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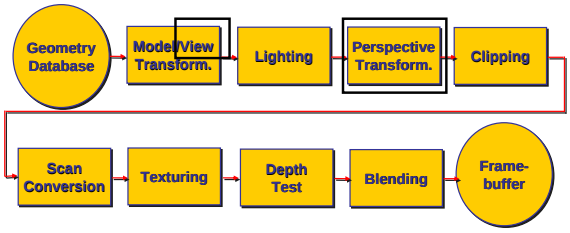
Chapter 6

Viewing/Perspective Transformations

transformations

Rendering Pipeline



- Specify view point (change of coordinate system)
- Project from 3D to 2D (introduce perspective)

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Rendering Pipeline

Scene graph Object geometry

Modelling Transforms

Viewing Transform

Projection Transform

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Rendering Pipeline

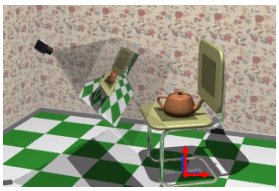
Scene graph Object geometry

Modelling Transforms

Viewing Transform

Projection Transform

- result
  - all vertices of scene in shared 3D world coordinate system



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Rendering Pipeline

Scene graph Object geometry

Modelling Transforms

Viewing Transform

Projection Transform

- result
  - scene vertices in 3D view (camera) coordinate system

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Rendering Pipeline

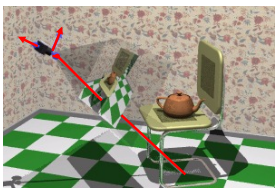
Scene graph Object geometry

Modelling Transforms

Viewing Transform

Projection Transform

- result
  - 2D screen coordinates of clipped vertices



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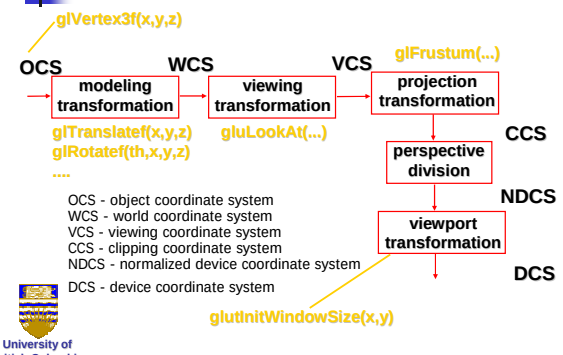
Projective Rendering Pipeline



OCS - object coordinate system
WCS - world coordinate system
VCS - viewing coordinate system
CCS - clipping coordinate system
NDCS - normalized device coordinate system
DCS - device coordinate system

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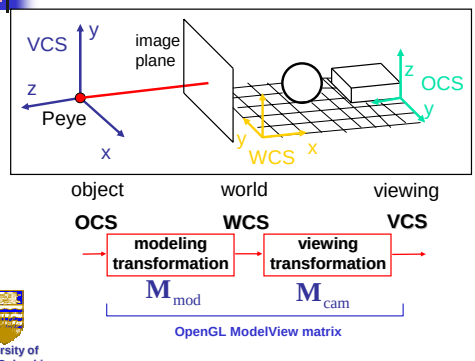
Viewing Transformation



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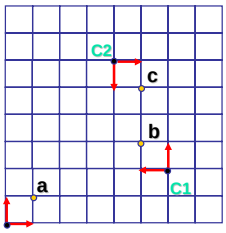
World vs. Camera Coordinates



$a = (1,1)_W$

$b = (1,1)_{C1} = (5,3)_W$

$c = (1,1)_{C2} = (1,3)_{C1} = (5,5)_W$

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OpenGL Basic Viewing

- starting spot - OpenGL
  - camera at world origin
    - probably inside an object
  - y axis is up
  - looking down negative z axis
    - why? RHS with x horizontal, y vertical, z out of screen
- To position – coordinate frame change
- Intuitive description
  - eye point, look-at point, up vector

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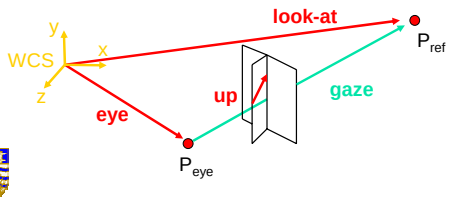
Camera Description

- arbitrary viewing position
  - eye point, look-at direction, up vector



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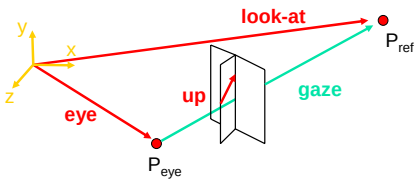
From World to View Coordinates: W2V

