CPSC 314
Midterm 1

February 14, 2018

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: Solutions

Student Number: ______

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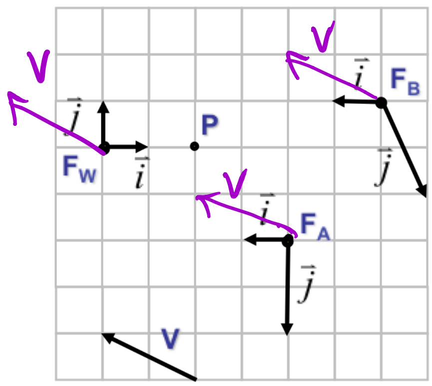
This midterm has 4 questions, for a total of 41 points.

1. Coordinate Frames



$$\begin{array}{cccc} & i_A & j_A & O_A \\ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_W = & \multicolumn{3}{c}{\begin{bmatrix} -1 & 0 & 4 \\ 0 & -2 & -2 \\ 0 & 0 & 1 \end{bmatrix}} \end{array} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_B = \begin{bmatrix} 1 & 1 & 3.5 \\ 0 & 1 & 1.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

(a) (3 points) Express point P in each of the three coordinate frames.

$P_W(2,0)$ $P_A(2,-1)$ $P_B(4.5,0.5)$

(b) (3 points) Express vector V in each of the three coordinate frames.

$V_W(-2,1)$ $V_A(2,-0.5)$ $V_B(1.5,-0.5)$

(c) (2 points) Find the 3×3 affine transformation matrix which takes a point from F_A and expresses it in terms of F_W. I.e., determine M, where $P_W = MP_A$. Write your answer in the space to the right of the diagram above.

(d) (2 points) Find the 3×3 affine transformation matrix which takes a point from F_A and expresses it in terms of F_B. I.e., determine M, where $P_B = MP_A$. Write your answer in the space to the right of the diagram above.

(e) (2 points) Develop a sequence of rotations, translates, and scales to construct the same 3×3 affine transformation as in part (c), i.e., $P_W = MP_A$. Express your solution as an algebraic composition of transformation matrices, in whatever order your prefer, e.g., $P_W = \text{Trans}(-1, 2, 0)\text{Rot}(z, 45°)\text{Scale}(2, 3, 1)P_A$.

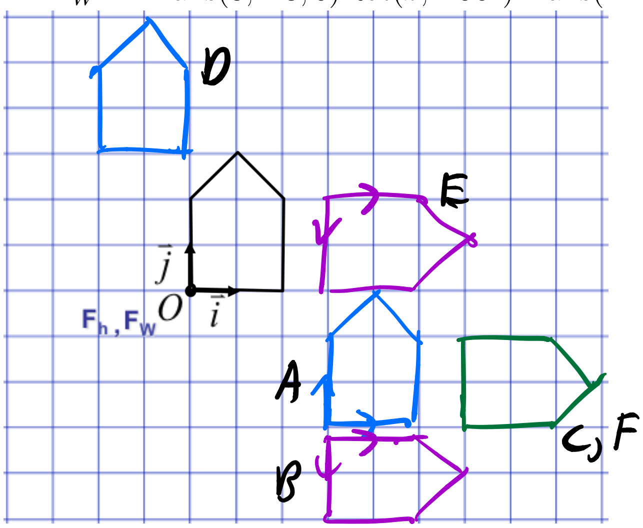
$P_W = \text{Trans}(4,-2,0)\,\text{Scale}(-1,-2,1)\,P_A$

or $P_W = \text{Trans}(4,-2,0)\,\text{Rot}(z,180°)\,\text{Scale}(1,2,1)$

or $P_W = \text{Rot}(z,180°)\,\text{Trans}(-4,2,0)\,\text{Scale}(1,2,1)$

2. Transformations
Consider the house which is shown below in its untransformed state, e.g., $F_{house} = F_W$. Suppose that a transformed house is generated as given by the following algebraic composition of transformations matrices:

$$P_W = \text{Trans}(3, -3, 0)\text{Rot}(z, -90°)\text{Trans}(-2, 3, 0)P_{house}.$$



(a) (2 points) Interpret the transformations in a left-to-right order, and draw the intermediate and final transformed houses. Label these in the order A,B,C.

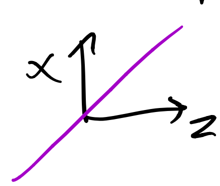
(b) (2 points) Interpret the transformations in a right-to-left order, and draw the intermediate and final transformed houses. Label these in the order D,E,F.

