CPSC 314
Midterm 1

February 17, 2017

> Answer the questions in the spaces provided on the question sheets. If you
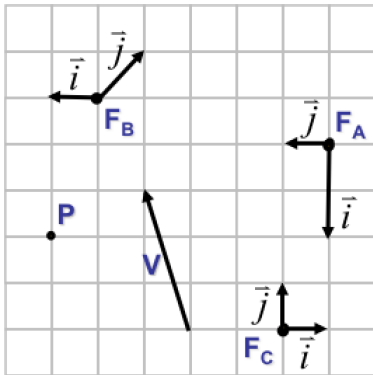> run out of room for an answer, continue on the back of the page.

Name: _____

Student Number: _____

Question 1	/ 10
Question 2	/ 10
Question 3	/ 8
Question 4	/ 11
TOTAL	/ 39

This exam has 4 questions, for a total of 39 points.

1. Coordinate Frames

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_C = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_B$$

(a) (3 points) Express point P in each of the three coordinate frames.

(b) (3 points) Express vector V in each of the three coordinate frames.

(c) (2 points) Find the 3×3 homogeneous transformation matrix which takes a point from F_A and expresses it in terms of F_C. I.e., determine M, where $P_C = M P_A$.

(d) (2 points) Find the 3×3 homogeneous transformation matrix which takes a point from F_B and expresses it in terms of F_A. I.e., determine M, where $P_A = M P_B$.

2. Transformations
   Consider the following sequence of transformations.
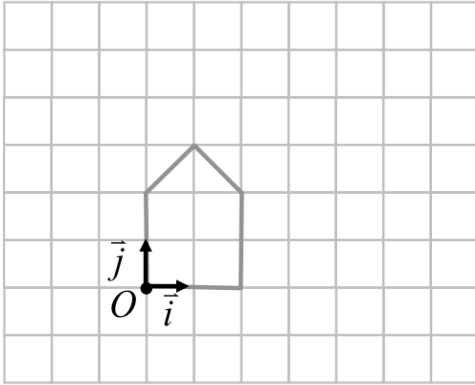
```
A: M.translate(1,3,0);
B: M.rotate(-90,0,0,1);
C: M.translate(2,2,0);
D: M.scale(2,1,0);
```

(a) (4 points) On the grid given above, sketch the house that would be drawn us-
    ing the transformation matrix, M, after each of the given transformations. La-
    bel these A, B, C, and D. Assume that matrix M is initialized to the identity
    matrix, and that each of the `translate()`, `rotate()`, `scale()` transformations
    right-multiplies the matrix M, as usual.

(b) (2 points) Give the 4×4 transformation matrix, M, that is produced by the above
    sequence.

(c) (3 points) What do each of the following transformations matrices do? Give your
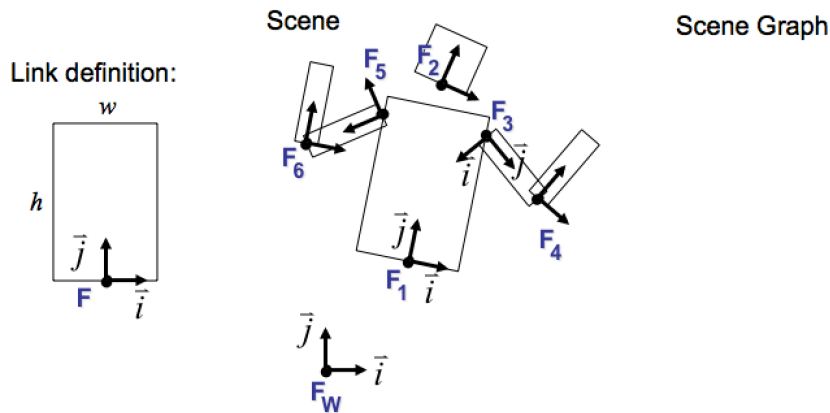   answer in terms of scales, rotations, and translations.

$$M = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) (1 point) Give the inverse of the first of the matrices given above.

3. Scene Graphs

(a) (2 points) In the space above, sketch a scene graph for the simple character model. Place the world frame, F_W, at the root of your scene graph. Label the transformations involved, using M_i to designate the matrix, and use arrows to indicate the direction of the change-of-basis transformation, i.e., an arrow from F_A to F_B indicates that the given matrix transforms a point expressed in coordinate frame F_A to coordinate frame F_B.

(b) (1 point) Give an algebraic expression for the compound transformation that transforms a point from frame F_4 to world coordinates, F_W.

(c) (1 point) Give an algebraic expression for the compound transformation that transforms a point from frame F_2 to world coordinates, F_6.

(d) (2 points) Assume that the body has a width of 3 units and a height 5 units. Give an approximate sequence of translate and rotate commands that would correspond to M_3, i.e., locating F_3 with respect to F_1, for the position and orientation shown in the above diagram.

(e) (2 points) Suppose that each link is drawn by making a call to a function `drawLink(M,w,h)`. This draws a link of width `w` and height `h`, as shown in the left figure, and using matrix `M` as the relevant local-to-world transformation matrix. The function will does this by drawing a translated-and-scaled version of a unit square, as shown in the code below. Complete the pseudocode below.

```
drawLink(M,w,h) {
  Msq = M;                 // make a copy of matrix M
    // add your transformations below;  example transformations:
    // Msq.translate(a,b,c)    This computes  Msq = Msq*Translate(a,b,c)
    // Msq.rotateZ(theta)      This computes  Msq = Msq*RotateZ(theta)
    // Msq.scale(a,b,c)        This computes  Msq = Msq*Scale(a,b,c)




  drawSquare(Msq);    //  draws a unit square; corners at (0,0) and (1,1)
}
```

4. Projection Transformations

   (a) (2 points) Sketch a two-point perspective projection of a cube.

   (b) (3 points) Sketch a side view, i.e., yz-plane, of a perspective view volume defined by `near=1, far=6, top=0.5, bot=-0.5, left=-1, right=1`.

   (c) (1 point) Express the point (8,-4,3,2), given in homogenous coordinates, in cartesian coordinates.

(d) (2 points) Sketch where the VCS point $P(-1, 1, -2)$ would appear in the final display window when used with the following projection matrix. Draw your final display window as a simple square. Show your work.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

(e) (1 point) The viewing transformation is often specified using an eye point, a reference "look-at" point, and an up vector. When does this not produce a well-defined result?

(f) (1 point) Which points are problematic to compute for a perspective projection?

(g) (1 point) If we triple the distance of an object from the camera what happens to the image size of an object for (a) a perspective projection; and (b) an orthographics projection?