- Setup *uvw* basis with origin at eye
- translate **eye** to origin
- rotate *uvw* to align with *xyz*



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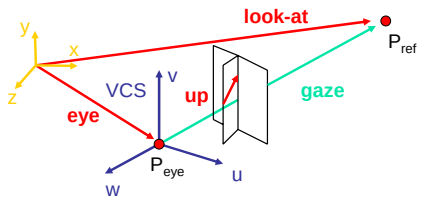
Deriving W2V Transformation

- Gaze is (look-at – eye)
- Use opposite of gaze $\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$



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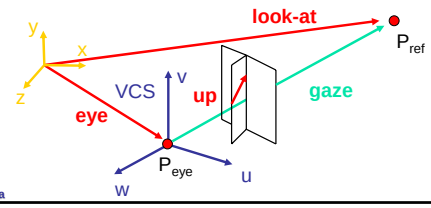
Deriving W2V Transformation

- U - perpendicular to W and Up vector
- V – perpendicular to W, V

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u}$$



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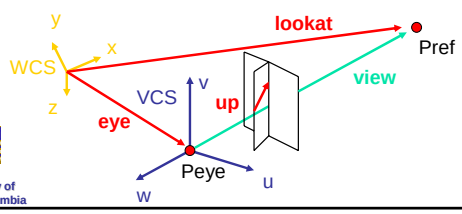
Deriving W2V Transformation

- translate **eye** to origin

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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Deriving W2V Transformation

- rotate from WCS *xyz* into *uvw* coordinate system with matrix that has rows u, v, w

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|}$$

$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$

$$\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$

$$\mathbf{R} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- reminder:
  - rotate from uvw to xyz coord sys with matrix M that has columns u,v,w
  - rotate from xyz coord sys to uvw coord sys with matrix M^T that has rows u,v,w

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Deriving W2V Transformation

- M=RT

$$\mathbf{R} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{world->view} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{u}\cdot\mathbf{e} \\ v_x & v_y & v_z & -\mathbf{v}\cdot\mathbf{e} \\ w_x & w_y & w_z & -\mathbf{w}\cdot\mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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OpenGL Viewing Transformation

```
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
```

- postmultiplies current matrix, so to be safe initialize first

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

- demo: Nate Robins tutorial *projection*

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Moving the Camera or the World?

- two equivalent operations
  - move camera one way vs. move world other way
- example
  - initial OpenGL camera: at origin, looking along -z axis
  - create a unit square parallel to camera at z = -10
  - translate in z by 3 possible in two ways
    - camera moves to z = -3
      - Note OpenGL models viewing in left-hand coordinates
    - camera stays put, but square moves to -7
  - resulting image same either way
    - possible difference: are lights specified in world or view coordinates?

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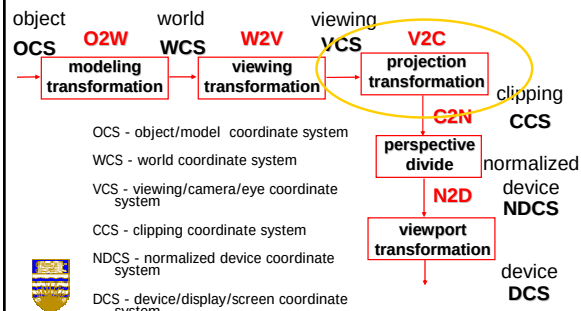
Projective Rendering Pipeline



OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

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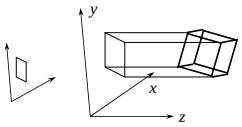
Projection Transformations



- Question: How to draw 3D object on 2D screen?
- Answer:
  - Project transformed object along Z axis onto XY plane - and from there to screen (space)
  - Canonical projection:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- "ignore" *z* coordinate – use *x* and *y* coordinates for screen coordinates

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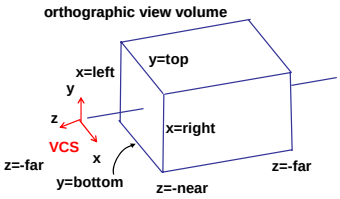
View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test



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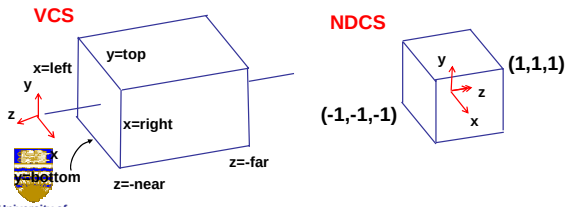
Understanding Z

- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)



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Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity for perspective (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0...1)
    - avoid/reduce numerical precision artifacts for distant objects

