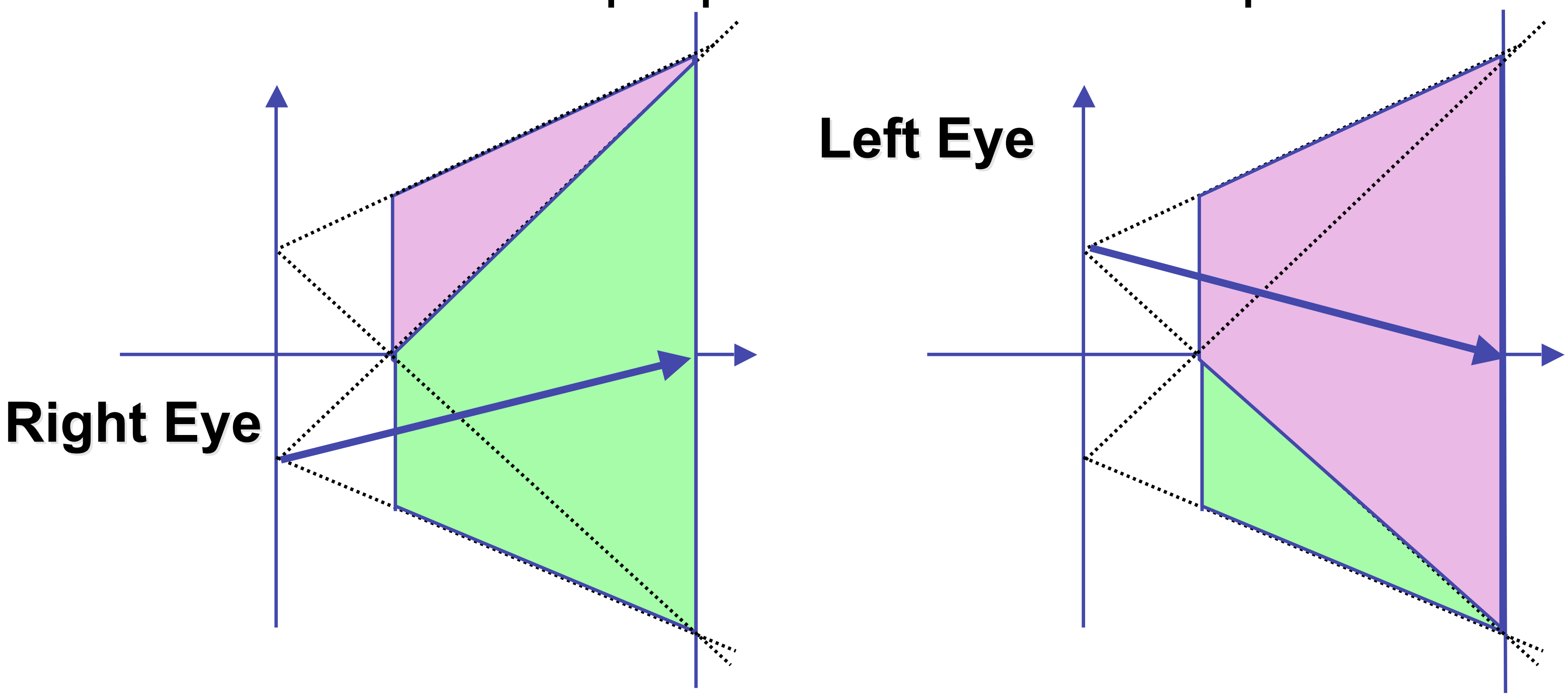
Asymmetric Frusta

- our formulation allows asymmetry
  - why bother?



Asymmetric Frusta

- our formulation allows asymmetry
  - why bother? binocular stereo
    - view vector not perpendicular to view plane

