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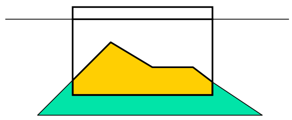
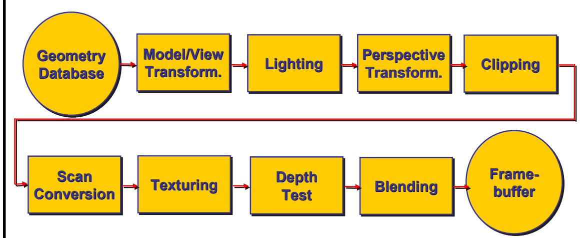
# Chapter 6

## Clipping





# The Rendering Pipeline

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# Line/Polygon Clipping

**Problem:**
Given a set of 2*D* lines/polygons and a window, clip the lines/polygons to their regions that are *inside* the window.

**Objectives**

- Efficiency
- Display in portion of screen (rectangular window)

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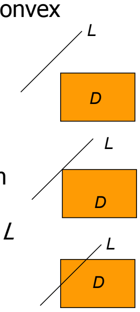
# Analytic Solution

- *Intersection* of convex regions is convex
  - Why?
- *L* & *D* are *convex* - intersection is convex
  - single connected segment of *L*
- **Question:** Can boundary of two convex shapes intersect more than twice?
- Clipping - compute intersection of *L* with four boundary segments of window *D*

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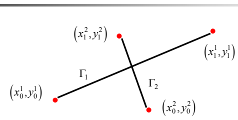
# Line-Line Intersection




$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} \quad t \in [0,1] \qquad G_2 = \begin{cases} x^2(r) = x_0^2 + (x_1^2 - x_0^2)r \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} \quad r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r*,*t*)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2 + (x_1^2 - x_0^2)r$$
$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

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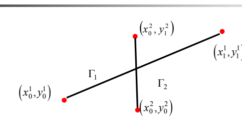
# Intersection with vertical/horizontal lines




$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} t \in [0,1] \quad G_2 = \begin{cases} x^2(r) = x_0^2 \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r*,*t*)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2$$
$$t = \frac{x_0^2 - x_0^1}{x_1^1 - x_0^1}$$
$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

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## Cohen-Sutherland Algorithm

**Purpose:**
Fast treatment of line segments that are trivially inside/outside window.

*P* = (*x*,*y*) - point to be classified against window *D*

| | | |
|---|---|---|
| 0101 | 0100 | 0110 |
| 0001 | 0000 | 0010 |
| 1001 | 1000 | 1010 |

**Idea:** Assign to P a binary code consisting of a bit for each edge of *D*, using lookup table:

| *bit* | 1 | 0 |
|---|---|---|
| 1 | $y < y_{min}$ | $y \geq y_{min}$ |
| 2 | $y > y_{max}$ | $y \leq y_{max}$ |
| 3 | $x > x_{max}$ | $x \leq x_{max}$ |
| 4 | $x < x_{min}$ | $x \geq x_{min}$ |

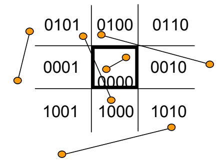


## Cohen-Sutherland Algorithm (cont'd)

Given *L* from $(x_0, y_0)$ to $(x_1, y_1)$
& rectangle D.

**If** bitwise **and** of the codes of $(x_0, y_0)$ and $(x_1, y_1)$ is not zero, or the bitwise **or** is zero,

**then** *L* can be trivially handled (it is either totally outside or totally inside D).

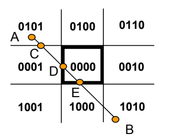



**Why?**



## Cohen-Sutherland Algorithm (cont'd)

$\textbf{C-S-Clip}(P_0 = (x_0, y_0), P_1 = (x_1, y_1), x_{min}, x_{max}, y_{min}, y_{max})$
$C_0 \Leftarrow \text{code}(P_0)$;  $C_1 \Leftarrow \text{code}(P_1)$;
if $((C_0 \text{ and } C_1) \mathrel{!}= 0)$ then return;
if $((C_0 \text{ or } C_1) == 0)$ then draw$(P_0, P_1)$;
else if (OutsideWindow($P_0$)) then
begin
*Edge* $\Leftarrow$ Window boundary of leftmost non-zero bit of $C_0$;
$P_2 \Leftarrow \overline{P_0, P_1} \cap Edge$;
$\textbf{C-S-Clip}(P_2, P_1, x_{min}, x_{max}, y_{min}, y_{max})$;
end
else
*Edge* $\Leftarrow$ Window boundary of leftmost non-zero bit of $C_1$;
$P_2 \Leftarrow \overline{P_0, P_1} \cap Edge$;
$\textbf{C-S-Clip}(P_0, P_2, x_{min}, x_{max}, y_{min}, y_{max})$;
end




AB → CB → DB → DE

| *bit* | 1 | 0 |
|---|---|---|
| 1 | $y < y_{min}$ | $y \geq y_{min}$ |
| 2 | $y > y_{max}$ | $y \leq y_{max}$ |
| 3 | $x > x_{max}$ | $x \leq x_{max}$ |
| 4 | $x < x_{min}$ | $x \geq x_{min}$ |



## Triangle Clipping

- How does intersection of rectangle & triangle looks like?
- How to expand clipping to triangles?
  - Hint: it is convex



## Other Geometric Problems

- Questions: How can these ideas be used to design an algorithm for checking if:
  - a point is inside a (convex) polygon ?
  - a (convex) polygon is inside/intersects/outside a (convex) polygon ?