CPSC 314
Assignment 4 (7%)

Due Friday March 9, 2018

Answer the questions in the spaces provided on the question sheets. If you run out of space for an answer, use separate pages and staple them to your assignment.

Name: Solutions

Student Number: ______

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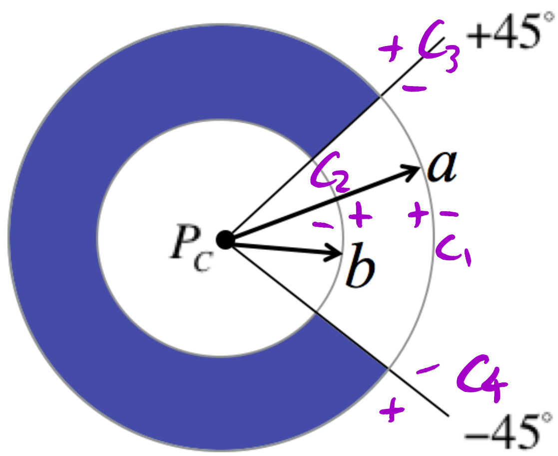
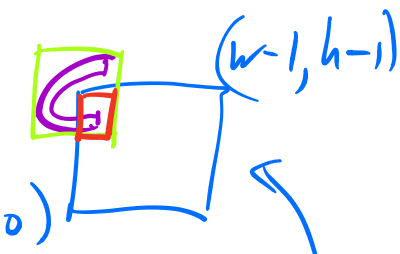
TOTAL	/ 28

1. (6 points) Scan Conversion with implicit functions
   Write pseudocode for scan converting the letter 'C' as defined below. The 'C' is centred at location $P_C(x_C, y_C)$, and has an outer and inner radius of a and b, respectively. You can assume that $b < a$. Use implicit equations to develop your solution. Do not use any trigonometric functions. Assume that a pixel is set on using `Image[x][y]=1`, and that the image spans from `(0,0)` in the bottom left to `(w-1,h-1)` in the top right, giving an image of `w` $\times$ `h` pixels. If part of the letter 'C' is located off-screen, the remainder should still be properly rendered.





$x_{min} = \max(0, x_c - a)$
$y_{min} = \max(0, y_c - a)$
$x_{max} = \min(w-1, x_c + a)$
$y_{max} = \min(h-1, y_c + a)$

Compute bounding box, and clip this to the display window

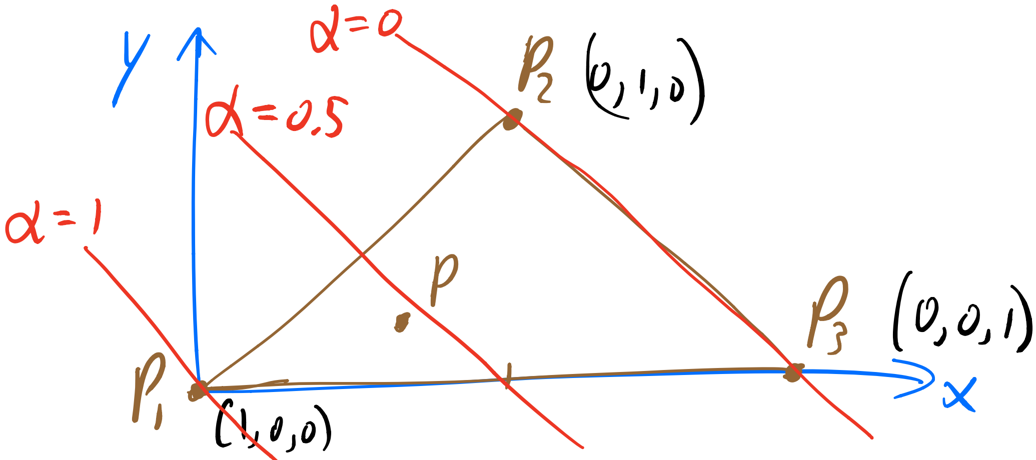
```
for(y = ymin; y ≤ ymax; y++){
    for(x = xmin; x ≤ xmax; x++){
        C1 = a^2 − (x−xc)^2 − (y−yc)^2
        C2 = (x−xc)^2 + (y−yc)^2 − b^2
        C3 = (y−yc) − (x−xc)
        C4 = (x−xc) + (y−yc)
        if C1>0 and C2>0 and (C3>0 or C4>0)
            then Image[x][y] = 1
    }
}
```

could also use ≥0

2. Barycentric coordinates and interpolation

A triangle has device coordinates $P_1(0,0)$, $P_2(50,50)$, $P_3(100,0)$. We wish to be able to interpolate a value v for an arbitrary point $P(x,y)$ in the triangle, given the values at the vertices.