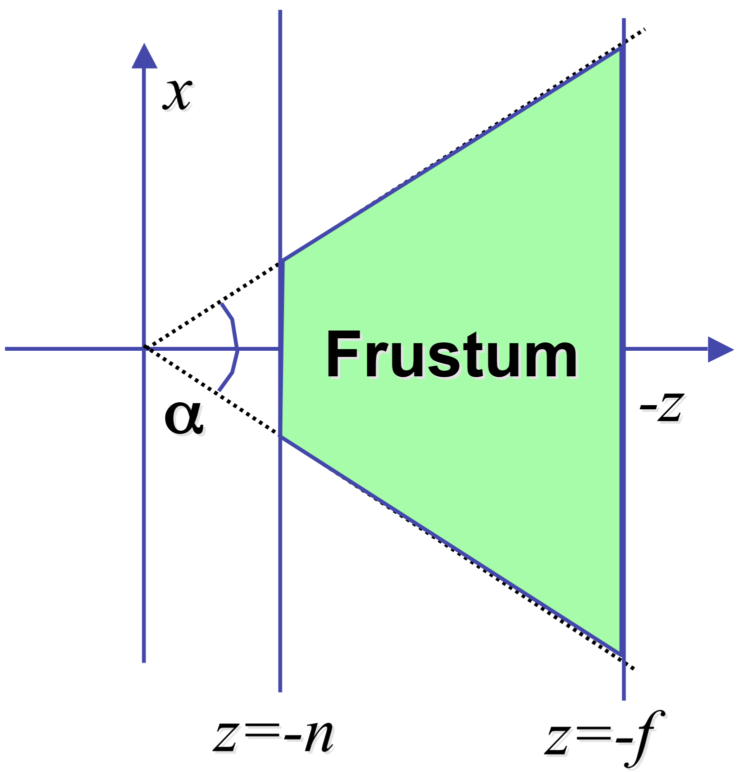


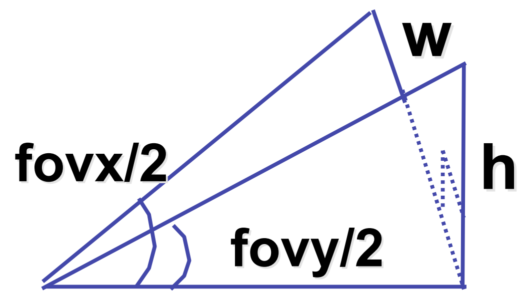
Simpler Formulation

- left, right, bottom, top, near, far
  - nonintuitive
  - often overkill
- look through window center
  - symmetric frustum
- constraints
  - left = -right, bottom = -top

Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
  - determines FOV in other direction
  - also set near, far (reasonably intuitive)





Perspective OpenGL

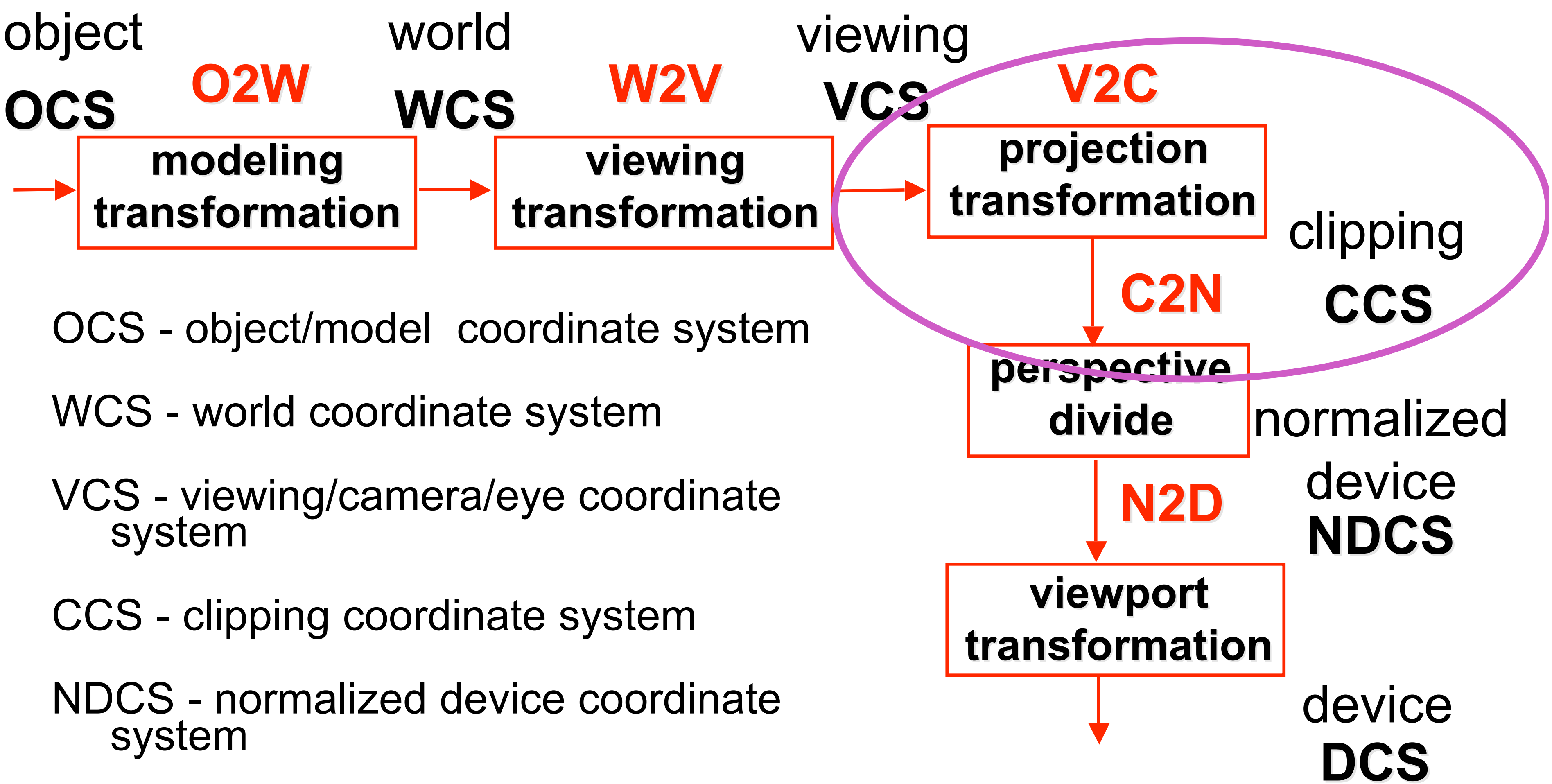
```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();

glFrustum(left,right,bot,top,near,far);
```
or
```
glPerspective(fovy,aspect,near,far);
```