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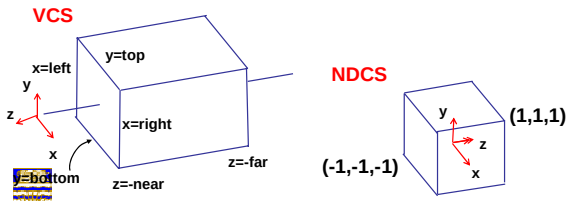
Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad top \to 1$$
$$bottom \to -1$$



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Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad top \to 1$$
$$bottom \to -1$$

- Solve two eq.

$$1 = a \cdot top + b \qquad a = \frac{2}{top - bottom}$$
$$-1 = a \cdot bottom + b \qquad b = \frac{-top - bottom}{top - bottom}$$

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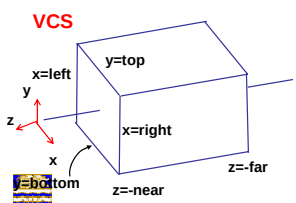
Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad y = top \to y' = 1$$
$$y = bot \to y' = -1$$



$$a = \frac{2}{top - bot}$$
$$b = -\frac{top + bot}{top - bot}$$

same idea for right/left, far/near

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Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

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Orthographic Derivation

- scale translate, reflect for new coord sys

$$P' = \begin{bmatrix} \boxed{\frac{2}{right - left}} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \boxed{\frac{2}{top - bot}} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \boxed{\frac{-2}{far - near}} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

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Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & \boxed{-\frac{right + left}{right - left}} \\ 0 & \frac{2}{top - bot} & 0 & \boxed{-\frac{top + bot}{top - bot}} \\ 0 & 0 & \frac{-2}{far - near} & \boxed{-\frac{far + near}{far - near}} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

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Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{\boxed{-}2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

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Orthographic OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```

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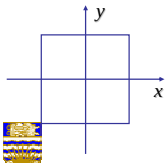
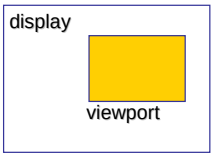
NDC to Viewport Transformation

- generate pixel coordinates
  - map x, y from range –1...1 (NDC) to pixel coordinates on the display
  - involves 2D scaling and translation





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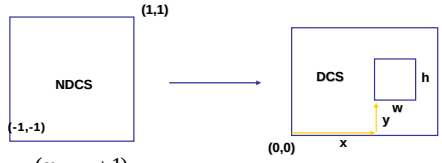
NDC to Viewport Transformation

- 2D scaling and translation



$$x_{DCS} = w\frac{(x_{NDCS}+1)}{2} + x$$

$$y_{DCS} = h\frac{(y_{NDCS}+1)}{2} + y$$

$$z_{DCS} = \frac{(z_{NDCS}+1)}{2}$$

OpenGL

```
glViewport(x,y,w,h);
```

default:

```
glViewport(0,0,w_wnd,h_wnd);
```

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Origin Location

- yet more possibly confusing conventions
  - OpenGL: lower left
  - most window systems: upper left
- often have to flip your y coordinates
  - when interpreting mouse position

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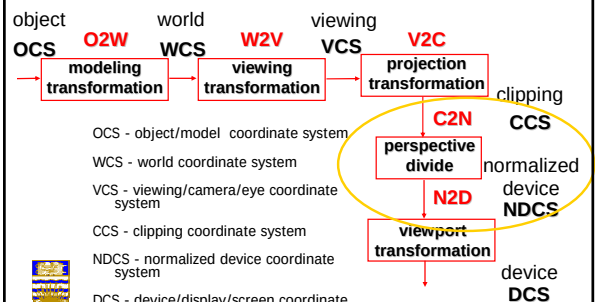
Projective Rendering Pipeline



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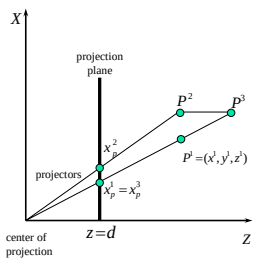
Perspective Projection

- Viewing is from *point at finite distance*
- Without loss of generality:
  - Viewpoint at origin
  - Viewing plane is $z=d$
- Given P=(x,y,z) triangle similarity gives:



$$\frac{x}{z}=\frac{x_p}{d} \text{ and } \frac{y}{z}=\frac{y_p}{d} \Rightarrow x_p=\frac{x}{z/d} \text{ and } y_p=\frac{y}{z/d}$$

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Perspective Projection (cont'd)

- In matrix notation with homogeneous coordinates:

$$P(x,y,z,1)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x\\y\\z\\z/d\end{bmatrix}.$$