(a) (1 point) Sketch the triangle and the point $P(30,10)$.



(b) (3 points) For each of the edges: (i) Write an explicit line equation for each of the edges, i.e., $y = f(x)$. (ii) Rearrange the terms of each of these equations to trivially transform these into implicit line equations, i.e., $f(x,y) = 0$; (iii) Lastly, give a scaled implicit line equation so that the implicit line equation evaluates to 1 at the third vertex, i.e., the vertex not on the line segment.

P_1P_2: $y = x$ $\Rightarrow$ $0 = x - y$ $\quad F_{12}(x,y) = x - y$

P_1P_3: $y = 0$ $\quad 0 = y$ $\quad F_{13}(x,y) = y$

P_2P_3: $y = 100 - x$ $\quad 0 = 100 - x - y$ $\quad F_{23}(x,y) = 100 - x - y$

$F_{12}(P_3) = x_3 - y_3 = 100$

$F_{13}(P_2) = y_2 = 50$

$F_{23}(P_1) = 100 - x_1 - y_1 = 100$

$F'_{12}(x,y) = F_{12}(x,y)/F(P_3) = \frac{x}{100} - \frac{y}{100}$

$F'_{13}(x,y) = F_{13}(x,y)/F(P_2) = y/50$

$F'_{23}(x,y) = F_{23}(x,y)/F(P_1) = 1 - \frac{x}{100} - \frac{y}{100}$

(c) (2 points) The barycentric coordinates of a point $P(x, y)$ in a triangle are given by $P = \alpha P_1 + \beta P_2 + \gamma P_3$, where $\alpha + \beta + \gamma = 1$.
(i) On your diagram above, label each of the vertices with their (α, β, γ) values.
(ii) Sketch and label lines corresponding to $\alpha = 0, \alpha = 0.5, \alpha = 1$ in the above diagram.

(d) (2 points) Using your work above, give expressions for computing α, β, γ for a point $P(x, y)$ in the triangle. Verify that $\alpha + \beta + \gamma = 1$ holds true for your expressions.

$$\alpha = F'_{23}(x,y) = 1 - \frac{x}{100} - \frac{y}{100}$$
$$\beta = F'_{13}(x,y) = + \frac{y}{50}$$
$$\gamma = F'_{12}(x,y) = \frac{x}{100} - \frac{y}{100}$$

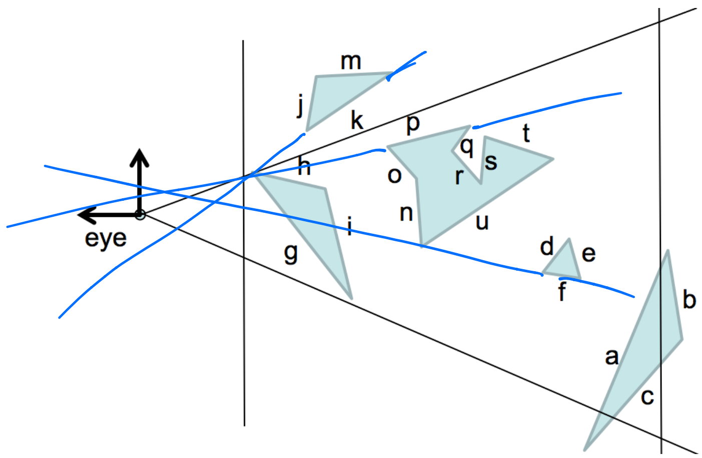
$\alpha + \beta + \gamma = 0$ ✓

(e) (1 point) Compute barycentric coordinates for $P(30, 10)$ and use them to compute v for that point, given the following known values for v at the vertices: $v_1 = 10, v_2 = 20, v_3 = 60$.

$$\alpha = 1 - 0.3 - 0.1 = 0.6$$
$$\beta = 0.2 = 0.2$$
$$\gamma = 0.3 - 0.1 = 0.2$$

$$V = \alpha V_1 + \beta V_2 + \gamma V_3$$
$$= (0.6)(10) + (0.2)(20) + (0.2)(60)$$
$$= 6 + 4 + 12$$
$$\boxed{= 22}$$

3. Visibility and Culling
Consider the scene below, shown as a side-view of VCS. Assume that all the objects shown are solid, and that the labelled lines represent polygonal faces of the objects.