Demo: Frustum vs. FOV

- Nate Robins tutorial (take 2):
  - http://www.xmission.com/~nate/tutors.html

Projective Rendering Pipeline



OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

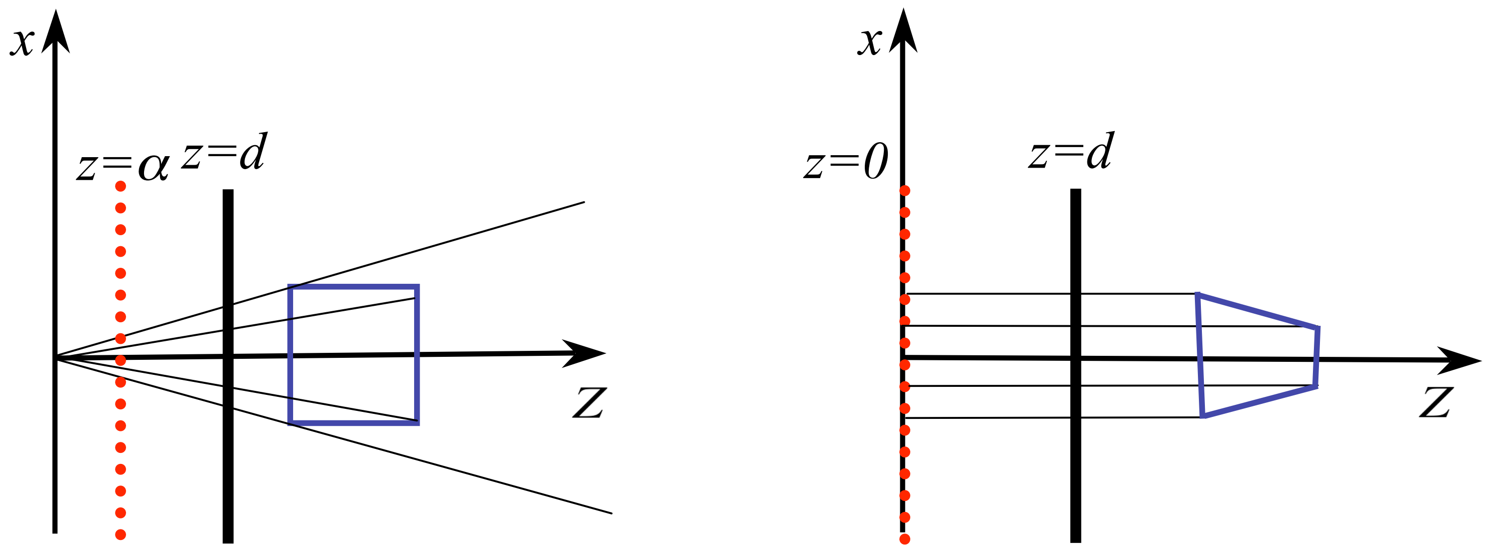
CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