(c) (2 points) Give an algebraic expression for the inverse transformation, i.e., a transformation that would take a point from F_W to F_{house}.

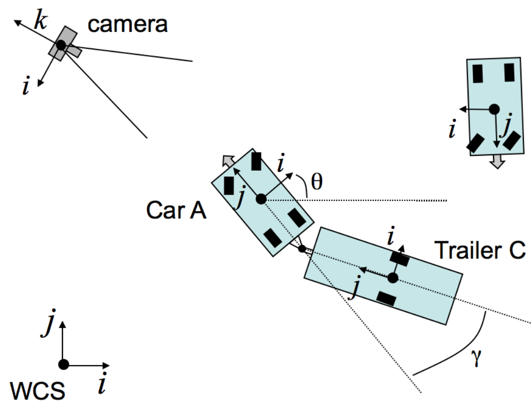
$$P_{house} = \text{Trans}(2,3,0)\,\text{Rot}(z, 90°)\,\text{Trans}(-3,3,0)\,P_W$$

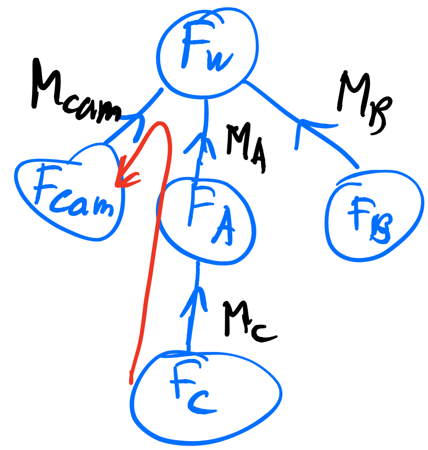
(d) (4 points) Give 4×4 transformation matrices that perform the following:

Trans(3, 4, 5)	Scale(7, 8, 9)	Rot(y, 180°)	Reflection about the plane $x = z$
$\begin{bmatrix} 1 & & & 3 \\ & 1 & & 4 \\ & & 1 & 5 \\ & & & 1 \end{bmatrix}$	$\begin{bmatrix} 7 & & & \\ & 8 & & \\ & & 9 & \\ & & & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



3. Scene Graphs





(a) (2 points) In the space above to the right, sketch a scene graph for the given scene. Place the world frame, F_W, at the root of your scene graph, and treat the camera as any other object. Label the frames as F_{WCS},F_A, F_B, F_C, and F_{cam}. Do not add coordinate frames for the various wheels. Add labels to the edges, using M_i to designate the transformation matrix that takes a point in frame i to its parent frame. Use arrows to indicate the direction of the change-of-basis transformation.

(b) (1 point) Give an algebraic expression in terms of the matrices M_i for the compound transformation that transforms a point from frame F_C to camera coordinates, F_{cam}.

$$P_{cam} = M_{cam}^{-1} M_A M_C P_C$$

(c) (1 point) Give the sequence of geometric transformations (as translates and rotates) that you would use to build matrix M_C, i.e., that connects the Trailer to its parent coordinate frame. Use the given i and j basis vectors to estimate approximate numbers for the required translations. Assume that $z = 0$ for all objects.

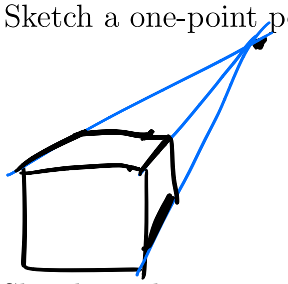
$$P_A = Trans(0,-2,0)\ Rot(z,\gamma)\ Trans(0,-2.5,0)\ P_C$$

(d) (1 point) How many numbers would a keyframe need in order to specify the positions and orientations of both cars and the trailer? Assume that they only move in the xy-plane.

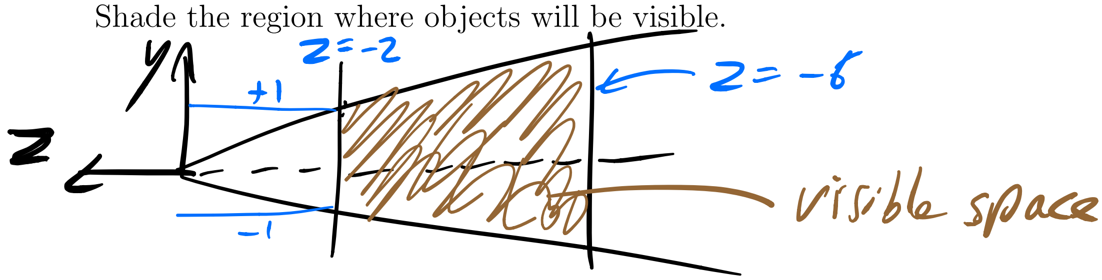
$x_A, y_A, \theta_A, \gamma, x_B, y_B, \theta_B$ = 7 numbers (degrees of freedom)

4. Projection Transformations

(a) (2 points) Sketch a one-point perspective projection of a cube.