(a) (2 points) List, in alphabetical order, the polygons that would be culled by view frustum culling. Must be fully outside w.r.t. one of the planes

b, j, k, m

(b) (2 points) List, in alphabetical order, the polygons that would be culled by back-face culling. Note: consider both types of culling independently of each other.
Eye should be "above" plane of polygon, i.e, on visible side

b, c, e, h, i, k, m, p, q, r, t, u   note: p, f are a close call

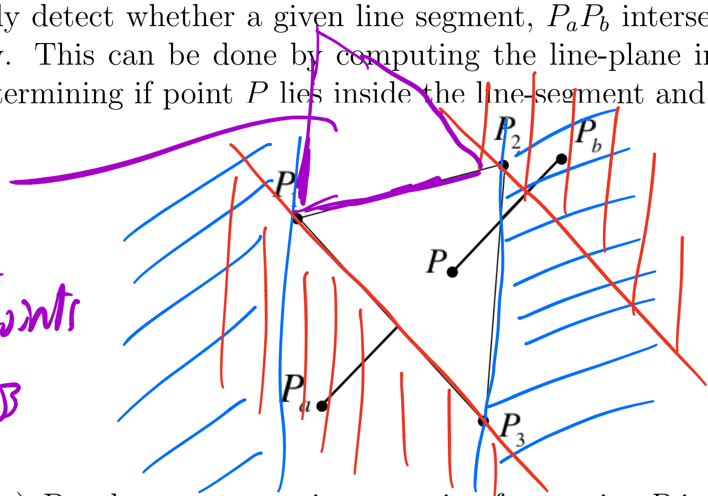
(c) (2 points) After view-frustum culling and back-face culling, list in alphabetical order the remaining faces that would be completely removed by z-buffer tests.

a (possibly, depending on g)
d, f, n, o, s

In the end, only g is visible
(and possibly part of a)

4. In a number of 3D graphics and 3D simulation algorithms, it is useful to be able to quickly and efficiently detect whether a given line segment, P_aP_b intersects a given triangle, as shown below. This can be done by computing the line-plane intersection at point P, and then determining if point P lies inside the line-segment and inside the triangle.

need to eliminate these points using: $0 \le 1-\alpha-\beta$



$0 \le \alpha \le 1$
$0 \le \beta \le 1$

diagram for 4(d)

(a) (2 points) Develop a parametric expression for a point P in the triangle using only two of the three barycentric coordinates, i.e., P(α,β), where α and β are the usual barycentric coordinates, i.e., $P = \alpha P_1 + \beta P_2 + \gamma P_3$.

$$P(\alpha,\beta) = \alpha P_1 + \beta P_2 + (1-\alpha-\beta)P_3$$
$$= P_3 + \alpha(P_1-P_3) + \beta(P_2-P_3) \quad \text{(optional)}$$

(b) (1 point) Give a parametric expression for a point $P(t)$ on the line segment.

$$P(t) = P_a + t(P_b - P_a) \quad \text{(not unique!)}$$

(c) (2 points) Give a set of equations that can be solved for the three unknown parameters, α, β, t that uniquely define the intersection point, P.

$$P(\alpha,\beta) = P(t)$$
$$P_3 + \alpha(P_1-P_3) + \beta(P_2-P_3) = P_a + t(P_b-P_a)$$
$$\alpha(P_1-P_3) + \beta(P_2-P_3) - t(P_b-P_a) = P_a - P_3$$

$$\begin{bmatrix} (P_1-P_3) & (P_2-P_3) & (P_b-P_a) \end{bmatrix}\begin{bmatrix}\alpha \\ \beta \\ t\end{bmatrix} = \begin{bmatrix} (P_a-P_3) \end{bmatrix}$$

optional compact form

(d) (2 points) Given values for α, β, t, show:
(i) how to compute the coordinates of P; (ii) how to test whether the line segment does or does-not intersect the triangle.

(i) back substitute into either line eqn or plane eqn
e.g. $P = P_a + t(P_b - P_a)$

(ii) verify that all the following hold true:
$0 \le t \le 1$
$0 \le \alpha \le 1$
$0 \le \beta \le 1$
$0 \le 1-\alpha-\beta$ ← also needed (see diagram)