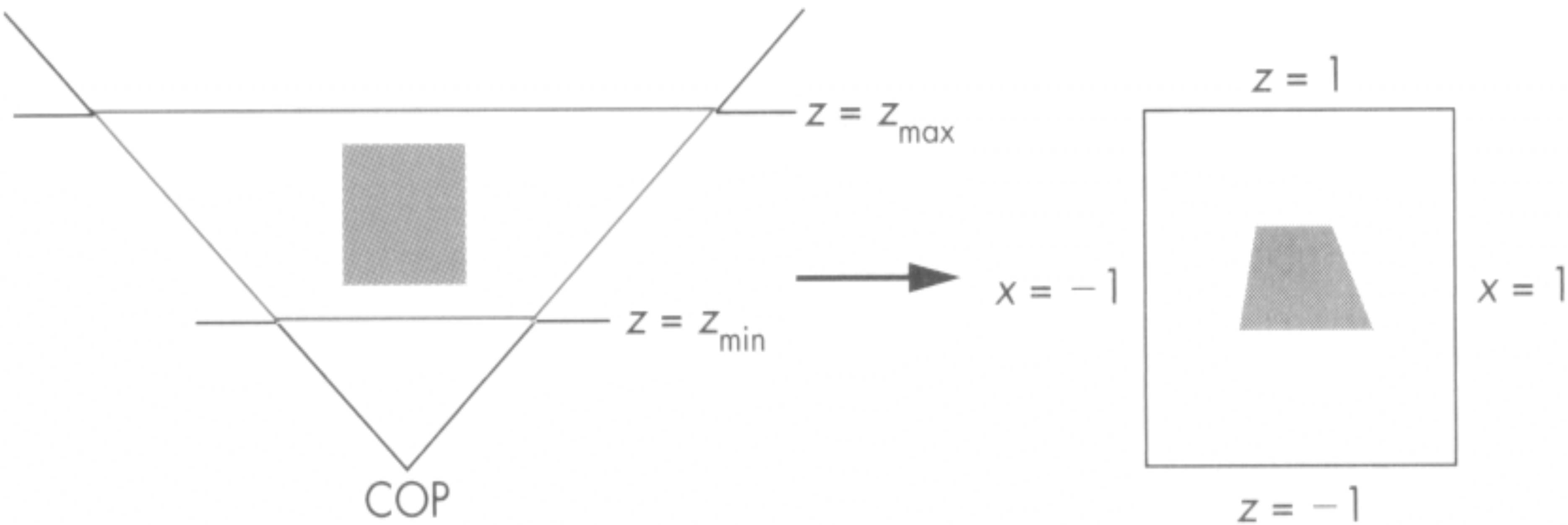
Projection Normalization

- warp perspective view volume to orthogonal view volume
  - render all scenes with orthographic projection!
  - aka perspective warp

