Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: 

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This exam has 7 questions, for a total of 59 points.
1. Coordinate Frames

(a) (3 points) Express point \( P \) in each of the three coordinate frames.

(b) (3 points) Express vector \( V \) in each of the three coordinate frames.

(c) (2 points) Find the \( 3 \times 3 \) transformation matrix which takes a point from \( F_A \) and expresses it in terms of \( F_W \). I.e., determine \( M \), where \( P_w = M P_A \). Write your answer in the appropriate space to the right of the diagram.

(d) (2 points) Find the \( 3 \times 3 \) transformation matrix which takes a point from \( F_B \) and expresses it in terms of \( F_A \). I.e., determine \( M \), where \( P_A = M P_B \). Write your answer in the appropriate space to the right of the diagram.

(e) (2 points) A transformation matrix is generated using the code fragment shown below. Suppose that the house below, which is drawn below in its untransformed state, is transformed and drawn using matrix \( M \) after each of the (cumulative) steps below. Sketch and label the four versions of the house.

\[
\begin{bmatrix}
 x \\
 y \\
 1
\end{bmatrix}_W =
\begin{bmatrix}
 x \\
 y \\
 1
\end{bmatrix}_A
\]

\[
\begin{bmatrix}
 x \\
 y \\
 1
\end{bmatrix}_A =
\begin{bmatrix}
 x \\
 y \\
 1
\end{bmatrix}_B
\]

\[
\text{M=identity();}
\text{A: M=M*Translate(-3,3,0);}
\text{B: M=M*Rotate(-90,0,0,1);}
\text{C: M=M*Translate(2,0,0);}
\text{D: M=Rotate(180,0,0,1)*M;}
\]
2. Local Illumination

(a) (6 points) Sketch the ambient, diffuse, and specular components for the scene below, as would be computed by the Phong illumination model, i.e., \( I = I_a k_a + I_L k_d (N \cdot L) + I_L k_s (R \cdot V)^n \). Assume that the eye is directly above the scene at a large distance, and use the values \( I_a = I_L = 1, k_a = 0.2, k_d = 0.5, k_s = 1, n = 200 \).

![Diagram of light and I(x)](image)

(b) (2 points) Which terms in the Phong illumination model need to have their values clamped, i.e., bounded in value, and why?

(c) (3 points) A spot light casts light into the scene as defined by a position \( P_L \), a direction, \( D \), and an angle, \( \alpha \). Only points that lie within angle \( \alpha \) of the direction, \( D \), are illuminated. Give the pseudocode to implement a spotlight shader for a point \( P \). For points lit by the spotlight, you can abstract the Phong illumination computation as \( I = \text{phong}(N, L, V) \), i.e., focus on the aspects specific to the spotlight.

![Diagram of spotlight and phong function](image)
3. Short answer

(a) (1 point) Convert the point \( P(x, y, z, h) = (1, 2, 2, 2) \) from homogeneous coordinates to cartesian coordinates:

(b) (1 point) What are the three parameters that are commonly used to define the viewing transformation, i.e., to specify the position and orientation of the camera in the world?

(c) (1 point) Give a parametric equation for a line passing through points \( P_1 \) and \( P_2 \).

(d) (1 point) The point \( P_{VCS}(4, 4, -6) \) projects to a point \( P' \) on an image plane placed at \( z_{VCS} = -2 \). Give the \( x, y, z \) coordinates of \( P' \).

(e) (1 point) Fill in the blank: Realistic rendering of materials such as milk and skin requires modeling \underline{scattering}.

(f) (1 point) Fill in the blank: When computing the illumination at a given surface point during ray tracing, we cast a \underline{ray} towards the light source to see if it is visible for that point.

(g) (1 point) Circle one: In a fragment shader, texture coordinates would typically be the following kind of shader variable: \underline{attribute uniform varying}.

(h) (1 point) True or False: Decreasing (narrowing) the field of view will result in a zooming effect, i.e., the objects at the center of the image will appear larger.

(i) (1 point) True or False: If a scene consists of closed 3D objects, then back-face culling can save us from having to render approximately half of the polygons that are in the view volume.

(j) (1 point) True or False: Given three points in 3D-space that define a triangle, there is a unique implicit equation of the form \( Ax+By+Cz + D = 0 \) that embeds the triangle. I.e., for any given plane, there is only one possible set of values for \( A, B, C, D \).

(k) (1 point) True or False: Ray tracing works by tracing light paths forward from the light into the scene until they hit the image plane.

(l) (1 point) True or False: Multiple bounces of light between the ceiling, walls, and floor of a room can be correctly modeled by path tracing but not with projective rendering or ray-tracing.

(m) (1 point) True or False: If \( f(P) \) defines a perspective projection transformation, and \( P = \alpha P_0 + \beta P_1 + \gamma P_2 \), then \( f(P) = \alpha f(P_0) + \beta f(P_1) + \gamma f(P_2) \), i.e., the projection of a point on a triangle, as modeled using barycentric coordinates, is the same as the barycentric combination of the projected vertices.
4. (4 points) Texture Mapping

The texture map, shown below on the left, is used to render the rectangle and triangle shown on the right, with the given \((u,v)\) texture coordinates assigned to their vertices. Sketch the resulting appearance. Assume that a REPEAT texture mode is being used in both dimensions.

5. (4 points) Scan Conversion

Give the pseudocode needed to scan-convert a circle centred at location \((a,b)\) and having radius \(r\). Use \texttt{SetPixel}(x,y) to set a pixel.
6. (6 points) Graphics Pipeline

Various computations in the graphics pipeline are done in the vertex shader (VS), fragment shader (FS), or as part of the default fixed function (FF) computations. Circle the most applicable location for each of the following computations:

<table>
<thead>
<tr>
<th>VS</th>
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VS FS FF multiplication by the projection matrix
VS FS FF z-buffer test
VS FS FF back face culling
VS FS FF Phong illumination
VS FS FF lookup of a texturemap colour using (u,v) coordinates
VS FS FF transformation of normals

7. Miscellaneous

(a) (2 points) Explain how to compute the intersection of the 3D ray \( C + tV \) with a sphere of radius 1 located at the origin.

(b) (2 points) List two things you learned about computer graphics and animation from Robert Bridson’s guest lecture.

(c) (2 points) Ray tracing makes recursive calls to trace the contributions of reflective and refractive light paths in a scene. What are the typical termination conditions for this recursion, i.e., when do we stop tracing new rays?
(d) (3 points) A simple way to draw the shadow of a polygon is to use a transformation matrix $M$ that computes the projected shadow location, $P'(x', y', z')$, for each polygon vertex $P(x, y, z)$, according to $P' = MP$. A dark-coloured “shadow polygon” can then be drawn using the projected vertices. This is illustrated below, for a given lighting direction, $L$, and a ground plane, $y = 0$.

First, use similar triangles to develop an equation for $x'$ as a function of $P(x, y, z)$ and $L(L_x, L_y, L_z)$. Then develop a similar expression for $z'$. Last, develop the matrix $M$ that implements the desired projection.