Announcements

- Reminder important dates – still to come
  - Assignment 1 due: Oct 14
  - Assignment 2 due: Nov 4
  - Assignment 3 due:
    - Theory: Nov 25
    - Programming: Nov 28
  - Quiz 1: Oct 20
  - Quiz 2: Nov 10
- A1Q3: “Given a line segment $S=(\text{P}_0, \text{P}_1)$ in 2D and a point $\text{P}$, write an algorithm to find if the point is on the line segment.”
Chapter 8

Scan Conversion (part 2)- Drawing Polygons on Raster Display
Rasterizing Polygons/Triangles

- Basic surface representation in rendering
- Why?
  - Lowest common denominator
    - Can approximate any surface with arbitrary accuracy
      - All polygons can be broken up into triangles
  - Guaranteed to be:
    - Planar
    - Triangles - Convex
- Simple to render
  - Can implement in hardware
Triangulation

- Convex polygons easily triangulated
- Concave polygons present a challenge
- Convexity - formal definition:

Object $S$ is convex iff for any two points $P, Q \in S$, $tP + (1-t)Q \subseteq S$, $t \in [0,1]$. 
OpenGL Triangulation

- Simple convex polygons
  - break into triangles, trivial
  - `glBegin(GL_POLYGON) ... glEnd()`

- Concave or non-simple polygons
  - break into triangles, more effort
  - `gluNewTess(), gluTessCallback(), ...`
Polygon Rasterization

- Assumptions – well behaved
  - simple - no self intersections
  - simply connected
  - (no holes)

- Solutions
  - Flood fill
  - Scan line
  - Implicit test
Formulation

- Input
  - polygon $P$ with rasterized edges
- Problem: Fill its interior with specified color on graphics display
Flood Fill Algorithm

- **Input**
  - polygon $P$ with rasterized edges
  - $P = (x, y) \in P$ point inside $P$
Flood Fill

FloodFill (Polygon P, int x, int y, Color C)
if not (OnBoundary (x, y, P) or Colored (x, y, C))
begin
  PlotPixel (x, y, C);
  FloodFill (P, x + 1, y, C);
  FloodFill (P, x, y + 1, C);
  FloodFill (P, x, y - 1, C);
  FloodFill (P, x - 1, y, C);
end ;
Flood Fill

- Drawbacks?
Flood Fill - Drawbacks

- How do we find a point inside?

- Pixels visited up to 4 times to check if already set

- Need per-pixel flag indicating if set already
  - clear for every polygon!
Scanline Algorithm

- Observation: Each intersection of straight line with boundary moves it from/into polygon
- Detect (& set) pixels inside polygon boundary (simple closed curve) with set of horizontal lines (pixel apart)
ScanConvert (Polygon \( P \), Color \( C \))
For \( y := 0 \) to ScreenYMax do
\( I \Leftarrow \) Points of intersections of edges of \( P \) with line \( Y = y \);
Sort \( I \) in increasing \( X \) order and
Fill with color \( C \) alternating segments;
end;

- Limit to *bounding box* to speed up
- Other enhancements….
Bounding Box

\((x_{\text{min}}, y_{\text{min}})\)

\((x_{\text{max}}, y_{\text{max}})\)
Scanline is more efficient for specific polygons – trapezoids (triangles)

\[
\text{scanTrapezoid}(x_L, x_R, y_B, y_T, x'_L, x'_R)
\]
for (y=yB; y<=yT; y++) {
    xl = intersect(Y=y,(xL,x' L));
    xr = intersect(Y=y,(xR,x' R));
    for (x=xl; x<=xl; x++)
        setPixel(x,y);
}
Edge Walking

- Exploit continuous L and R edges

\[
\text{scanTrapezoid}(x_L, x_R, y_B, y_T, \Delta x_L, \Delta x_R)
\]
for (y = yB; y <= yT; y++) {
    for (x = xL; x <= xR; x++)
        setPixel(x, y);
    xL += DxL;
    xR += DxR;
}

Edge Walking
Edge Walking Triangles

- Split triangles into two regions with continuous left and right edges

\[
\text{scanTrapezoid}(x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}) \\
\text{scanTrapezoid}(x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}})
\]
Edge Walking Triangles

- Issues
  - Many small triangles
    - setup cost is non-trivial
  - Clipping triangles produces non-triangles
Define a triangle from implicit edge equations:
Computing Edge Equations

- Computing $A, B, C$ from $(x_1, y_1), (x_2, y_2)$

\[
Ax_1 + By_1 + C = 0 \\
Ax_2 + By_2 + C = 0
\]

- Two equations, three unknowns
- Express $A, B$ in terms of $C$
Computing Edge Equations

\[
\begin{bmatrix}
  x_1 & y_1 \\
  x_2 & y_2 \\
\end{bmatrix}
\begin{bmatrix}
  A \\
  B \\
\end{bmatrix}
= -C
\begin{bmatrix}
  1 \\
  1 \\
\end{bmatrix}
\]

\[
 A = \frac{-C - By_1}{x_1}
\]

\[
 B(x_1y_2 - x_2y_1) = C(x_2 - x_1)
\]

(special case if \( x_1 = 0 \))

- Choose \( C = x_2y_1 - x_1y_2 \) for convenience
- Then \( A = y_2 - y_1 \) and \( B = x_1 - x_2 \)
  - Our original implicit formula
- Note – in literature you can find same equation multiplied by \(-1\)
  - Changes sides
Edge Equations