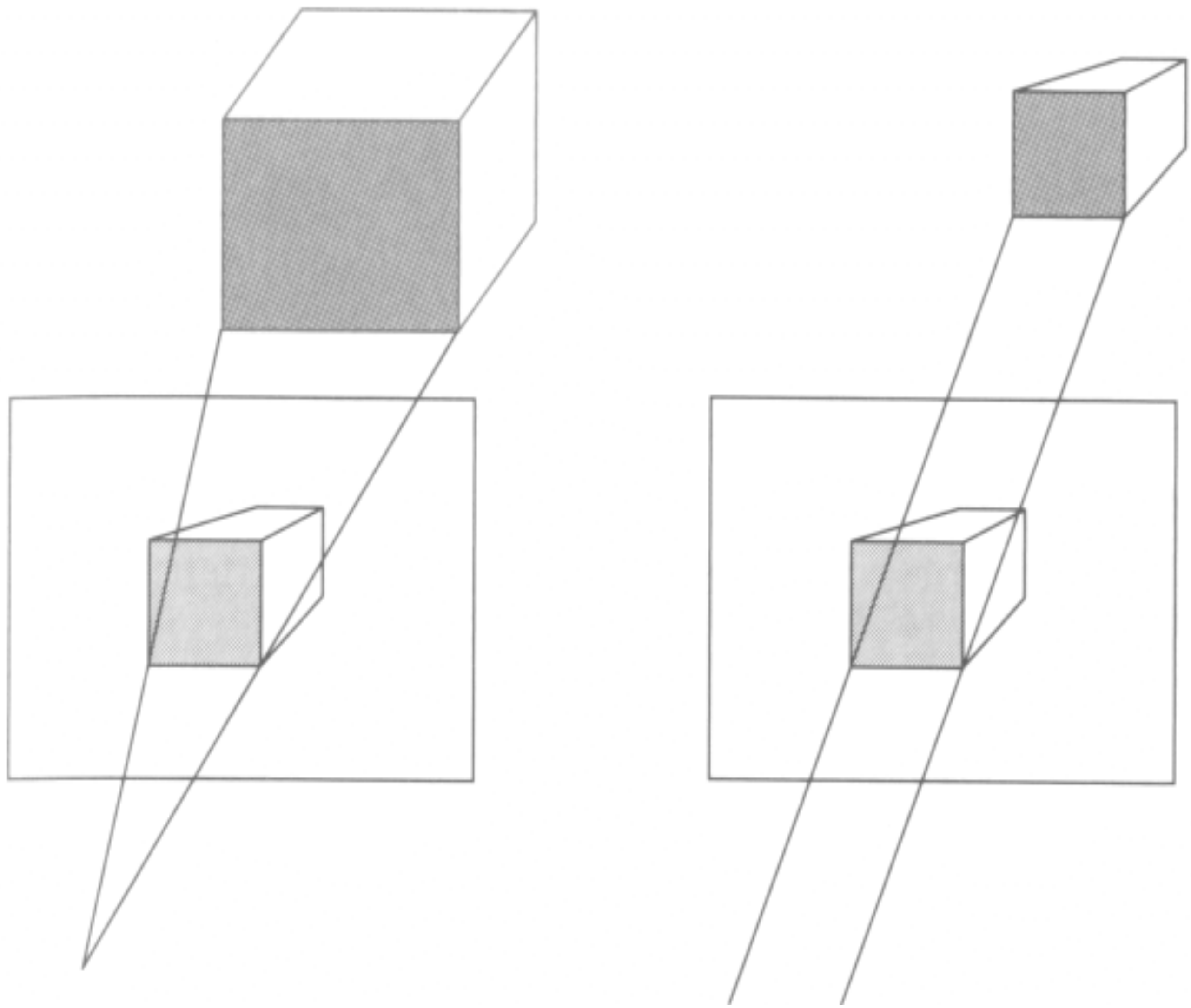


Perspective Normalization

- perspective viewing frustum transformed to cube
- orthographic rendering of cube produces same image as perspective rendering of original



Predistortion



Demos

- Tuebingen applets from Frank Hanisch
  - http://www.gris.uni-tuebingen.de/projects/grdev/doc/html/etc/AppletIndex.html#Transformationen

Projective Rendering Pipeline

object **OCS** → **O2W** → **modeling transformation** → world **WCS** → **W2V** → **viewing transformation** → viewing **VCS** → **V2C** → **projection transformation** → clipping **CCS** → **C2N** → **perspective divide** → normalized device **NDCS** → **N2D** → **viewport transformation** → device **DCS**

OCS - object/model coordinate system

WCS - world coordinate system

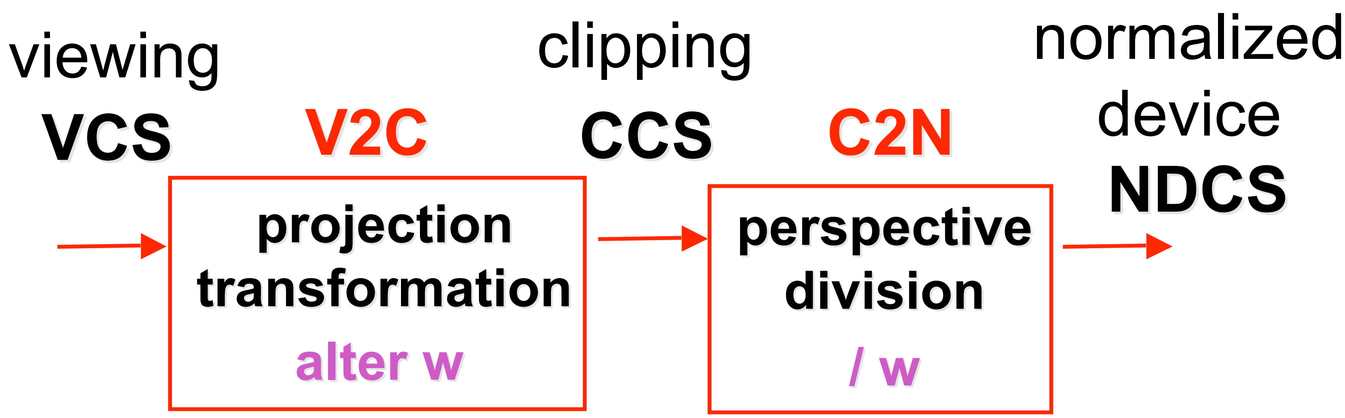
VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

Separate Warp From Homogenization



- warp requires only standard matrix multiply
  - distort such that orthographic projection of distorted objects is desired persp projection
    - w is changed
  - clip after warp, before divide
  - division by w: homogenization