

# CPSC 314

## Midterm

October 14, 2014

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

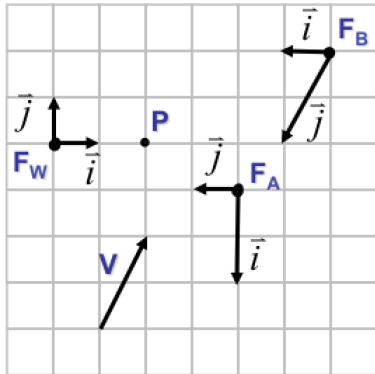
Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

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This exam has 6 questions, for a total of 41 points.

## 1. Coordinate Frames



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_W = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_B$$

(a) (3 points) Express point  $P$  in each of the three coordinate frames.

(b) (3 points) Express point  $V$  in each of the three coordinate frames.

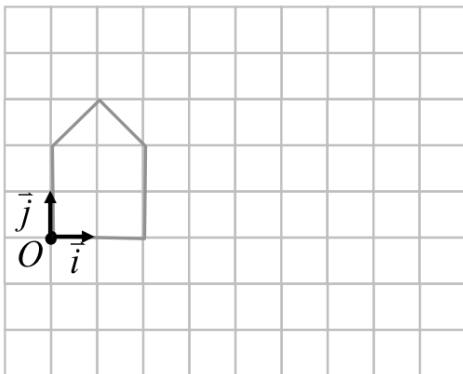
(c) (2 points) Find the  $3 \times 3$  homogeneous transformation matrix which takes a point from  $F_A$  and expresses it in terms of  $F_W$ . I.e., determine  $M$ , where  $P_W = MP_A$ .

(d) (2 points) Find the  $3 \times 3$  homogeneous transformation matrix which takes a point from  $F_B$  and expresses it in terms of  $F_A$ . I.e., determine  $M$ , where  $P_A = MP_B$ .

## 2. (5 points) Graphics Pipeline

Sketch a block diagram of the sequence of transformations in the graphics pipeline. Label the coordinate frames and the relevant transformation matrices. Abbreviations are fine.

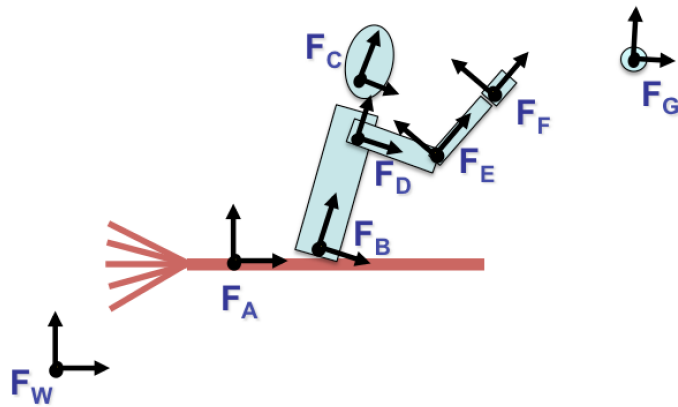
## 3. Transformations Consider the following sequence of transformations.



A: `M.translate(2,0,0);`  
 B: `M.rotate(-90,0,0,1);`  
 C: `M.translate(-1,3,0);`  
 D: `M.scale(2,1,0);`

- (a) (3 points) On the grid given above, sketch the house that results from the given sequence of calls to `drawHouse()`. Label these *A*, *B*, *C*, and *D*. Assume that the house as shown is drawn with the matrix *M* initialized to the identity matrix.
- (b) (1 point) If the individual transformation matrices are labeled  $M_A$ ,  $M_B$ ,  $M_C$ , and  $M_D$ , give an expression, in terms of these matrices, for the final compound transformation matrix,  $M$ , that is produced by the above sequence.

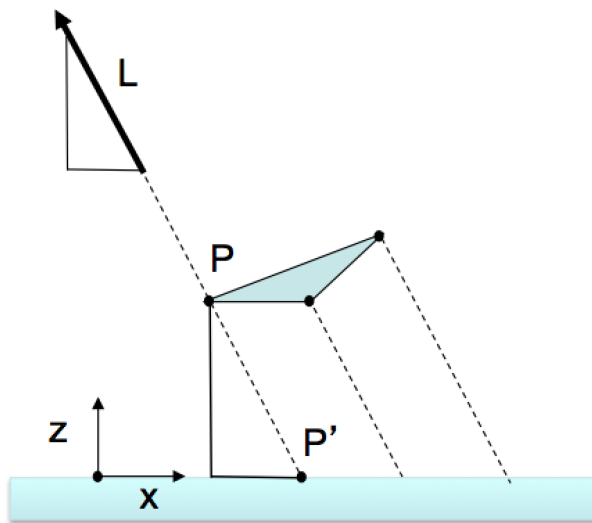
## 4. Scene Graphs



- (a) (3 points) Sketch the scene graph for the *Harry Potter* scene shown above, where he is reaching to catch the elusive *snitch* (ball). Label the transformations involved, using  $M_A$  to designate the matrix that takes points from coordinate frame  $F_A$  to its parent coordinate frame, etc. Include  $F_{VCS}$ ,  $F_{WCS}$ , and  $M_{view}$  in your scene graph. Assume that the *snitch* and the broom are defined with respect to  $F_W$ , and that the torso,  $F_B$ , is defined with respect to the broom,  $F_A$ .
- (b) (2 points) Give an expression for the compound transformation that would indicate where the center of the *snitch* is located in terms of the hand coordinate frame,  $F_F$ .

## 5. Casting Shadows

A simple way to draw the shadow of a polygon is to use a transformation matrix  $M$  that computes the projected shadow location,  $P'(x', y', z')$ , for each polygon vertex  $P(x, y, z)$  according to  $P' = MP$ . A dark-coloured “shadow polygon” can then be drawn using the projected vertices. As illustrated below for the  $xz$ -plane, a directional light source is defined by a vector,  $L(L_x, L_y, L_z)$ , that points in the direction of the light source.



- (a) (5 points) Determine the projected shadow location,  $P'(x', y', z')$ , as a function of the point,  $P(x, y, z)$ , and the lighting direction,  $L(L_x, L_y, L_z)$ . Hint: First use similar triangles to develop an expression for  $x'$ . Similarly, develop expressions for  $y'$  and  $z'$ . Use the space above for your work.
- (b) (3 points) What is the name of the type of projection that is illustrated above? Give the transformation matrix,  $M$ , that would implement the projection, based on the expressions you developed in part (a).

## 6. Short Answer

- (a) (1 point) Express the point  $(6,4,3,2)$ , given in homogenous coordinates, in cartesian coordinates.
- (b) (2 points) Sketch a side view, i.e., in the  $yz$ -plane, of the perspective view frustum that would be specified by `left=-1`, `right=1`, `bot=-1`, `top=1`, `near=4`, `far=10`.
- (c) (1 point) If the near value were doubled for the above example, what would the observed effect be on the resulting image?
- (d) (3 points) What do each of the following transformations matrices do? Give your answer in terms of scales, rotations, and translations. If it also implements a reflection, also indicate that (and show your work).

$$M = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (e) (2 points) Sketch where the VCS point  $P(2, -1, -2)$  would appear in the final display window (draw this as a simple square) when used with the following projection matrix. Show your work.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$