- Given \( P_0, P_1, P_2 \), what are our three edges?
- Half-spaces defined by the edge equations must share the same sign on the interior of the triangle
  - Consistency (Ex: \([P_0 P_1], [P_1 P_2], [P_2 P_0]\))
- How do we make sure that sign is positive?
  - Test & flip if needed \((A= -A, B= -B, C= -C)\)
Edge Equations: Code

- **Basic structure of code:**
  - Setup: compute edge equations, bounding box
  - (Outer loop) For each scanline in bounding box...
  - (Inner loop) ...check each pixel on scanline:
    - evaluate edge equations
    - draw pixel if all three are positive
Edge Equations: Code

```c
findBoundingBox(&xmin, &xmax, &ymin, &ymax);
setupEdges (&a0,&b0,&c0,&a1,&b1,&c1,&a2,&b2,&c2);

for (int y = yMin; y <= yMax; y++) {
    for (int x = xMin; x <= xMax; x++) {
        float e0 = a0*x + b0*y + c0;
        float e1 = a1*x + b1*y + c1;
        float e2 = a2*x + b2*y + c2;
        if (e0 > 0 && e1 > 0 && e2 > 0)
            Image[x][y] = TriangleColor;
    }
}
```
// more efficient inner loop
for (int y = yMin; y <= yMax; y++) {
    float e0 = a0*xMin + b0*y + c0;
    float e1 = a1*xMin + b1*y + c1;
    float e2 = a2*xMin + b2*y + c2;
    for (int x = xMin; x <= xMax; x++) {
        if (e0 > 0 && e1 > 0 && e2 > 0)
            Image[x][y] = TriangleColor;
            e0 += a0;    e1+= a1;    e2 += a2;
    }
}
Triangle Rasterization Issues

- *Exactly which pixels should be lit?*
  - Pixels inside triangle edges

- *What about pixels exactly on the edge?*
  - Draw - BUT order of triangles matters (it shouldn’t)
  - Don’t draw - BUT gaps possible between triangles

- Need consistent (if arbitrary) rule
  - Example: draw pixels on left or top edge, but not on right or bottom edge
Triangle Rasterization Issues

- Sliver
Triangle Rasterization Issues

- Moving Slivers
Triangle Rasterization Issues

- Shared Edge Ordering
Interpolation – access triangle interior

- Interpolate between vertices:
  - $z$
  - $r, g, b$ - colour components
  - $u, v$ - texture coordinates
  - $N_x, N_y, N_z$ - surface normals

- Equivalent
  - Bilinear interpolation
  - Barycentric coordinates
Barycentric Coordinates

- Area
  \[ A = \frac{1}{2} \left\| \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} \right\| \]

- Barycentric coordinates
  \[ a_1 = A_{P_2P_3P} / A, \quad a_2 = A_{P_3P_1P} / A, \]
  \[ a_3 = A_{P_1P_2P} / A, \]
  \[ P = a_1 P_1 + a_2 P_2 + a_3 P_3 \]
Barycentric Coordinates

- weighted combination of vertices

\[ P = a_1 \cdot P_1 + a_2 \cdot P_2 + a_3 \cdot P_3 \]

\[ a_1 + a_2 + a_3 = 1 \]

\[ 0 \leq a_1, a_2, a_3 \leq 1 \]
Barycentric Coords: Alternative formula

For point $P$ on scanline:

$$P_L = P_2 + \frac{d_1}{d_1 + d_2} (P_3 - P_2)$$

$$= (1 - \frac{d_1}{d_1 + d_2}) P_2 + \frac{d_1}{d_1 + d_2} P_3 =$$

$$= \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3$$
Computing Barycentric Coords

Similarly:

\[ P_R = P_2 + \frac{b_1}{b_1 + b_2} (P_1 - P_2) \]

\[ = (1 - \frac{b_1}{b_1 + b_2})P_2 + \frac{b_1}{b_1 + b_2}P_1 = \]

\[ = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1 \]
Computing Barycentric Coords

- combining

\[ P = \frac{c_2}{c_1 + c_2} \cdot P_L + \frac{c_1}{c_1 + c_2} \cdot P_R \]

\[ P_L = \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \]

\[ P_R = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1 \]

- gives

\[ P = \frac{c_2}{c_1 + c_2} \left( \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3 \right) + \frac{c_1}{c_1 + c_2} \left( \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1 \right) \]
Computing Barycentric Coords

thus

\[ P = a_1 \cdot P_1 + a_2 \cdot P_2 + a_3 \cdot P_3 \]

with

\[
\begin{align*}
  a_1 &= \frac{c_1}{c_1 + c_2} \frac{b_1}{b_1 + b_2} \\
  a_2 &= \frac{c_2}{c_1 + c_2} \frac{d_2}{d_1 + d_2} + \frac{c_1}{c_1 + c_2} \frac{b_2}{b_1 + b_2} \\
  a_3 &= \frac{c_2}{c_1 + c_2} \frac{d_1}{d_1 + d_2}
\end{align*}
\]
Computing Barycentric Coords

- Can verify barycentric properties

\[ a_1 + a_2 + a_3 = 1 \]

\[ 0 \leq a_1, a_2, a_3 \leq 1 \]
Bilinear Interpolation

- Interpolate quantity along $L$ and $R$ edges, as a function of $y$
  - then interpolate quantity as a function of $x$