- In Euclidean coordinates:

$$\left(\frac{x}{z/d},\frac{y}{z/d},\frac{z}{z/d}\right)=\left(\frac{x}{z/d},\frac{y}{z/d},d\right)=(x_p,y_p,d).$$

- P singular: $det(P)=0$

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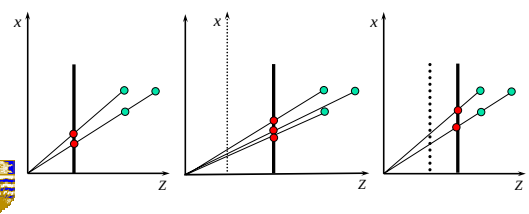
Perspective Projection (cont'd)

- What is (if any) is the difference between:
  - Moving projection plane
  - Moving viewpoint (center of projection)?

camtrans



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How to make non-degenerate?

$$P(x,y,z,1)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x\\y\\z\\z/d\end{bmatrix}.$$

$$\left(\frac{x}{z/d},\frac{y}{z/d},\frac{z}{z/d}\right)=\left(\frac{x}{z/d},\frac{y}{z/d},d\right)=(x_p,y_p,d).$$

- z' monotonically increasing function of z

$$P(x,y,z,1)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&a&b\\0&0&1/d&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x\\y\\az+b\\z/d\end{bmatrix}. \qquad z'=da+\frac{b}{z}$$

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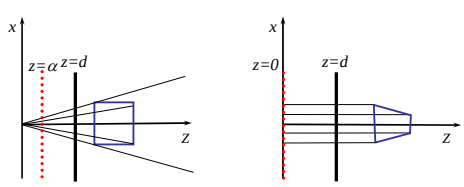
Perspective Warp –Geometric Meaning

- Warp viewing frustum (world)
- Then use parallel projection



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Perspective Warp

- Matrix formulation

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\frac{d}{d-\alpha}&\frac{-\alpha d}{d-\alpha}\\0&0&\frac{1}{d}&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}=\begin{bmatrix}x\\y\\(z-\alpha)d/(d-\alpha)\\z/d\end{bmatrix} \qquad \begin{bmatrix}x_p\\y_p\\z_p\end{bmatrix}=\begin{bmatrix}\frac{x}{z/d}\\\frac{y}{z/d}\\\frac{d^2}{d-\alpha}\left(1-\frac{\alpha}{z}\right)\end{bmatrix}$$

- Preserves relative depth (third coordinate)
- What does $\alpha=0$ mean?

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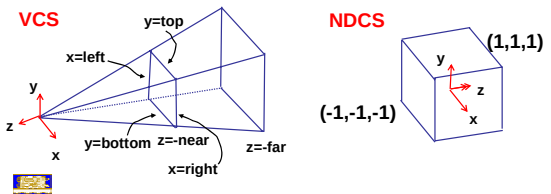
OpenGL Perspective Derivation



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Perspective Derivation

earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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Perspective Derivation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = Ex + Az$$
$$y' = Fy + Bz$$
$$z' = Cz + D$$
$$w' = -z$$

$$x = left \rightarrow x'/w' = 1$$
$$x = right \rightarrow x'/w' = -1$$
$$y = top \rightarrow y'/w' = 1$$
$$y = bottom \rightarrow y'/w' = -1$$
$$z = -near \rightarrow z'/w' = 1$$
$$z = -far \rightarrow z'/w' = -1$$

$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{-z},$$

$$1 = F\frac{y}{-z} + B\frac{z}{-z}, \quad 1 = F\frac{y}{-z} - B, \quad 1 = F\frac{top}{-(-near)} - B,$$

$$1 = F\frac{top}{near} - B$$

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Perspective Derivation

- similarly for other 5 planes
- 6 planes, 6 unknowns

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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Perspective Example

view volume
- left = -1, right = 1
- bot = -1, top = 1
- near = 1, far = 4

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

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Perspective Example

$$\begin{bmatrix} 1 \\ -1 \\ -5z_{VCS}/3 - 8/3 \\ -z_{VCS} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -5/3 & -8/3 \\ & & -1 & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ z_{VCS} \\ 1 \end{bmatrix}$$

/ w

$$x_{NDCS} = -1/z_{VCS}$$
$$y_{NDCS} = 1/z_{VCS}$$
$$z_{NDCS} = \frac{5}{3} + \frac{8}{3z_{VCS}}$$

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Perspective OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();

glFrustum(left,right,bot,top,near,far);
```
or
```
glPerspective(fovy,aspect,near,far);
```
- symmetric version

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Another Transformations Quiz

- What does each transformation preserve?

	lines	parallel lines	distance	angles	normals	convexity	conics
scaling							
rotation							
translation							
shear							
perspective							

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