(b) (3 points) Sketch a side view, i.e., yz-plane, of a perspective view volume defined by `near=2, far=6, top=1, bot=-1, left=-1, right=1`. Label the key dimensions. Shade the region where objects will be visible.



(c) (1 point) If we decrease the field of view for a perspective view frustum, what does that do to an object at the center of an image?

magnified, i.e., zoom in

(d) (1 point) Express the point $(x, y, z, h) = (9, -3, 3, 6)$ in cartesian coordinates.

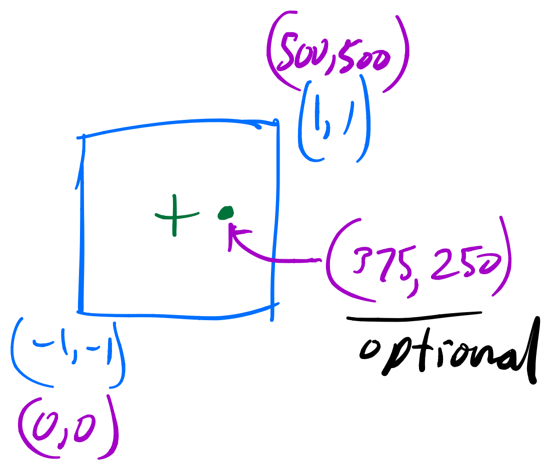
$(x/h, y/h, z/h) = (3/2, -1/2, 1/2)$

(e) (2 points) Sketch where the VCS point $P(2, 0, -4)$ would appear in a final image having dimensions of 500×500 pixel, when used with the following projection matrix. Draw your final display window as a simple square. Show your work.

$$\begin{bmatrix} 2 \\ 0 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

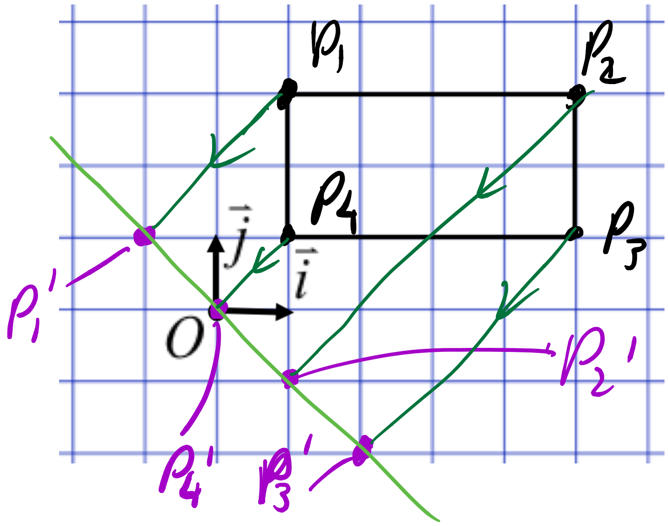
$/h$

$$\begin{bmatrix} 0.5 \\ 0 \\ 1 \end{bmatrix} \text{NDCS}$$



(f) (3 points) The following 3×3 affine matrix is applied to points in the 2D plane. What does this matrix do? Illustrate your answer on the diagram below.

$$\begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$P_1(1,3) \to P_1'(-1,1)$
$P_2(5,3) \to P_2'(1,-1)$
$P_3(5,1) \to P_3'(2,-2)$
$P_4(1,1) \to P_4'(0,0)$

orthographic projection onto the line $y=-x$

(g) (2 points) Suppose that a vertex shader stores separate 4×4 matrices for $M_{\text{modelview}}$ and M_{proj}. A given point, P_{obj}, is transformed by the vertex shader. Give the algebraic expression for the computation of the vertex shader. How many multiplications and additions does the vertex shader perform in order to transform one point? Assume that the matrices are arbitrary 4×4 matrices, i.e., that there are no special optimizations being done.

$$P_{ccs} = M_{proj} \, M_{modelview} \, P_{obj}$$

$$\begin{bmatrix} \square \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \quad & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

one element: $4\times$, $3+$

four elements: $\underbrace{16\times, 12+}_{\text{one matrix-vector multiply}}$

two matrix-vector mult:

if assume $h=1$:
$28\times, 24+$

optional

$\boxed{32\times, 24+}$